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# **CFRP-PC Training Course**

## **Design Example**

### **FSB 12x57**

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*For Training Purposes Only, NOT To Be Used For Construction*

UNIVERSITY of **HOUSTON**  
CULLEN COLLEGE of ENGINEERING  
Department of Civil & Environmental Engineering

# DESIGN OF DECKED FSB-12x57 PRETENTIONED (INTERIOR) BEAM WITH STRAIGHT CFRP CABLES

Geomertry from "Rehabilitation of Bridges over Cow Key Channel"

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## CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FSB 12x57

### **1. REFERENCES**

LRFD: AASHTO LRFD Bridge Design Specifications, 8th Edition, 2017

AASHTO CFRP: Guide Specifications for the Design of Concrete Bridge Beams Prestressed with Carbon Fiber-Reinforced Polymer (CFRP) Systems, 1st Edition, 2018

AASHTO GFRP: AASHTO LRFD Bridge Design Guide Specifications for GFRP-Reinforced Concrete, 2nd Edition, 2018

SDG: FDOT Structures Manual, Volume 1 -Structures Design Guidelines

FRPG: FDOT Structures Manual, Volume 4 -Fiber Reinforced Polymer Guidelines

FDOT Standard Specifications For Road and Bridge Construction, January 2020

ACI 440.1R: Guide for the Design and Construction of Structural Concrete Reinforced with FRP Bars, 2015

ACI 440.4R: Prestressing Concrete Structures with FRP Tendons, 2011

NCHRP Report 907: Design of Concrete Bridge Beams Prestressed with CFRP systems

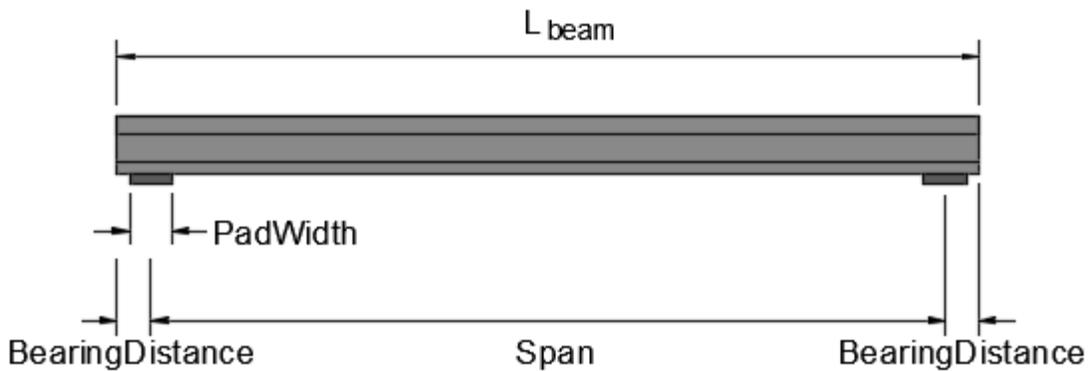
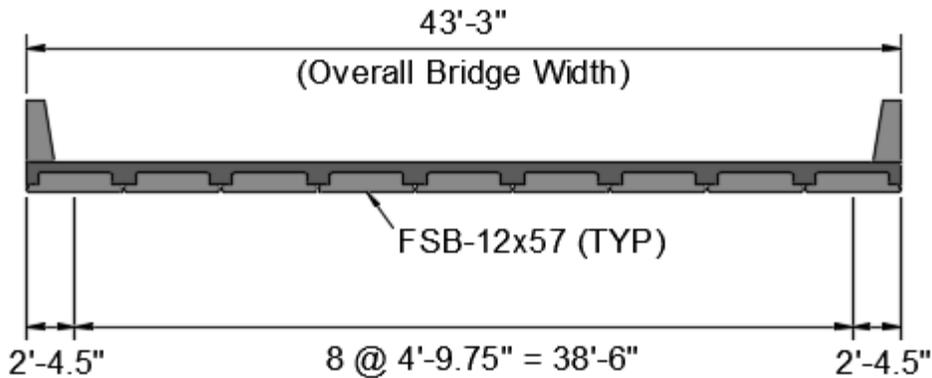
FDOT Design Example: LRFD Design Example #1, Prestressed Precast Concrete Beam Bridge Design

FDOT Design Example: LRFDPBeamV5.2 (MathCAD)

## 2. GEOMETRY AND MATERIAL INPUTS

### 2.1 Bridge Layout and Dimensions

$$k_{cf} = \frac{\text{kip}}{\text{ft}^3}$$



(FDOT Design example  
LRFDPBeamV5.2-CFRP)

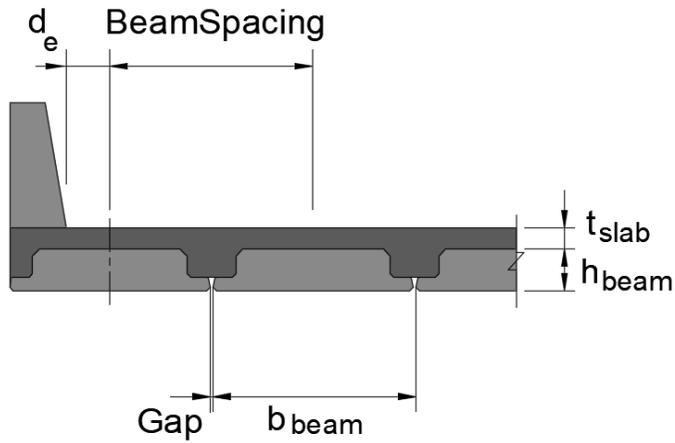
$$\text{Span} := 467 \text{ in} = 38.917 \cdot \text{ft}$$

Clear span

$$\text{BearingDistance} := 6.5 \text{ in}$$

$$L_{\text{beam}} := \text{Span} + 2 \cdot \text{BearingDistance} = 40 \cdot \text{ft}$$

$$\text{PadWidth} := 8 \text{ in}$$



$$\text{Gap} := 0.75 \text{ in}$$

$$\text{BeamSpacing} := 4 \text{ ft} + 9.75 \text{ in} = 57.75 \cdot \text{in}$$

$$\text{NumberOfBeam} := 9$$

$$\text{TotalBridgeWidth} := \text{BeamSpacing} \cdot \text{NumberOfBeam} - \text{Gap} = 43.25 \cdot \text{ft}$$

$$d_e := 9.5 \text{ in} = 0.792 \text{ ft}$$

**2.2 FSB 12-57 Beam**

$A_{\text{beam}} := 581.76\text{in}^2$	Cross section area
$\text{Perimeter}_{\text{beam}} := 132.33\text{in}$	Perimeter of beam section
$I_{xx.\text{beam}} := 7084\text{in}^4$	Moment of inertia
$I_{yy.\text{beam}} := 118622\text{in}^4$	
$y_{\text{beam.top}} := 6.35\text{in}$	Distance from top of beam to N.A.
$y_{\text{beam.bot}} := 5.65\text{in}$	Distance from bottom of beam to N.A.
$h_{\text{beam}} := 12\text{in}$	Depth, thickness of beam
$h_{\text{beam.top}} := 8\text{in}$	Depth of upper part of beam
$h_{\text{beam.bot}} := 4\text{in}$	Depth of lower part of beam
$b_{\text{beam}} := 57\text{in}$	Width of the beam
$b_{\text{beam.bot}} := b_{\text{beam}} = 57\cdot\text{in}$	Width of lower part of beam
$b_{\text{beam.top}} := (b_{\text{beam}} - 6\text{in}\cdot 2) = 45\cdot\text{in}$	Width of upper part of beam
$A_{\text{beam.top}} := h_{\text{beam.top}}\cdot b_{\text{beam.top}} = 360\cdot\text{in}^2$	Area of upper part of beam
$A_{\text{beam.bot}} := h_{\text{beam.bot}}\cdot b_{\text{beam.bot}} = 228\cdot\text{in}^2$	Area of lower part of beam

$$f_{c.beam} := 8.5 \text{ksi}$$

Concrete strength

$$f_{ci.beam} := 6 \text{ksi}$$

Concrete strength, at release

$$f_{r.beam} := 0.24 \cdot \left( \frac{f_{c.beam}}{\text{ksi}} \right)^{0.5} \text{ksi} = 0.7 \cdot \text{ksi}$$

Modulus of rupture  
(SDG 1.4.1B)

$$\gamma_{concrete} := 0.15 \text{kcf}$$

Unit weight of reinforced concrete

$$\gamma_{beam} := \gamma_{concrete} = 0.15 \cdot \text{kcf}$$

Unit weight of reinforced deck concrete

$$K_1 := 1.0$$

Correction factor for aggregate  
(SDG 1.4.1A)

$$w_c := 0.145 \text{kcf}$$

Unit weight of concrete

$$E_{c.beam} := 120000 \cdot K_1 \cdot \left( \frac{w_c}{\text{kcf}} \right)^2 \cdot \left( \frac{f_{c.beam}}{\text{ksi}} \right)^{0.33} \cdot \text{ksi} = 5112 \cdot \text{ksi}$$

Elastic modulus of beam concrete  
(LRFD 5.4.2.4-1)

$$E_{ci.beam} := 120000 \cdot K_1 \cdot \left( \frac{w_c}{\text{kcf}} \right)^2 \cdot \left( \frac{f_{ci.beam}}{\text{ksi}} \right)^{0.33} \cdot \text{ksi} = 4557 \cdot \text{ksi}$$

Elastic modulus of beam concrete, at  
release  
(LRFD 5.4.2.4-1)

### 2.3 Cast in Place Deck

$$t_{\text{slab}} := 6 \text{ in}$$

Thickness of slab

$$t_{\text{slab.top}} := t_{\text{slab}} = 6 \cdot \text{in}$$

Thickness of slab, top

$$b_{\text{slab.top}} := \text{BeamSpacing} + \text{Gap} = 58.5 \cdot \text{in}$$

Slab width

$$t_{\text{slab.bot}} := h_{\text{beam.top}} = 8 \cdot \text{in}$$

Thickness of slab, bottom part

$$b_{\text{slab.bot}} := b_{\text{beam.bot}} - b_{\text{beam.top}} + \text{Gap} = 12.75 \cdot \text{in}$$

Slab width, bottom part

$$A_{\text{slab.top}} := b_{\text{slab.top}} \cdot t_{\text{slab.top}} = 351 \cdot \text{in}^2$$

Area of slab, top part

$$A_{\text{slab.bot}} := b_{\text{slab.bot}} \cdot t_{\text{slab.bot}} = 102 \cdot \text{in}^2$$

Area of slab, bottom part

$$A_{\text{slab}} := A_{\text{slab.top}} + A_{\text{slab.bot}} = 453 \cdot \text{in}^2$$

Area of slab, total

$$f_{\text{c.slub}} := 5.5 \text{ ksi}$$

Concrete strength

$$\gamma_{\text{slab}} := \gamma_{\text{concrete}} = 0.15 \cdot \text{kcf}$$

Unit weight of reinforced deck concrete

$$E_{\text{c.slub}} := 120000 \cdot K_1 \cdot \left( \frac{w_c}{\text{kcf}} \right)^2 \cdot \left( \frac{f_{\text{c.slub}}}{\text{ksi}} \right)^{0.33} \cdot \text{ksi} = 4428 \cdot \text{ksi}$$

Elastic modulus of deck concrete

(LRFD 5.4.2.4-1)

$$n_c := \frac{E_{\text{c.beam}}}{E_{\text{c.slub}}} = 1.154$$



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DESIGN EXAMPLE: FSB 12x57**

$$y_{\text{slab.top}} := \frac{A_{\text{slab.top}} \cdot \frac{t_{\text{slab.top}}}{2} + A_{\text{slab.bot}} \cdot \left( t_{\text{slab.top}} + \frac{t_{\text{slab.bot}}}{2} \right)}{A_{\text{slab}}} = 4.576 \cdot \text{in}$$

Distance between top fiber of slab to the c.g of the slab

$$y_{\text{slab.bot}} := t_{\text{slab.top}} + t_{\text{slab.bot}} - y_{\text{slab.top}} = 9.424 \cdot \text{in}$$

Distance between bottom fiber of slab to the c.g of the slab

**2.4 GFRP Reinforcing Bar**

$GFRPBarSize := 4$	Size of GFRP bar
$d_{GFRP} := \frac{GFRPBarSize}{8} \text{ in} = 0.5 \cdot \text{in}$	Nominal diameter of GFRP bar <span style="float: right;">s</span>
$A_{GFRP} := \frac{1}{4} \cdot \pi \cdot d_{GFRP}^2 = 0.196 \cdot \text{in}^2$	Area of one GFRP bar
$f_{fu,GFRP}^* := \frac{21.6 \text{ kip}}{A_{GFRP}} = 110.008 \cdot \text{ksi}$	Tensile strength of GFRP bar (FDOT Standard Specifications, Section 932, Table 3-1)
$C_{E,GFRP} := 0.7$	Environmental reduction factor for GFRP (AASHTO-GFRP Table 2.6.1.2-1) Here assume "concrete exposed to earth and weather"
$f_{fu,GFRP} := C_{E,GFRP} \cdot f_{fu,GFRP}^* = 77.006 \cdot \text{ksi}$	Design tensile strength
$E_{GFRP} := 6500 \text{ ksi}$	Elastic modulus of GFRP bar (No less than 6500 ksi per FDOT Standard Specifications, Section 932)
$\phi_{bend} = f_{fu} \cdot \left( 0.11 + 0.05 \frac{r}{d_b} \right)$	Strength reduction for bent FRP stirrup (ACI 440.4R 5.4)
$\phi_{bend,GFRP} := 0.6$	take 0.6 in this design example
$n_b := \frac{E_{GFRP}}{E_{c,beam}} = 1.271$	
$d_{slab.rebar} := 2.5 \text{ in}$	Distance from top of slab to centroid of the slab reinforcement (longitudinal)



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### 2.5 Prestressing CFRP Strand

*[Assume 0.6in diameter CFRP Cable; Properties refer to FDOT Standard Specification Section 933 Table 1-2, 7-strand-15.2mm CFRP prestressing strand]*

$StrandType_{CFRP} := 1$	CFRP strand type: 1 for Cable; 2 for Bar
$D_p := 0.6in$	Diameter of CFRP prestressing tendon
$A_{pf} := 0.179in^2$	Effective cross-sectional area of each tendon
$E_p := 22480ksi$	Elastic modulus of prestressing tendon
$P_u := 61kip$	Ultimate tensile force for each tendon
$f_{pu} := \frac{P_u}{A_{pf}} = 341 \cdot ksi$	Ultimate tensile strength Design tensile strength , $f_{pu}$ , is defined later in "other inputs"
$C_{E.CFRP} := 1$	Environmental reduction factor for CFRP (AASHTO CFRP Table 1.4.1.2-1)
$f_{pu} := C_{E.CFRP} \cdot f_{pu} = 341 \cdot ksi$	Design tensile strength of CFRP tendon
$\epsilon_{pu} := \frac{f_{pu}}{E_p} = 0.015$	Design tensile strain
$f_{pi.limit} := \begin{cases} 0.70 \cdot f_{pu} & \text{if } StrandType_{CFRP} = 1 \\ 0.65 \cdot f_{pu} & \text{if } StrandType_{CFRP} = 2 \end{cases} = 238.5 \cdot ksi$	Jacking stress limit (AASHTO CFRP Table 1.9.1.1)
$f_{pi} := f_{pi.limit} = 239 \cdot ksi$	Jacking stress
$P_i := f_{pi} \cdot A_{pf} = 42.7 \cdot kip$	Jacking force



### 3. SECTION PROPERTIES

#### 3.1 Section Properties - Non-Composite Section (FSB)

$$h_g := h_{\text{beam}} = 12 \cdot \text{in}$$

Overall depth

$$b_{g,\text{top}} := b_{\text{beam.top}} = 45 \cdot \text{in}$$

Beam width, top

$$b_{g,\text{bot}} := b_{\text{beam.bot}} = 57 \cdot \text{in}$$

Beam width, bottom

$$A_g := A_{\text{beam}} = 581.76 \cdot \text{in}^2$$

Total beam area

$$I_g := I_{xx,\text{beam}} = 7.084 \times 10^3 \cdot \text{in}^4$$

Moment inertia

$$y_{g,\text{top}} := y_{\text{beam.top}} = 6.35 \cdot \text{in}$$

Distance from top fiber (beam) to N.A

$$y_{g,\text{bot}} := y_{\text{beam.bot}} = 5.65 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A

$$S_{g,\text{top}} := \frac{I_g}{y_{g,\text{top}}} = 1.116 \times 10^3 \cdot \text{in}^3$$

$$S_{g,\text{bot}} := \frac{I_g}{y_{g,\text{bot}}} = 1.254 \times 10^3 \cdot \text{in}^3$$

**3.2 Section Properties - Composite Section (FSB+Deck)**

$$b_{\text{slab.top.comp}} := b_{\text{slab.top}} \cdot \frac{E_{\text{c.slub}}}{E_{\text{c.beam}}} = 50.672 \cdot \text{in} \quad \text{Transformed slab width, in composite section, top part}$$

$$b_{\text{slab.bot.comp}} := b_{\text{slab.bot}} \cdot \frac{E_{\text{c.slub}}}{E_{\text{c.beam}}} = 11.044 \cdot \text{in} \quad \text{Transformed slab width, in composite section, bottom part}$$

$$A_{\text{slab.top.comp}} := b_{\text{slab.top.comp}} \cdot t_{\text{slab.top}} = 304.031 \cdot \text{in}^2 \quad \text{Transformed slab area, in composite section, top part}$$

$$A_{\text{slab.bot.comp}} := b_{\text{slab.bot.comp}} \cdot t_{\text{slab.bot}} = 88.351 \cdot \text{in}^2 \quad \text{Transformed slab area, in composite section, bottom part}$$

$$A_{\text{slab.comp}} := A_{\text{slab.top.comp}} + A_{\text{slab.bot.comp}} = 392.383 \cdot \text{in}^2 \quad \text{Total area of slab, in composite section}$$

$$A_{\text{g.comp}} := A_{\text{beam}} + A_{\text{slab.comp}} = 974.143 \cdot \text{in}^2 \quad \text{Area}$$

$$h_{\text{g.comp}} := t_{\text{slab}} + h_{\text{beam}} = 18 \cdot \text{in} \quad \text{Overall height of the section}$$

$$y_{\text{g.comp.bot}} := \frac{A_{\text{slab.top.comp}} \cdot \left( h_{\text{g.comp}} - \frac{t_{\text{slab.top}}}{2} \right) + A_{\text{slab.bot.comp}} \cdot \left( h_{\text{g.comp}} - t_{\text{slab.top}} - \frac{t_{\text{slab.bot}}}{2} \right) + A_{\text{beam}} \cdot y_{\text{g.bot}}}{A_{\text{g.comp}}} = 8.781 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A

$$y_{\text{g.comp.top}} := h_{\text{g.comp}} - y_{\text{g.comp.bot}} = 9.219 \cdot \text{in} \quad \text{Distance from top fiber (slab) to N.A}$$

$$y_{\text{g.comp.topbeam}} := h_{\text{beam}} - y_{\text{g.comp.bot}} = 3.219 \cdot \text{in} \quad \text{Distance from top fiber (beam) to N.A}$$

$$\begin{aligned}
 I_{g,comp} &:= \frac{1}{12} \cdot b_{slab,top,comp} \cdot t_{slab,top}^3 \dots && = 2.598 \times 10^4 \cdot \text{in}^4 \\
 &+ A_{slab,top,comp} \cdot \left( y_{g,comp,top} - \frac{t_{slab,top}}{2} \right)^2 \dots \\
 &+ \frac{1}{12} \cdot b_{slab,bot,comp} \cdot t_{slab,bot}^3 \dots \\
 &+ A_{slab,bot,comp} \cdot \left( y_{g,comp,top} - t_{slab,top} - \frac{t_{slab,bot}}{2} \right)^2 \dots \\
 &+ I_g + A_g \cdot (y_{g,comp,bot} - y_{g,bot})^2
 \end{aligned}$$

Moment inertia

$$S_{g,comp,top} := \frac{I_{g,comp}}{y_{g,comp,top}} = 2.819 \times 10^3 \cdot \text{in}^3$$

$$S_{g,comp,bot} := \frac{I_{g,comp}}{y_{g,comp,bot}} = 2.959 \times 10^3 \cdot \text{in}^3$$

$$S_{g,comp,topbeam} := \frac{I_{g,comp}}{y_{g,comp,topbeam}} = 8.072 \times 10^3 \cdot \text{in}^3$$

$$e_g := y_{g,top} + t_{slab,top} - y_{slab,top} = 7.774 \cdot \text{in}$$

Distance between the c.g. of the basic beam and c.g of the slab

## **4. LOAD AND LOAD DISTRIBUTION**

### **4.1 Dead Load**

$$w_{\text{beam}} := A_g \cdot \gamma_{\text{beam}} = 0.606 \cdot \frac{\text{kip}}{\text{ft}}$$

Self weight of beam

$$w_{\text{slab}} := A_{\text{slab}} \cdot \gamma_{\text{slab}} = 0.472 \cdot \frac{\text{kip}}{\text{ft}}$$

Self weight of slab

$$w_{\text{barrier.each}} := 0.408 \frac{\text{kip}}{\text{ft}}$$

Traffic railing barrier DL, average of two ,  
"rehabilitation of Bridges over Cow Key  
Channel, Bridge 900086"

$$w_{\text{barrier}} := \frac{2 \cdot w_{\text{barrier.each}}}{\text{NumberOfBeam}} = 0.091 \cdot \frac{\text{kip}}{\text{ft}}$$

For purpose of this design example, all  
barrier loads are equally distributed to the  
total number of beams

$$w_{\text{forms}} := 0$$

Stay-in-place forms

$$w_{\text{fws}} := 0.015 \frac{\text{kip}}{\text{ft}}$$

Future wearing surface

$$w_{\text{utility}} := 0.142 \frac{\text{kip}}{\text{ft}}$$

Utility and sidewalk DL, "rehabilitation of  
Bridges over Cow Key Channel, Bridge  
900086"

*for interior beams*

**DC Dead load**  
(LRFD 3.3.2)

$$M_{\text{sf.beam}}(x) := \frac{w_{\text{beam}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{beam}} \cdot x^2}{2}$$

Self weight of beam  
(x distance from support)

$$V_{\text{sf.beam}}(x) := \frac{w_{\text{beam}} \cdot \text{Span}}{2} - w_{\text{beam}} \cdot x$$

$$M_{\text{sf.slab}}(x) := \frac{w_{\text{slab}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{slab}} \cdot x^2}{2}$$

Self weight of slab

$$V_{\text{sf.slab}}(x) := \frac{w_{\text{slab}} \cdot \text{Span}}{2} - w_{\text{slab}} \cdot x$$

$$M_{\text{forms}}(x) := \frac{w_{\text{forms}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{forms}} \cdot x^2}{2} \quad \text{Stay-in-place forms}$$

$$V_{\text{forms}}(x) := \frac{w_{\text{forms}} \cdot \text{Span}}{2} - w_{\text{forms}} \cdot x$$

$$M_{\text{barrier}}(x) := \frac{w_{\text{barrier}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{barrier}} \cdot x^2}{2} \quad \text{Barriers}$$

$$V_{\text{barrier}}(x) := \frac{w_{\text{barrier}} \cdot \text{Span}}{2} - w_{\text{barrier}} \cdot x$$

Total DC dead load

$$w_{\text{DC}} := w_{\text{beam}} + w_{\text{slab}} + w_{\text{forms}} + w_{\text{barrier}} = 1.169 \cdot \frac{\text{kip}}{\text{ft}}$$

$$M_{\text{DC}}(x) := M_{\text{sf.beam}}(x) + M_{\text{sf.slab}}(x) + M_{\text{forms}}(x) + M_{\text{barrier}}(x)$$

$$V_{\text{DC}}(x) := V_{\text{sf.beam}}(x) + V_{\text{sf.slab}}(x) + V_{\text{forms}}(x) + V_{\text{barrier}}(x)$$

**DW Dead load**  
(LRFD 3.3.2)

$$M_{\text{fws}}(x) := \frac{w_{\text{fws}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{fws}} \cdot x^2}{2} \quad \text{Future wearing surface} \\ \text{(x distance from support)}$$

$$V_{\text{fws}}(x) := \frac{w_{\text{fws}} \cdot \text{Span}}{2} - w_{\text{fws}} \cdot x$$

$$M_{\text{utility}}(x) := \frac{w_{\text{utility}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{utility}} \cdot x^2}{2} \quad \text{Utility}$$

$$V_{\text{utility}}(x) := \frac{w_{\text{utility}} \cdot \text{Span}}{2} - w_{\text{utility}} \cdot x$$



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DESIGN EXAMPLE: FSB 12x57**

$$w_{DW} := w_{fws} + w_{utility} = 0.157 \cdot \frac{\text{kip}}{\text{ft}}$$

Total DW dead load

$$M_{DW}(x) := M_{fws}(x) + M_{utility}(x)$$

$$V_{DW}(x) := V_{fws}(x) + V_{utility}(x)$$

**4.2 Live Load**

$$IM_{\text{fatigue}} := 1.15$$

Impact factor (dynamic load effects) for fatigue limit states

(LRFD Table 3.6.2.1-1)

$$IM := 1.33$$

Impact factor (dynamic load effects) for limit states other than fatigue and fracture

(LRFD Table 3.6.2.1-1)

$$w_{\text{lane}} := 0.64 \frac{\text{kip}}{\text{ft}}$$

Uniformly distributed design lane load

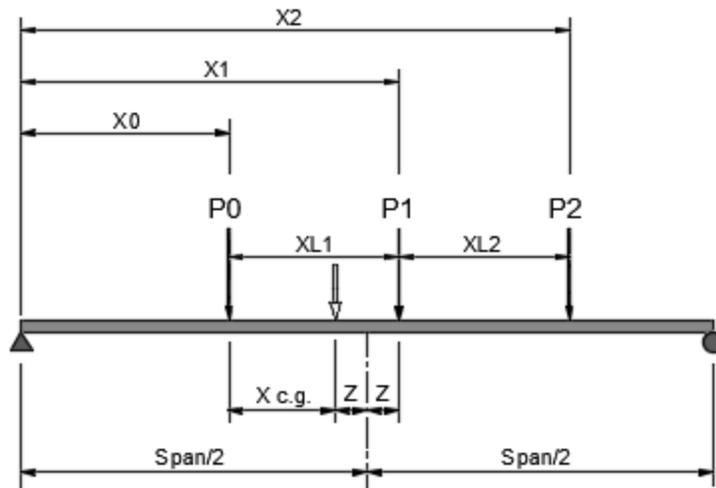
(LRFD 3.6.1.2.4)

$$M_{\text{lane}}(x) := \frac{w_{\text{lane}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{lane}} \cdot x^2}{2}$$

Moment and shear introduced by design lane load

x distance from left support

$$V_{\text{lane}}(x) := \frac{w_{\text{lane}} \cdot \text{Span}}{2} - w_{\text{lane}} \cdot x$$



Maximum Live Load Moment

(FDOT design examples)

----- Live Load (except for fatigue limit state) -----

$$P := \begin{pmatrix} 32 \\ 32 \\ 8 \end{pmatrix} \cdot \text{kip}$$

$$XL := \begin{pmatrix} 0 \\ 14 \\ 14 \end{pmatrix} \cdot \text{ft}$$

HL-93 Truck load, axle loads and spacing  
(LRFD 3.6.1.2.2)

$$x_{cg} := \frac{P_0 \cdot XL_0 + P_1 \cdot (XL_0 + XL_1) + P_2 \cdot (XL_0 + XL_1 + XL_2)}{P_0 + P_1 + P_2} = 9.333 \text{ ft}$$

Center of gravity for axle loads

$$z := \frac{XL_1 - x_{cg}}{2} = 2.333 \cdot \text{ft}$$

Distance from c.g. of axle loads to centerline of span

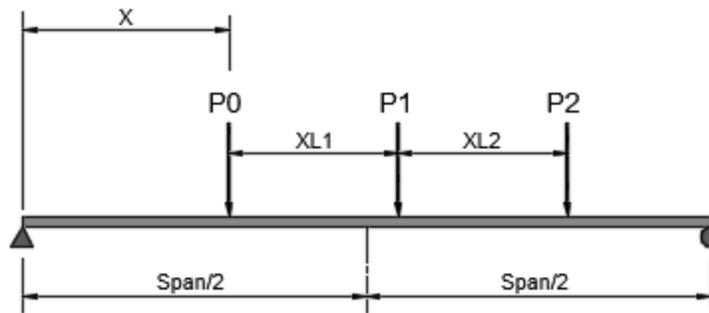
$$X_{left} := \frac{\text{Span}}{2} - z - x_{cg} = 7.792 \text{ ft}$$

$$X := \begin{pmatrix} X_{left} \\ X_{left} + XL_1 \\ X_{left} + XL_1 + XL_2 \end{pmatrix} = \begin{pmatrix} 7.792 \\ 21.792 \\ 35.792 \end{pmatrix} \text{ ft}$$

Distance from left support to axle loads

**Live load moments and shear - One HL-93 truck**

**Case 1**



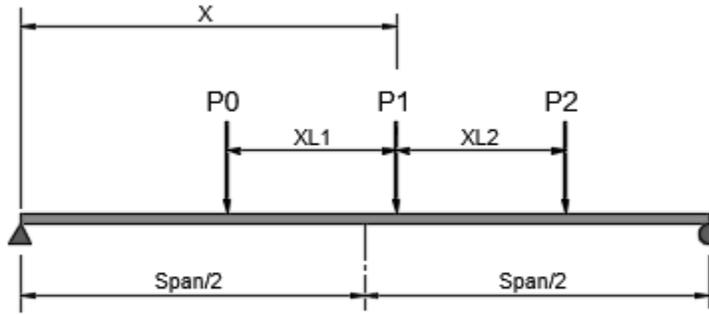
**CASE 1**

(FDOT design examples)

$$M_{truck1}(x) := P_0 \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x - XL_1) \cdot x}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_1 - XL_2) \cdot x}{\text{Span}}$$

$$V_{truck1}(x) := P_0 \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x - XL_1)}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_1 - XL_2)}{\text{Span}}$$

**Case 2**



**CASE 2**

(FDOT design examples)

$$M_{\text{truck2}}(x) := P_0 \cdot \frac{(\text{Span} - x) \cdot (x - XL_1)}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_2) \cdot x}{\text{Span}}$$

$$V_{\text{truck2}}(x) := P_0 \cdot \frac{-(x - XL_1)}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_2)}{\text{Span}}$$

$$M_{\text{truck}}(x) := \max(M_{\text{truck1}}(x), M_{\text{truck2}}(x))$$

Moment and shear introduced by Truck HL-93

$$V_{\text{truck}}(x) := \max(V_{\text{truck1}}(x), V_{\text{truck2}}(x))$$

$$M_{\text{LL.IM}}(x) := M_{\text{truck}}(x) \cdot \text{IM} + M_{\text{lane}}(x)$$

Live load (truck and lane) moment and shear, including impact (IM is for limit states other than fatigue)

$$V_{\text{LL.IM}}(x) := V_{\text{truck}}(x) \cdot \text{IM} + V_{\text{lane}}(x)$$

$$M_{\text{LL}}(x) := M_{\text{truck}}(x) + M_{\text{lane}}(x)$$

Live load (truck and lane) moment and shear, excluding impact

$$V_{\text{LL}}(x) := V_{\text{truck}}(x) + V_{\text{lane}}(x)$$

----- Live Load for fatigue limit state -----

$$P_f := \begin{pmatrix} 32 \\ 32 \\ 8 \end{pmatrix} \cdot \text{kip} \quad X_{L_f} := \begin{pmatrix} 0 \\ 30 \\ 14 \end{pmatrix} \cdot \text{ft} \quad \begin{array}{l} \text{HL-93 Truck load, axle loads and spacing} \\ \text{(LRFD 3.6.1.2.2)} \end{array}$$

$$x_{cg.f} := \frac{P_{f_0} \cdot X_{L_{f_0}} + P_{f_1} \cdot (X_{L_{f_0}} + X_{L_{f_1}}) + P_{f_2} \cdot (X_{L_{f_0}} + X_{L_{f_1}} + X_{L_{f_2}})}{P_{f_0} + P_{f_1} + P_{f_2}} = 18.222 \text{ ft}$$

Center of gravity for axle loads

$$z_f := \frac{X_{L_{f_1}} - x_{cg.f}}{2} = 5.889 \cdot \text{ft}$$

Distance from c.g. of axle loads to centerline of span

$$X_{left.f} := \frac{\text{Span}}{2} - z_f - x_{cg.f} = -4.653 \text{ ft}$$

$$X_f := \begin{pmatrix} X_{left.f} \\ X_{left.f} + X_{L_{f_1}} \\ X_{left.f} + X_{L_{f_1}} + X_{L_{f_2}} \end{pmatrix} = \begin{pmatrix} -4.653 \\ 25.347 \\ 39.347 \end{pmatrix} \text{ ft}$$

Distance from left support to axle loads

**Live load moments and shear - One HL-93 truck**

**Case 1**

$$M_{truck1.f(x)} := P_{f_0} \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x - X_{L_{f_1}}) \cdot x}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - X_{L_{f_1}} - X_{L_{f_2}}) \cdot x}{\text{Span}}$$

$$V_{truck1.f(x)} := P_{f_0} \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x - X_{L_{f_1}})}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - X_{L_{f_1}} - X_{L_{f_2}})}{\text{Span}}$$

**Case 2**

$$M_{\text{truck2.f}(x)} := P_{f_0} \cdot \frac{(\text{Span} - x) \cdot (x - XL_{f_1})}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - XL_{f_2}) \cdot x}{\text{Span}}$$

$$V_{\text{truck2.f}(x)} := P_{f_0} \cdot \frac{-(x - XL_{f_1})}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - XL_{f_2})}{\text{Span}}$$

$$M_{\text{truck.f}(x)} := \max(M_{\text{truck1.f}(x)}, M_{\text{truck2.f}(x)})$$

Moment and shear introduced by Truck HL-93, for fatigue limit state

$$V_{\text{truck.f}(x)} := \max(V_{\text{truck1.f}(x)}, V_{\text{truck2.f}(x)})$$

$$M_{\text{LL.IM.f}(x)} := M_{\text{truck.f}(x)} \cdot IM_{\text{fatigue}}$$

Fatigue Live load (truck) moment and shear

$$V_{\text{LL.IM.f}(x)} := V_{\text{truck.f}(x)} \cdot IM_{\text{fatigue}}$$

at midspan

$$M_{\text{LL.IM.f}}\left(\frac{\text{Span}}{2}\right) = 189 \cdot \text{kip} \cdot \text{ft}$$

$$V_{\text{LL.IM.f}}\left(\frac{\text{Span}}{2}\right) = 30 \cdot \text{kip}$$

**4.3 Live Load Distribution Factors**

(LRFD 4.6.2.2)

Live load on the deck must be distributed to the precast prestressed beams. AASHTO provides factors for the distribution of live load into the beams. The factors can be used if the following criteria is met:

- (1) Width of deck is constant
- (2) Number of beams is not less than four
- (3) Beams are parallel and have approximately the same stiffness
- (4) The overhang minus the barrier width does not exceed 3.0 feet
- (5) Curvature in plan is less than the limit specified in LRFD 4.6.1.2.4
- (6) Cross-section is consistent with one of the cross-sections shown in Table 4.6.2.2.1-1

In this example, all the conditions are satisfied

$$k_{\text{dist}} := \max(1.5, 2.5 \cdot \text{NumberOfBeam}^{-0.2}) = 1.611 \quad (\text{LRFD Table 4.6.2.2.2b-1})$$

$$b_{\text{dist}} := b_{\text{beam}} = 57 \cdot \text{in}$$

$$d_{\text{dist}} := h_{\text{beam}} + t_{\text{slab}} = 18 \cdot \text{in}$$

$$L_{\text{dist}} := \text{Span} = 38.917 \text{ ft}$$

$$I_{\text{dist}} := \frac{b_{\text{dist}} \cdot d_{\text{dist}}^3}{12} = 2.77 \times 10^4 \cdot \text{in}^4$$

$$J_{\text{dist}} := \frac{(b_{\text{dist}} \cdot d_{\text{dist}})^4}{40 \cdot \left( \frac{b_{\text{dist}} \cdot d_{\text{dist}}^3}{12} + \frac{d_{\text{dist}} \cdot b_{\text{dist}}^3}{12} \right)} = 9.068 \times 10^4 \cdot \text{in}^4 \quad (\text{LRFD C4.6.2.2.1-3})$$

Type f cross-section in LRFD Table 4.6.2.2.1

**Moment distribution factor** for interior beams

When one design lane loaded

$$g_{m,1} = k \cdot \left( \frac{b}{33.3L} \right)^{0.5} \cdot \left( \frac{I}{J} \right)^{0.25} \quad (\text{LRFD Table 4.6.2.2.2b-1})$$

$$g_{m.1} := k_{\text{dist}} \cdot \left( \frac{\frac{b_{\text{dist}}}{\text{in}}}{33.3 \frac{L_{\text{dist}}}{\text{ft}}} \right)^{0.5} \cdot \left( \frac{I_{\text{dist}}}{J_{\text{dist}}} \right)^{0.25} = 0.251$$

Distribution factor for moment in interior beams when one lane is loaded

When two or more design lane loaded

$$g_{m.2} = k \cdot \left( \frac{b}{305} \right)^{0.6} \cdot \left( \frac{b}{12.0 \cdot L} \right)^{0.2} \cdot \left( \frac{I}{J} \right)^{0.06} \quad (\text{LRFD Table 4.6.2.2.2b-1})$$

$$g_{m.2} := k_{\text{dist}} \cdot \left( \frac{\frac{b_{\text{dist}}}{\text{in}}}{305} \right)^{0.6} \cdot \left( \frac{\frac{b_{\text{dist}}}{\text{in}}}{12.0 \cdot \frac{L_{\text{dist}}}{\text{ft}}} \right)^{0.2} \cdot \left( \frac{I_{\text{dist}}}{J_{\text{dist}}} \right)^{0.06} = 0.360$$

Distribution factor for moment in interior beams when two or more lanes are loaded

$$g_m := \max(g_{m.1}, g_{m.2}) = 0.36$$

Distribution factor for moment in interior beams

Verify if the distribution factor satisfies LRFD criteria for "Range of Applicability"

$$g_{m.\text{check}} := \begin{cases} b \leftarrow 35\text{in} \leq b_{\text{dist}} \leq 60\text{in} & = \text{"OK"} \\ L \leftarrow 20\text{ft} \leq L_{\text{dist}} \leq 120\text{ft} \\ N_b \leftarrow 5 \leq \text{NumberOfBeam} \leq 20 \\ \text{"OK"} & \text{if } (b \cdot L \cdot N_b) = 1 \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$g_{m.\text{check}} = \text{"OK"}$$

**Shear distribution factor** for interior beams

$$g_v = \left( \frac{b}{156} \right)^{0.4} \cdot \left( \frac{b}{12.0 \cdot L} \right)^{0.1} \cdot \left( \frac{I}{J} \right)^{0.05} \cdot \min\left(1, \frac{b}{48}\right) \quad (\text{LRFD Table 4.6.2.2.3a-1})$$

$$g_v := \left( \frac{b_{\text{dist}}}{156} \right)^{0.4} \cdot \left( \frac{b_{\text{dist}}}{12.0 \cdot \frac{L_{\text{dist}}}{\text{ft}}} \right)^{0.1} \cdot \left( \frac{I_{\text{dist}}}{J_{\text{dist}}} \right)^{0.05} \cdot \min \left( 1, \frac{b_{\text{dist}}}{48} \right) = 0.511$$

$$g_v = 0.511$$

Distribution factor for shear in interior beams

Verify if the distribution factor satisfies LRFD criteria for "Range of Applicability"

$$g_{v.\text{check}} := \begin{cases} b \leftarrow 35\text{in} \leq b_{\text{dist}} \leq 60\text{in} & = \text{"OK"} \\ L \leftarrow 20\text{ft} \leq L_{\text{dist}} \leq 120\text{ft} \\ N_b \leftarrow 5 \leq \text{NumberOfBeam} \leq 20 \\ J \leftarrow 40000 \cdot \text{in}^4 \leq J_{\text{dist}} \leq 610000 \cdot \text{in}^4 \\ I \leftarrow 25000 \cdot \text{in}^4 \leq I_{\text{dist}} \leq 610000 \cdot \text{in}^4 \\ \text{"OK"} \quad \text{if } (b \cdot L \cdot N_b \cdot J \cdot I) = 1 \\ \text{"NG"} \quad \text{otherwise} \end{cases}$$

$$g_{v.\text{check}} = \text{"OK"}$$

**Summary of live load distribution factors, interior beams**

$$g_m = 0.36$$

Distribution factor for moment in interior beams

$$g_v = 0.511$$

Distribution factor for shear in interior beams

~~$$M_{LL,IM}(x) := g_m \cdot M_{LL,IM}(x)$$~~

Re-define  
Live load (truck and lane) moment and shear, including impact (IM is for limit states other than fatigue)  
Consider LL distribution factor

~~$$V_{LL,IM}(x) := g_v \cdot V_{LL,IM}(x)$$~~

$$\underline{M_{LL}}(x) := g_m \cdot M_{LL}(x)$$

Re-define  
 Live load (truck and lane) moment and shear, excluding impact  
 Consider LL distribution factor

$$\underline{V_{LL}}(x) := g_v \cdot V_{LL}(x)$$

$$\underline{M_{LL.IM.f}}(x) := \frac{g_m}{1.2} \cdot M_{LL.IM.f}(x)$$

Re-define  
 Live load (truck) moment and shear, including impact (IM is for fatigue)  
 Consider LL distribution factor

$$\underline{V_{LL.IM.f}}(x) := \frac{g_v}{1.2} \cdot V_{LL.IM.f}(x)$$

*Noted that the distribution factor calculated in LRFD 4.6.2.2 for a single lane loaded already includes the 1.2 "multiple presence factor" for a single lane, therefore, this value may be used for the service and strength limit states. However, multiple presence factors should not be used for the fatigue limit state. Therefore, the multiple presence factor of 1.2 for the single lane is required to be removed from the value calculated above to determine the factors used for the fatigue limit state*

#### 4.4 Summary of Moments and Shear Forces

Strength1 = 1.25DC + 1.5DW + 1.75LL	Strength I limit state
-------------------------------------	------------------------

Service1 = 1.0DC + 1.0DW + 1.0LL	Service I limit state
----------------------------------	-----------------------

Service3 = 1.0DC + 1.0DW + 0.8LL	Service III limit state
----------------------------------	-------------------------

Fatigue1 = 1.75LL <sub>fatigue</sub>	Fatigue I limit state
--------------------------------------	-----------------------

$M_{Str1}(x) := 1.25 \cdot M_{DC}(x) + 1.5 \cdot M_{DW}(x) + 1.75 \cdot M_{LL.IM}(x)$	Strength I limit state
---	------------------------

$M_{Srv1}(x) := 1.0 \cdot M_{DC}(x) + 1.0 \cdot M_{DW}(x) + 1.0 \cdot M_{LL.IM}(x)$	Service I limit state
---	-----------------------

$M_{Srv3}(x) := 1.0 \cdot M_{DC}(x) + 1.0 \cdot M_{DW}(x) + 0.8 \cdot M_{LL.IM}(x)$	Service III limit state
---	-------------------------

$M_{fatigue}(x) := 1.75 \cdot M_{LL.IM.f}(x)$	Fatigue I limit state
---	-----------------------

$V_{Str1}(x) := 1.25 \cdot V_{DC}(x) + 1.5 \cdot V_{DW}(x) + 1.75 \cdot V_{LL.IM}(x)$	Strength I limit state
---	------------------------

$V_{Srv1}(x) := 1.0 \cdot V_{DC}(x) + 1.0 \cdot V_{DW}(x) + 1.0 \cdot V_{LL.IM}(x)$	Service I limit state
---	-----------------------

$V_{Srv3}(x) := 1.0 \cdot V_{DC}(x) + 1.0 \cdot V_{DW}(x) + 0.8 \cdot V_{LL.IM}(x)$	Service III limit state
---	-------------------------

$V_{fatigue}(x) := 1.75 \cdot V_{LL.IM.f}(x)$	Fatigue I limit state
---	-----------------------

at mispan

$M_{DC.mid} := M_{DC} \left( \frac{Span}{2} \right) = 221 \cdot kip \cdot ft$	Due to DC
---	-----------

$M_{DW.mid} := M_{DW} \left( \frac{Span}{2} \right) = 29.722 \cdot kip \cdot ft$	Due to DW
--	-----------

$$M_{LL.IM.mid} := M_{LL.IM} \left( \frac{\text{Span}}{2} \right) = 245 \cdot \text{kip} \cdot \text{ft} \quad \text{Due to LL}$$

$$M_{LL.IM.fatigue.mid} := M_{LL.IM.f} \left( \frac{\text{Span}}{2} \right) = 57 \cdot \text{kip} \cdot \text{ft} \quad \text{Due to LL}$$

$$M_{Str1.mid} := 1.25 \cdot M_{DC.mid} + 1.5 \cdot M_{DW.mid} + 1.75 \cdot M_{LL.IM.mid} = 750 \cdot \text{kip} \cdot \text{ft}$$

Strength I limit state

$$M_{Srv1.mid} := 1.0 \cdot M_{DC.mid} + 1.0 \cdot M_{DW.mid} + 1.0 \cdot M_{LL.IM.mid} = 496 \cdot \text{kip} \cdot \text{ft}$$

Service I limit state

$$M_{Srv3.mid} := 1.0 \cdot M_{DC.mid} + 1.0 \cdot M_{DW.mid} + 0.8 \cdot M_{LL.IM.mid} = 447 \cdot \text{kip} \cdot \text{ft}$$

Service III limit state

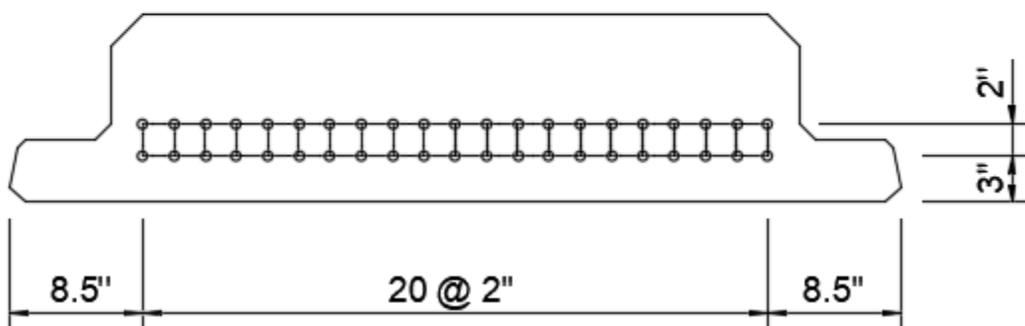
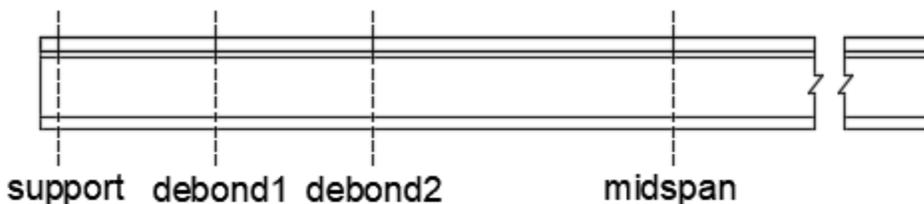
$$M_{fatigue.mid} := 1.75 \cdot M_{LL.IM.fatigue.mid} = 99 \cdot \text{kip} \cdot \text{ft}$$

Fatigue I limit state

**5. STRAND PATTERN**

for FSB 12x57 Beam

The pattern of strands need to be adjusted according to limit states



support := 0 ft

Distance from left support

debond1 := 0 ft

Debond place1, "0" if no need debonding

debond2 := 0 ft

Debond place2, "0" if no need debonding

$$\text{midspan} := \frac{\text{Span}}{2} = 19.458 \text{ ft}$$

Max number of strands in each row, from bottom up

Number of strands in each row, from bottom up

$$n_{\text{max}} := \binom{21}{21}$$

Adjust

$$n := \binom{18}{0}$$

**Adjust the numbers to satisfy Service III and Strength I limit states (Sections 9 and 11 in this example)**

*Note: Service I can be satisfied by debonding (Section 8 in this example)*

$$n_{\text{total}} := \sum_{i=0}^{\text{rows}(n)-1} n_i = 18$$

Total number of strands

$$A_p := n \cdot A_{pf} = \binom{3.222}{0} \cdot \text{in}^2$$

Total area of strands in each row

$$A_{p,\text{total}} := \sum_{i=0}^{\text{rows}(n)-1} A_{p_i} = 3.222 \cdot \text{in}^2$$

Total area of strands

$$d_p := \binom{h_g - 3\text{in}}{h_g - 5\text{in}} = \binom{9}{7} \cdot \text{in}$$

Distance from top beam section to the center of each strand row

$$d_{p,\text{strands.c.g}} := \frac{\sum_{i=0}^{\text{rows}(n)-1} (A_{p_i} \cdot d_{p_i})}{A_{p,\text{total}}} = 9 \cdot \text{in}$$

Distance from top beam section to the gravity center of strand group

$$d_{p,\text{strands.c.g.bot}} := h_g - d_{p,\text{strands.c.g}} = 3 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group

## **6. TRANSFORMED SECTION PROPERTIES**

### **6.1 Transformed Section Properties - Non-Composite Section (FSB+Strands)**

$$n_p = 4.397 \quad \text{Elastic modulus ratio, } E_p/E_c$$

$$A_{g, tr} := A_g + (n_p - 1)A_{p, total} = 592.706 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g, tr, top} := \frac{A_g \cdot y_{g, top} + \sum_{i=0}^{\text{rows}(n)-1} [A_{p_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g, tr}} = 6.399 \cdot \text{in}$$

Distance from top fiber (beam) to N.A

$$y_{g, tr, bot} := h_g - y_{g, tr, top} = 5.601 \cdot \text{in} \quad \text{Distance from bottom fiber (beam) to N.A}$$

$$I_{g, tr} := I_g + A_g \cdot (y_{g, tr, top} - y_{g, top})^2 + \sum_{i=0}^{\text{rows}(n)-1} [(n_p - 1) \cdot A_{p_i} \cdot (d_{p_i} - y_{g, tr, top})^2] = 7.159 \times 10^3 \cdot \text{in}^4$$

$$S_{g, tr, top} := \frac{I_{g, tr}}{y_{g, tr, top}} = 1.119 \times 10^3 \cdot \text{in}^3 \quad \text{Moment inertia}$$

$$S_{g, tr, bot} := \frac{I_{g, tr}}{y_{g, tr, bot}} = 1.278 \times 10^3 \cdot \text{in}^3$$

$$e_{g, tr, cg, strands} := y_{g, tr, bot} - d_{p, strands, c.g, bot} = 2.601 \cdot \text{in}$$

eccentricity of strand group

### **6.2 Transformed Section Properties - Composite Section (FSB+Slab+Strands+Bars)**

*Assume #4 GFRP bar at 12" spacing, longitudinal direction in the slab*

$$d_{slab, rebar} = 2.5 \cdot \text{in} \quad \text{Distance from top of slab to centroid of the slab reinforcement (longitudinal)}$$

$$n_b = 1.271 \quad \text{Elastic modulus ratio, } E_{grfp}/E_c$$

$$A_{slab, rebar} := \frac{b_{slab, top}}{12 \text{in}} \cdot A_{GFRP} = 0.957 \cdot \text{in}^2 \quad \text{Total area of longitudinal reinforcement in slab in the section}$$

$$y_{\text{rebar}} := h_{\text{g.comp}} - d_{\text{slab.rebar}} = 15.5 \cdot \text{in}$$

Distance from c.g. of rebar group (in deck slab) to bottom of girder section

$$A_{\text{g.comp.tr}} := A_{\text{g.comp}} + (n_{\text{p}} - 1)A_{\text{p.total}} + (n_{\text{b}} - 1) \cdot A_{\text{slab.rebar}} = 985.348 \cdot \text{in}^2$$

Area

$$y_{\text{g.comp.tr.top}} := \frac{A_{\text{g.comp}} \cdot y_{\text{g.comp.top}} + \sum_{i=0}^{\text{rows}(n)-1} \left[ A_{\text{p}_i} \cdot (n_{\text{p}} - 1) \cdot d_{\text{p}_i} \right] \dots + (n_{\text{b}} - 1) \cdot A_{\text{slab.rebar}} \cdot d_{\text{slab.rebar}}}{A_{\text{g.comp.tr}}} = 9.215 \cdot \text{in}$$

Distance from top fiber (deck) to N.A

$$y_{\text{g.comp.tr.bot}} := h_{\text{g.comp}} - y_{\text{g.comp.tr.top}} = 8.785 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A

$$y_{\text{g.comp.tr.topbeam}} := y_{\text{g.comp.tr.top}} - t_{\text{slab.top}} = 3.215 \cdot \text{in}$$

Distance from top fiber of the beam to N.A

$$I_{\text{g.comp.tr}} := I_{\text{g.comp}} + A_{\text{g.comp}} \cdot (y_{\text{g.comp.tr.top}} - y_{\text{g.comp.top}})^2 \dots = 2.6 \times 10^4 \cdot \text{in}^4 + \sum_{i=0}^{\text{rows}(n)-1} \left[ (n_{\text{p}} - 1) \cdot A_{\text{p}_i} \cdot (d_{\text{p}_i} - y_{\text{g.comp.tr.top}})^2 \right] \dots + (n_{\text{b}} - 1) \cdot A_{\text{slab.rebar}} \cdot (d_{\text{slab.rebar}} - y_{\text{g.comp.tr.top}})^2$$

Moment inertia

$$S_{\text{g.comp.tr.top}} := \frac{I_{\text{g.comp.tr}}}{y_{\text{g.comp.tr.top}}} = 2.821 \times 10^3 \cdot \text{in}^3$$

$$S_{\text{g.comp.tr.bot}} := \frac{I_{\text{g.comp.tr}}}{y_{\text{g.comp.tr.bot}}} = 2.959 \times 10^3 \cdot \text{in}^3$$

$$S_{\text{g.comp.tr.topbeam}} := \frac{I_{\text{g.comp.tr}}}{y_{\text{g.comp.tr.topbeam}}} = 8.087 \times 10^3 \cdot \text{in}^3$$

$$e_{\text{g.comp.tr.cg.strands}} := y_{\text{g.comp.tr.bot}} - d_{\text{p.strands.c.g.bot}} = 5.785 \cdot \text{in}$$

Eccentricity of strand group

## **7. PRESTRESS LOSSES**

$$\Delta f_p = \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pTH}$$

Total prestress loss  
(AASHTO CFRP 1.9.2.1)

$\Delta f_{pES}$ : prestress loss due to elastic shortening

$\Delta f_{pLT}$ : long-term time-dependent prestress losses, due to shrinkage, creep, and relaxation

$\Delta f_{pTH}$ : prestress losses due to temperature change (Not considered in this example)

### **7.1 Prestress Loss due to Elastic Shortening**

$$\Delta f_{pES} = \frac{E_f}{E_{ct}} f_{cgp}$$

Prestress loss due to elastic shortening  
(AASHTO CFRP 1.4.2)

$$f_{pi} = 239 \cdot \text{ksi}$$

Jacking stress

$$F_{pi} := A_{p,\text{total}} \cdot f_{pi} = 768.6 \cdot \text{kip}$$

Total jacking force

$$e_{g,\text{tr.cg.strands}} = 2.601 \cdot \text{in}$$

Eccentricity of strand group to N.A. of the transformed non-composite section

$$M_{\text{sf.beam.midspan}} := M_{\text{sf.beam}} \left( \frac{\text{Span}}{2} \right) = 114.724 \cdot \text{kip} \cdot \text{ft}$$

Moment at midspan due to self weight of beam

$$f_{cgp} := \frac{F_{pi}}{A_{g,\text{tr}}} + \frac{F_{pi} \cdot e_{g,\text{tr.cg.strands}}^2}{I_{g,\text{tr}}} - \frac{M_{\text{sf.beam.midspan}} \cdot e_{g,\text{tr.cg.strands}}}{I_{g,\text{tr}}} = 1.523 \cdot \text{ksi}$$

Concrete stress at gravity center of the prestressing force at transfer

$$\Delta f_{pES} := \frac{E_p}{E_{ci,\text{beam}}} f_{cgp} = 7.512 \cdot \text{ksi}$$

Prestress loss due to elastic shortening  
(AASHTO CFRP 1.4.2)

$$f_{pt} := f_{pi} - \Delta f_{pES} = 231.035 \cdot \text{ksi}$$

Prestress stress at transfer

**7.2 Long-Term Time-Dependent Prestress Losses (Refined Estimation)**

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df}$$

Total long-term time-dependent losses  
id - time between transfer and deck placement  
df - time between deck placement and the final time

(LRFD 5.9.3.4.1-1)

**-----Time-Dependent losses between transfer and deck placement-----**

**Shrinkage of girder concrete (i->d)**

$$\Delta f_{pSR} = \epsilon_{bid} \cdot E_p \cdot K_{id} \quad \text{(LRFD 5.9.3.4)}$$

$$k_s := \max\left(1.45 - 0.13 \frac{A_g}{\text{Perimeter}_{\text{beam}} \cdot \text{in}}, 1.0\right) = 1.000$$

Factor for the effect of the volume-to-surface ratio of the component  
(LRFD 5.4.2.3.2)

$$k_{hs} := 2.00 - 0.014RH = 0.950$$

Humidity factor for shrinkage  
(LRFD 5.4.2.3.3)

$$k_{hc} := 1.56 - 0.008RH = 0.960$$

Humidity factor for creep  
(LRFD 5.4.2.3.2)

$$k_f := \frac{5}{1 + \frac{f_{ci.beam}}{\text{ksi}}} = 0.714$$

Factor for the effect of concrete strength  
(LRFD 5.4.2.3.2)

$$k_{td}(t_0, t_1) := \frac{t_1 - t_0}{12 \left( \frac{100 - 4 \cdot \frac{f_{ci.beam}}{\text{ksi}}}{\frac{f_{ci.beam}}{\text{ksi}} + 20} \right) + (t_1 - t_0)}$$

Time development factor  
(LRFD 5.4.2.3.2)

$$k_{td}(t_i, t_d) = 0.772$$

Time development factor  
Time from transfer to deck placement

$$k_{td}(t_i, t_f) = 0.997$$

Time development factor  
Time from transfer to final time

$$k_{td}(t_d, t_f) = 0.996$$

Time development factor  
Time from deck placement to final time

$$\epsilon_{bid} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}(t_i, t_d) \cdot 0.48 \cdot 10^{-3} = 2.516 \times 10^{-4}$$

Shrinkage strain of girder  
(LRFD 5.4.2.3.3)

$$\Psi_b(t_1, t_0) := 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td}(t_0, t_1) \cdot t_0^{-0.118}$$

Creep coefficient  
(LRFD 5.4.2.3.2)

$$\Psi_b(t_d, t_i) = 1.006$$

Time development factor  
Time from transfer to deck placement

$$\Psi_b(t_f, t_i) = 1.298$$

Time development factor  
Time from transfer to final time

$$\Psi_b(t_f, t_d) = 0.738$$

Time development factor  
Time from deck placement to final time

$$K_{id} := \frac{1}{1 + \frac{E_p}{E_{ci.beam}} \cdot \frac{A_{p.total}}{A_{g.tr}} \cdot \left( 1 + \frac{A_{g.tr} \cdot e_{g.tr.cg.strands}^2}{I_{g.tr}} \right) \cdot (1 + 0.7 \cdot \Psi_b(t_f, t_i))} = 0.926$$

Transformed section coefficient  
(LRFD 5.9.3.4.2)

$$\Delta f_{pSR} := \epsilon_{bid} \cdot E_p \cdot K_{id} = 5.237 \cdot \text{ksi}$$

Prestress loss due to  
Shrinkage of girder concrete  
(LRFD 5.9.3.4)

**Creep of girder concrete (i->d)**

$$\Delta f_{pCR} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \Psi_b(t_d, t_i) \cdot K_{id} = 7 \cdot \text{ksi}$$

Prestress loss due to  
Creep of girder concrete  
(LRFD 5.9.3.4)

**Relaxation of prestressing CFRP (i->d)**

$f_{pu} = 341 \cdot \text{ksi}$  Design tensile strength

$f_{pt} = 231 \cdot \text{ksi}$  Prestressing stress at transfer

$$\Delta f_{pR}(t_0, t_1) := \begin{cases} \left[ \left( 0.019 \frac{f_{pt}}{f_{pu}} - 0.0066 \right) \cdot \log[24 \cdot (t_1 - t_0)] \cdot f_{pu} \right] & \text{if StrandType}_{CFRP} = 1 \\ \left[ \left( 0.013 \frac{f_{pt}}{f_{pu}} - 0.006 \right) \cdot \log[24 \cdot (t_1 - t_0)] \cdot f_{pu} \right] & \text{if StrandType}_{CFRP} = 2 \end{cases}$$

Prestress loss due to relaxation  
(AASHTO CFRP 1.9.2.5.2)

$\Delta f_{pR1} := \Delta f_{pR}(t_i, t_d) = 7.397 \cdot \text{ksi}$

Prestress loss due to relaxation

Time from transfer to deck placement

**Total time-depedent prestress loss between transfer and deck placement**

$\Delta f_{pLT.id} := \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} = 19.634 \cdot \text{ksi}$

**-----Time-Dependent losses between deck placement to final time-----**

**Shrinkage of girder concrete (d->f)**

$\Delta f_{pSD} = \epsilon_{bdf} \cdot E_p \cdot K_{df}$  (LRFD 5.9.3.4.3)

$\epsilon_{bdf} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}(t_d, t_f) \cdot 0.48 \cdot 10^{-3} = 3.246 \times 10^{-4}$

Shrinkage strain of girder  
(LRFD 5.4.2.3.3)

$e_{g.comp.tr.cg.strands} = 5.785 \cdot \text{in}$

Eccentricity of prestressing force with  
respect to composite section  
(LRFD 5.9.3.4.3)

$$I_{g,comp.tr} = 2.6 \times 10^4 \cdot \text{in}^4$$

Moment inertia for transformed composite section

$$A_{g,comp.tr} = 985.348 \cdot \text{in}^2$$

Area for transformed composite section

$$K_{df} := \frac{1}{1 + \frac{E_p}{E_{ci,beam}} \cdot \frac{A_{p,total}}{A_{g,comp.tr}} \cdot \left( 1 + \frac{A_{g,comp.tr} \cdot e_{g,comp.tr} \cdot c_{g,strands}^2}{I_{g,comp.tr}} \right) \cdot (1 + 0.7 \cdot \Psi_b(t_f, t_i))} = 0.935$$

Transformed section coefficient  
(LRFD 5.9.3.4.3)

$$\Delta f_{pSD} := \epsilon_{bdf} \cdot E_p \cdot K_{df} = 6.82 \cdot \text{ksi}$$

Prestress loss due to  
Shrinkage of girder concrete  
(LRFD 5.9.3.4.3)

**Creep of girder concrete (d->f)**

$$P_{pLT,id} := \Delta f_{pLT,id} \cdot A_{p,total} = 63.261 \cdot \text{kip}$$

Prestress force loss between transfer to  
deck placement

$$M_{DL,nc} := M_{sf,slab} \left( \frac{\text{Span}}{2} \right) + M_{forms} \left( \frac{\text{Span}}{2} \right) = 89.332 \cdot \text{kip} \cdot \text{ft}$$

Permanent dead load moment at midspan  
(exclude self weight of beam) acting on  
non-composite section

$$M_{DL,comp} := M_{barrier} \left( \frac{\text{Span}}{2} \right) + M_{fws} \left( \frac{\text{Span}}{2} \right) + M_{utility} \left( \frac{\text{Span}}{2} \right) = 46.887 \cdot \text{kip} \cdot \text{ft}$$

Permanent dead load moment at midspan  
acting on composite section

$$\Delta f_{cd} := (-1) \left[ \frac{P_{pLT.id}}{A_{g.tr}} + \frac{P_{pLT.id} \cdot (e_{g.tr.cg.strands})^2}{I_{g.tr}} \dots \right] = -0.681 \cdot \text{ksi}$$

$$\left[ + \frac{M_{DL.nc} \cdot e_{g.tr.cg.strands}}{I_{g.tr}} + \frac{M_{DL.comp} \cdot e_{g.comp.tr.cg.strands}}{I_{g.comp.tr}} \right]$$

Change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight and superimposed loads

$$\Delta f_{pCD} := \frac{E_p}{E_{c.beam}} f_{cgp} \cdot (\Psi_b(t_f, t_i) - \Psi_b(t_d, t_i)) \cdot K_{df} + \frac{E_p}{E_{c.beam}} \Delta f_{cd} \cdot \Psi_b(t_f, t_d) \cdot K_{df} = -0.015 \cdot \text{ksi}$$

Prestress loss due to Creep of girder concrete (LRFD 5.9.3.4.3)

**Relaxation of prestressing CFRP (d->f)**

$$f_{pu} = 341 \cdot \text{ksi}$$

Design tensile strength

$$f_{pt} = 231 \cdot \text{ksi}$$

Prestress stress at transfer

$$\Delta f_{pR2} := \Delta f_{pR}(t_d, t_f) = 11.505 \cdot \text{ksi}$$

Prestress loss due to relaxation

Time from transfer to deck placement

**Shrinkage of deck concrete (d->f)**

$$k_{s.deck} := \max \left[ 1.45 - 0.13 \frac{A_{slab}}{(2b_{slab.top} + 2 \cdot t_{slab.bot}) \cdot \text{in}}, 1.0 \right] = 1.007$$

Factor for the effect of the volume-to-surface ratio of the component (LRFD 5.4.2.3.2)

$$k_{hs} = 0.950$$

Humidity factor for shrinkage (LRFD 5.4.2.3.3)

$$k_{hc} = 0.960$$

Humidity factor for creep  
(LRFD 5.4.2.3.2)

$$k_{f,deck} := \frac{5}{1 + \frac{f_{c,slab}}{\text{ksi}}} = 0.769$$

Factor for the effect of concrete strength  
(LRFD 5.4.2.3.2)

$$k_{td,deck}(t_0, t_1) := \frac{t_1 - t_0}{12 \left( \frac{100 - 4 \cdot \frac{f_{c,slab}}{\text{ksi}}}{\frac{f_{c,slab}}{\text{ksi}} + 20} \right) + (t_1 - t_0)}$$

Time development factor  
(LRFD 5.4.2.3.2)

$$k_{td,deck}(t_d, t_f) = 0.996$$

Time development factor  
Time from deck placement to final time

$$\epsilon_{ddf} := k_{s,deck} \cdot k_{hs} \cdot k_{f,deck} \cdot k_{td,deck}(t_d, t_f) \cdot 0.48 \cdot 10^{-3} = 3.520 \times 10^{-4}$$

Shrinkage strain of deck  
(LRFD 5.4.2.3.3)

$$\Psi_{b,deck}(t_1, t_0) := 1.9 \cdot k_{s,deck} \cdot k_{hc} \cdot k_{f,deck} \cdot k_{td,deck}(t_0, t_1) \cdot t_0^{-0.118}$$

Creep coefficient  
(LRFD 5.4.2.3.2)

$$\Psi_{b,deck}(t_f, t_d) = 0.8$$

Time development factor  
Time from deck placement to final time

$$e_{deck,comp} := y_{g,comp,tr,top} - y_{slab,top} = 4.638 \cdot \text{in}$$

Eccentricity of deck with respect to N.A. of transformed composite section

$$\Delta f_{cdf} := \frac{\epsilon_{ddf} \cdot A_{slab} \cdot E_{c,slab}}{1 + 0.7 \cdot \Psi_{b,deck}(t_f, t_d)} \cdot \left( \frac{1}{A_{g,comp.tr}} - \frac{e_{g.tr.cg.strands} \cdot e_{deck.comp}}{I_{g,comp.tr}} \right) = 0.249 \cdot \text{ksi}$$

Change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

$$\Delta f_{pSS} := \frac{E_p}{E_{c,beam}} \cdot \Delta f_{cdf} \cdot K_{df} \cdot (1 + 0.7 \cdot \Psi_b(t_f, t_d)) = 1.554 \cdot \text{ksi}$$

Prestress loss due to shrinkage of deck concrete

(LRFD 5.9.3.4.3)

**Total time-depedent prestress loss between deck placement and final time**

$$\Delta f_{pLT,df} := \Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS} = 16.756 \cdot \text{ksi}$$

**Total time-depedent prestress loss between deck placement and final time**

$$\Delta f_{pLT} := \Delta f_{pLT,id} + \Delta f_{pLT,df} = 36.39 \cdot \text{ksi}$$

**7.3 Summary -Prestress Loss**

$$\Delta f_p := \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pTH} = 43.902 \cdot \text{ksi}$$

Total prestress loss  
(AASHTO CFRP 1.9.2.1)

$$\text{Loss\%} := \frac{\Delta f_p}{f_{pi}} = 18.4\%$$

Total prestress loss percentage

$$\Delta f_{pES} = 7.512 \cdot \text{ksi}$$

Prestress loss due to elastic shortening

$$\Delta f_{pTH} = 0$$

Prestress loss due to temperature change (not considered)

$$\Delta f_{pLT} = 36.39 \cdot \text{ksi}$$

Total long-term time-dependent prestress loss

$$\Delta f_{pLT.id} = 19.634 \cdot \text{ksi}$$

Long-term time-dependent prestress loss between transfer and deck placement

$$\Delta f_{pLT.df} = 16.756 \cdot \text{ksi}$$

Long-term time-dependent prestress loss between deck placement and final time

$$f_{pi} = 239 \cdot \text{ksi}$$

Jacking stress

$$f_{pt} = 231 \cdot \text{ksi}$$

Prestressing stress at transfer

$$f_{pe} := f_{pi} - \Delta f_p = 195 \cdot \text{ksi}$$

Effect prestressing stress after all losses

$$\text{Check}_{pe.limit} := \begin{cases} \text{"OK"} & \text{if } f_{pe} \leq f_{pe.limit} \\ \text{"NG"} & \text{if } f_{pe} > f_{pe.limit} \end{cases}$$

Service stress limit (after all losses)

**Check<sub>pe.limit</sub> = "OK"**

(AASHTO CFRP Table 1.9.1.1)

**8. STRESS LIMITS FOR CONCRETE  
-TEMPORARY STRESSES BEFORE LOSSES (LRFD 5.9.2.3.1)**

for temporary stresses before losses (at release) (Compression +, Tension -)

$$f_{\text{compressive.allow.before.loss}} := 0.65 \cdot f_{\text{ci.beam}} = 3.900 \cdot \text{ksi}$$

Compressive stress limit for concrete  
(LRFD 5.9.2.3.1a)

*Note: assume minimum tension reinforcement satisfy SDG 4.3.1-1 and LRFD 5.9.2.3.1*

$$f_{\text{tensile.allow.15%.before.loss}} := -0.24 \cdot \left( \frac{f_{\text{ci.beam}}}{\text{ksi}} \right)^{0.5} \text{ ksi} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete in outer  
15% regions  
(FDOT SDG 4.3.1)

$$f_{\text{tensile.allow.70%.bond.before.loss}} := -0.24 \cdot \left( \frac{f_{\text{ci.beam}}}{\text{ksi}} \right)^{0.5} \text{ ksi} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete in middle  
70% regions, for bonded reinforcements  
(FDOT SDG 4.3.1, LRFD 5.9.2.3.1b)

$$f_{\text{tensile.allow.70%.unbond.before.loss}} := -\min \left[ 0.0948 \cdot \left( \frac{f_{\text{ci.beam}}}{\text{ksi}} \right)^{0.5} \text{ ksi}, 0.2 \text{ ksi} \right] = -0.2 \cdot \text{ksi}$$

Tensile stress limit for concrete in middle  
70% regions, for bonded reinforcements  
(FDOT SDG 4.3.1, LRFD 5.9.2.3.1b)

At release, loads include self-weight of beam and the prestressing force

$$P_{\text{pi}} := A_{\text{p.total}} \cdot f_{\text{pi}} = 768.6 \cdot \text{kip}$$

Total prestress force at release

**8.1 At Mid-Span Section**

midspan = 19.458 ft

$$f_{c.topbeam.midspan} := \frac{P_{pi}}{A_{g,tr}} + \frac{M_{sf.beam}(midspan) - P_{pi} \cdot e_{g,tr.cg.strands}}{S_{g,tr.top}} = 0.74 \cdot ksi$$

Concrete stress at top fiber of the beam at mid-span

$$f_{c.botbeam.midspan} := \frac{P_{pi}}{A_{g,tr}} - \frac{M_{sf.beam}(midspan) - P_{pi} \cdot e_{g,tr.cg.strands}}{S_{g,tr.bot}} = 1.784 \cdot ksi$$

Concrete stress at bottom fiber of the beam at mid-span

$$f_{compressive.allow.before.loss} = 3.900 \cdot ksi$$

$$f_{tensile.allow.70%.bond.before.loss} = -0.588 \cdot ksi$$

in the pretensioned beams, at the midspan, "bonded reinforcements"

$$Check_{top.midspan} := \begin{cases} \text{if } f_{c.topbeam.midspan} \geq 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } f_{c.topbeam.midspan} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \\ \text{if } f_{c.topbeam.midspan} < 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } -f_{c.topbeam.midspan} \leq -f_{tensile.allow.70%.bond.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \end{cases}$$

**Check<sub>top.midspan</sub> = "OK"**

Check concrete stress limits at the midspan, at release

$$Check_{bot.midspan} := \begin{cases} \text{if } f_{c.botbeam.midspan} \geq 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } f_{c.botbeam.midspan} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \\ \text{if } f_{c.botbeam.midspan} < 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } -f_{c.botbeam.midspan} \leq -f_{tensile.allow.70%.bond.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \end{cases}$$

**Check<sub>bot.midspan</sub> = "OK"**

Check concrete stress limits at the midspan, at release

### 8.2 At Support (transfer length) Section

After transfer length, the prestressing forces are transferred to concrete, so the section that needs to check is located at transfer length distance from the support

$$\text{support}_{\text{check}} := \text{support} + L_{\text{transfer}} = 2.5 \text{ ft}$$

$$f_{c,\text{topbeam.at.release2}} := \frac{P_{\text{pi}}}{A_{g,\text{tr}}} + \frac{M_{\text{sf.beam}}(\text{support}_{\text{check}}) - P_{\text{pi}} \cdot e_{g,\text{tr.cg.strands}}}{S_{g,\text{tr.top}}} = -0.194 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at support

$$f_{c,\text{botbeam.at.release2}} := \frac{P_{\text{pi}}}{A_{g,\text{tr}}} - \frac{M_{\text{sf.beam}}(\text{support}_{\text{check}}) - P_{\text{pi}} \cdot e_{g,\text{tr.cg.strands}}}{S_{g,\text{tr.bot}}} = 2.602 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the beam at support

$$f_{\text{compressive.allow.before.loss}} = 3.9 \cdot \text{ksi}$$

$$f_{\text{tensile.allow.15%.before.loss}} = -0.588 \cdot \text{ksi}$$

in the pretensioned beams, at the support, "bonded reinforcements"

$$\text{Check}_{\text{sp.top}} := \begin{cases} \text{if } f_{c,\text{topbeam.at.release2}} \geq 0 \text{ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } f_{c,\text{topbeam.at.release2}} \leq f_{\text{compressive.allow.before.loss}} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \\ \text{if } f_{c,\text{topbeam.at.release2}} < 0 \text{ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } -f_{c,\text{topbeam.at.release2}} \leq -f_{\text{tensile.allow.15%.before.loss}} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \end{cases}$$

in the pretensioned beams, at the support, "bonded reinforcements" check concrete stress limits at top fiber of beam

$$\text{Check}_{\text{sp.top}} = \text{"OK"}$$

$$\text{Check}_{\text{sp.bot}} := \begin{cases} \text{if } f_{c,\text{botbeam.at.release2}} \geq 0 \text{ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } f_{c,\text{botbeam.at.release2}} \leq f_{\text{compressive.allow.before.loss}} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \\ \text{if } f_{c,\text{botbeam.at.release2}} < 0 \text{ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } -f_{c,\text{botbeam.at.release2}} \leq -f_{\text{tensile.allow.15%.before.loss}} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \end{cases}$$

in the pretensioned beams, at the support, "bonded reinforcements" check concrete stress limits at bot fiber of beam

$$\text{Check}_{\text{sp.bot}} = \text{"OK"}$$

$$\text{Check}_{db} := \begin{cases} \text{"No Need to Debond Strands"} & \text{if } \text{Check}_{sp.top} = \text{"OK"} \wedge \text{Check}_{sp.bot} = \text{"OK"} \\ \text{"Need to Debond Strand"} & \text{otherwise} \end{cases}$$

**Check<sub>db</sub> = "No Need to Debond Strands"**

Need to debond strands?

**Debond some strands at support to satisfy concrete stress limites (if necessary)**

Adjust

$$n = \begin{pmatrix} 18 \\ 0 \end{pmatrix} \quad n_{max.debond} := n \cdot 40\% = \begin{pmatrix} 7.2 \\ 0 \end{pmatrix}$$

$$n_{dbs} := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

*Note: the number of partially debonded strands should NOT exceed 30% of the total number of strands (SDG 4.3.1); in any horizontal row, the number of debonded strands should NOT exceed 40% of the total strands in that row (LRFD 5.9.4.3.3.)*

$$n_{after.dbs} := n - n_{dbs} = \begin{pmatrix} 18 \\ 0 \end{pmatrix}$$

Number of bonded strands in each row after debonding at support dbs bottom up, 0...8

$$n_{after.dbs.total} := \sum_{i=0}^{rows(n)-1} n_{after.dbs_i} = 18$$

Remaining total number of bonded strands after debonding support

$$\text{Check}_{30\%} := \begin{cases} \text{"OK"} & \text{if } \frac{n_{total} - n_{after.dbs.total}}{n_{total}} \leq 30\% \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check if total debonded strands exceed 30% of total strands (SDG 4.3.1)

**Check<sub>30%</sub> = "OK"**

$$\text{Check}_{40\%} := \begin{cases} \text{"NG"} & \text{if for } i \in 0 .. rows(n) - 1 \\ & \left| \begin{array}{l} n_{dbs_i} > 40\% \cdot n_i \\ \text{break} \end{array} \right. \\ \text{"OK"} & \text{otherwise} \end{cases}$$

Check if debonded strands in each row exceed 40% of total strands in that row (LRFD 5.9.4.3.3)

**Check<sub>40%</sub> = "OK"**

$$P_{pi.dbs} := n_{after.dbs.total} \cdot A_{pf} \cdot f_{pi} = 768.6 \cdot \text{kip}$$

Total prestress force at release, after debonding support

$$A_{p.dbs} := n_{after.dbs} \cdot A_{pf} = \left( \frac{3.222}{0.000} \right) \cdot \text{in}^2$$

Total area of strands in each row, after debonding support

$$A_{p.total.dbs} := \sum_{i=0}^{rows(n)-1} A_{p.dbs_i} = 3.222 \cdot \text{in}^2$$

Total area of strands, after debonding support

$$d_{p.strands.c.g.dbs} := \frac{\sum_{i=0}^{rows(n)-1} (A_{p.dbs_i} \cdot d_{p_i})}{A_{p.total.dbs}} = 9 \cdot \text{in}$$

Distance from top beam section to the gravity center of strand group, after debonding support

$$d_{p.strands.c.g.bot.dbs} := h_g - d_{p.strands.c.g.dbs} = 3 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group, after debonding support

**Need to recalculate transformed section properties - Non-Composite (Girder+Strands), at support section, after debonding support**

$$A_{g.tr.dbs} := A_g + (n_p - 1) A_{p.total.dbs} = 592.706 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g.tr.top.dbs} := \frac{A_g \cdot y_{g.top} + \sum_{i=0}^{rows(n)-1} [A_{p.dbs_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g.tr.dbs}} = 6.399 \cdot \text{in}$$

Distance from top fiber (beam) to N.A after debonding support

$$y_{g.tr.bot.dbs} := h_g - y_{g.tr.top.dbs} = 5.601 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A after debonding support

$$I_{g.tr.dbs} := I_g + A_g \cdot (y_{g.tr.top.dbs} - y_{g.top})^2 \dots = 7.159 \times 10^3 \cdot \text{in}^4$$

$$+ \sum_{i=0}^{\text{rows}(n)-1} \left[ (n_p - 1) \cdot A_{p.dbs_i} \cdot (d_{p_i} - y_{g.tr.top.dbs})^2 \right]$$

Moment inertia, after debonding support

$$S_{g.tr.top.dbs} := \frac{I_{g.tr.dbs}}{y_{g.tr.top.dbs}} = 1.119 \times 10^3 \cdot \text{in}^3$$

$$S_{g.tr.bot.dbs} := \frac{I_{g.tr.dbs}}{y_{g.tr.bot.dbs}} = 1.278 \times 10^3 \cdot \text{in}^3$$

$$e_{g.tr.cg.strands.dbs} := y_{g.tr.bot.dbs} - d_{p.strands.c.g.bot.dbs} = 2.601 \cdot \text{in}$$

eccentricity of strand group  
after debonding support

**Check concrete stress limits at support section, after debonding support**

$$f_{c.topbeam.support} := \frac{P_{pi.dbs}}{A_{g.tr.dbs}} + \frac{M_{sf.beam}(\text{support}_{check}) - P_{pi.dbs} \cdot e_{g.tr.cg.strands.dbs}}{S_{g.tr.top.dbs}}$$

$$f_{c.topbeam.support} = -0.194 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at support

$$f_{c.botbeam.support} := \frac{P_{pi.dbs}}{A_{g.tr.dbs}} - \frac{M_{sf.beam}(\text{support}_{check}) - P_{pi.dbs} \cdot e_{g.tr.cg.strands.dbs}}{S_{g.tr.bot.dbs}}$$

$$f_{c.botbeam.support} = 2.602 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the beam at support

$$f_{compressive.allow.before.loss} = 3.9 \cdot \text{ksi}$$

$$f_{tensile.allow.15%.before.loss} = -0.588 \cdot \text{ksi}$$

in the pretensioned beams, at the support, "bonded reinforcements"

```

Checktop.dbs := if fc.topbeam.support ≥ 0ksi
                | "OK" if fc.topbeam.support ≤ fcompressive.allow.before.loss
                | "NG" otherwise
                | if fc.topbeam.support < 0ksi
                  | "OK" if -fc.topbeam.support ≤ -ftensile.allow.15%.before.loss
                  | "NG" otherwise
    
```

Check<sub>top.dbs</sub> = "OK"

Check concrete stress limits at the support, top fiber, at release

```

Checkbot.dbs := if fc.botbeam.support ≥ 0ksi
                | "OK" if fc.botbeam.support ≤ fcompressive.allow.before.loss
                | "NG" otherwise
                | if fc.botbeam.support < 0ksi
                  | "OK" if -fc.botbeam.support ≤ -ftensile.allow.15%.before.loss
                  | "NG" otherwise
    
```

Check<sub>bot.dbs</sub> = "OK"

Check concrete stress limits at the support, bottom fiber, at release

**8.3 At Debond1 Section (if necessary)**

$$\text{debond1} = 0$$

debond distance, defined at strand pattern

$$\frac{\text{debond1}}{\text{Span}} = 0.0\%$$

Check if this debond distance is within 15% span range to beam ends

	<b>Adjust</b>		
$n = \begin{pmatrix} 18 \\ 0 \end{pmatrix}$	$n_{\text{dbs}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$n_{\text{db1}} := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$n_{\text{after.db1}} := n - n_{\text{db1}} = \begin{pmatrix} 18 \\ 0 \end{pmatrix}$
			Number of bonded strands in each row after debond 1 bottom up, 0...8

*Note: the number of partially debonded strands should NOT exceed 30% of the total number of strands (SDG 4.3.1); in any horizontal row, the number of debonded strands should NOT exceed 40% of the total strands in that row (LRFD 5.9.4.3.3.)*

$n_{\text{after.db1.total}} := \sum_{i=0}^{\text{rows}(n)-1} n_{\text{after.db1}_i} = 18$	Remaining total number of bonded strands after debond 1
---	---

$P_{\text{pi.db1}} := n_{\text{after.db1.total}} \cdot A_{\text{pf}} \cdot f_{\text{pi}} = 768.6 \text{ kip}$	Total prestress force at release, after debond1
---	---

$A_{\text{p.db1}} := n_{\text{after.db1}} \cdot A_{\text{pf}} = \begin{pmatrix} 3.222 \\ 0.000 \end{pmatrix} \cdot \text{in}^2$	Total area of strands in each row, after debond1
---	--

$A_{\text{p.total.db1}} := \sum_{i=0}^{\text{rows}(n)-1} A_{\text{p.db1}_i} = 3.222 \cdot \text{in}^2$	Total area of strands, after debond1
--	--------------------------------------

$d_{\text{p.strands.c.g.db1}} := \frac{\sum_{i=0}^{\text{rows}(n)-1} (A_{\text{p.db1}_i} \cdot d_{\text{p}_i})}{A_{\text{p.total.db1}}} = 9 \cdot \text{in}$	Distance from top beam section to the gravity center of strand group, after debond1
--	---

$d_{\text{p.strands.c.g.bot.db1}} := h_g - d_{\text{p.strands.c.g.db1}} = 3 \cdot \text{in}$	Distance from bottom beam section to the gravity center of strand group, after debond1
--	--

**Need to recalculate transformed section properties - Non-Composite (Girder+Strands), in debond 1 region**

$$A_{g, \text{tr.db1}} := A_g + (n_p - 1)A_{p, \text{total.db1}} = 592.706 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g, \text{tr.top.db1}} := \frac{A_g \cdot y_{g, \text{top}} + \sum_{i=0}^{\text{rows}(n)-1} \left[ A_{p, \text{db1}_i} \cdot (n_p - 1) \cdot d_{p_i} \right]}{A_{g, \text{tr.db1}}} = 0.533 \text{ ft}$$

Distance from top fiber (beam) to N.A  
after debond1

$$y_{g, \text{tr.bot.db1}} := h_g - y_{g, \text{tr.top.db1}} = 0.467 \text{ ft}$$

Distance from bottom fiber (beam) to N.A  
after debond1

$$I_{g, \text{tr.db1}} := I_g + A_g \cdot (y_{g, \text{tr.top.db1}} - y_{g, \text{top}})^2 \dots = 7.159 \times 10^3 \cdot \text{in}^4$$

$$+ \sum_{i=0}^{\text{rows}(n)-1} \left[ (n_p - 1) \cdot A_{p, \text{db1}_i} \cdot (d_{p_i} - y_{g, \text{tr.top.db1}})^2 \right]$$

Moment inertia  
after debond1

$$S_{g, \text{tr.top.db1}} := \frac{I_{g, \text{tr.db1}}}{y_{g, \text{tr.top.db1}}} = 1.119 \times 10^3 \cdot \text{in}^3$$

$$S_{g, \text{tr.bot.db1}} := \frac{I_{g, \text{tr.db1}}}{y_{g, \text{tr.bot.db1}}} = 1.278 \times 10^3 \cdot \text{in}^3$$

$$e_{g, \text{tr.cg.strands.db1}} := y_{g, \text{tr.bot.db1}} - d_{p, \text{strands.c.g.bot.db1}} = 2.601 \cdot \text{in}$$

eccentricity of strand group  
after debond1

**Check concrete stress limits at debond1 section, after debond1**

$$f_{c, \text{topbeam.debond1}} := \frac{P_{p_i, \text{db1}}}{A_{g, \text{tr.db1}}} + \frac{M_{\text{sf.beam}}(\text{debond1}) - P_{p_i, \text{db1}} \cdot e_{g, \text{tr.cg.strands.db1}}}{S_{g, \text{tr.top.db1}}} = -0.49 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at  
debond1 section

$$f_{c.botbeam.debond1} := \frac{P_{pi.db1}}{A_{g.tr.db1}} - \frac{M_{sf.beam}(debond1) - P_{pi.db1} \cdot e_{g.tr.cg.strands.db1}}{S_{g.tr.bot.db1}} = 2.861 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the beam at debond1 section

$$f_{compressive.allow.before.loss} = 3.9 \cdot \text{ksi}$$

$$f_{tensile.allow.debond1} := \begin{cases} f_{tensile.allow.15\%.before.loss} & \text{if } \frac{debond1}{Span} \leq 15\% \\ f_{tensile.allow.70\%.bond.before.loss} & \text{if } \frac{debond1}{Span} > 15\% \end{cases} = -0.588 \cdot \text{ksi}$$

in the pretensioned beams, at the debond1 section, "bonded reinforcements"

$$Check_{top.db1} := \begin{cases} \text{if } f_{c.topbeam.debond1} \geq 0 \text{ksi} \\ \quad \begin{cases} \text{"OK"} & \text{if } f_{c.topbeam.debond1} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} & \text{otherwise} \end{cases} \\ \text{if } f_{c.topbeam.debond1} < 0 \text{ksi} \\ \quad \begin{cases} \text{"OK"} & \text{if } -f_{c.topbeam.debond1} \leq -f_{tensile.allow.debond1} \\ \text{"NG"} & \text{otherwise} \end{cases} \end{cases}$$

**Check<sub>top.db1</sub> = "OK"**

Check concrete stress limits at the debond1 section, top fiber, at release

$$Check_{bot.db1} := \begin{cases} \text{if } f_{c.botbeam.debond1} \geq 0 \text{ksi} \\ \quad \begin{cases} \text{"OK"} & \text{if } f_{c.botbeam.debond1} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} & \text{otherwise} \end{cases} \\ \text{if } f_{c.botbeam.debond1} < 0 \text{ksi} \\ \quad \begin{cases} \text{"OK"} & \text{if } -f_{c.botbeam.debond1} \leq -f_{tensile.allow.debond1} \\ \text{"NG"} & \text{otherwise} \end{cases} \end{cases}$$

**Check<sub>bot.db1</sub> = "OK"**

Check concrete stress limits at the debond1 section, bottom fiber, at release

**8.4 At Debond2 Section (if necessary)**

$$\text{debond2} = 0$$

debond distance, defined at strand pattern

$$\frac{\text{debond2}}{\text{Span}} = 0.0\%$$

$$n = \begin{pmatrix} 18 \\ 0 \end{pmatrix} \quad n_{\text{dbs}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad n_{\text{db1}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Adjust

$$n_{\text{db2}} := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

make all 0,  
means no debonding after  
debond2

*Note: the number of partially debonded strands should NOT exceed 30% of the total number of strands (SDG 4.3.1); in any horizontal row, the number of debonded strands should NOT exceed 40% of the total strands in that row (LRFD 5.9.4.3.3.)*

$$n_{\text{after.db2}} := n - n_{\text{db2}} = \begin{pmatrix} 18 \\ 0 \end{pmatrix}$$

Number of bonded strands in each row  
after debond 2  
bottom up, 0...8

$$n_{\text{after.db2.total}} := \sum_{i=0}^{\text{rows}(n)-1} n_{\text{after.db2}_i} = 18$$

Remaining total number of bonded strands  
after debond 2

$$P_{\text{pi.db2}} := n_{\text{after.db2.total}} \cdot A_{\text{pf}} \cdot f_{\text{pi}} = 768.6 \cdot \text{kip}$$

Total prestress force at release, after  
debond2

$$A_{\text{p.db2}} := n_{\text{after.db2}} \cdot A_{\text{pf}} = \begin{pmatrix} 3.222 \\ 0.000 \end{pmatrix} \cdot \text{in}^2$$

Total area of strands in each row, after  
debond2

$$A_{\text{p.total.db2}} := \sum_{i=0}^{\text{rows}(n)-1} A_{\text{p.db2}_i} = 3.222 \cdot \text{in}^2$$

Total area of strands, after debond2

$$d_{\text{p.strands.c.g.db2}} := \frac{\sum_{i=0}^{\text{rows}(n)-1} (A_{\text{p.db2}_i} \cdot d_{\text{p}_i})}{A_{\text{p.total.db2}}} = 9 \cdot \text{in}$$

Distance from top beam section to the  
gravity center of strand group, after  
debond2

$$d_{p.strands.c.g.bot.db2} := h_g - d_{p.strands.c.g.db2} = 3 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group, after debond2

**Need to recalculate transformed section properties - Non-Composite (Girder+Strands), in debond 2 region**

$$A_{g.tr.db2} := A_g + (n_p - 1)A_{p.total.db2} = 592.706 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g.tr.top.db2} := \frac{A_g \cdot y_{g.top} + \sum_{i=0}^{rows(n)-1} [A_{p.db2_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g.tr.db2}} = 6.399 \cdot \text{in}$$

Distance from top fiber (beam) to N.A after debond2

$$y_{g.tr.bot.db2} := h_g - y_{g.tr.top.db2} = 5.601 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A after debond2

$$I_{g.tr.db2} := I_g + A_g \cdot (y_{g.tr.top.db2} - y_{g.top})^2 + \sum_{i=0}^{rows(n)-1} [(n_p - 1) \cdot A_{p.db2_i} \cdot (d_{p_i} - y_{g.tr.top.db2})^2] \dots = 7.159 \times 10^3 \cdot \text{in}^4$$

Moment inertia after debond2

$$S_{g.tr.top.db2} := \frac{I_{g.tr.db2}}{y_{g.tr.top.db2}} = 1.119 \times 10^3 \cdot \text{in}^3$$

$$S_{g.tr.bot.db2} := \frac{I_{g.tr.db2}}{y_{g.tr.bot.db2}} = 1.278 \times 10^3 \cdot \text{in}^3$$

$$e_{g.tr.cg.strands.db2} := y_{g.tr.bot.db2} - d_{p.strands.c.g.bot.db2} = 2.601 \cdot \text{in}$$

eccentricity of strand group after debond2

**Check concrete stress limits at debond2 section, after debond2**

$$f_{c,topbeam,debond2} := \frac{P_{pi,db2}}{A_{g,tr,db2}} + \frac{M_{sf,beam}(debond2) - P_{pi,db2} \cdot e_{g,tr,cg,strands,db2}}{S_{g,tr,top,db2}} = -0.49 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at debond2 section

$$f_{c,botbeam,debond2} := \frac{P_{pi,db2}}{A_{g,tr,db2}} - \frac{M_{sf,beam}(debond2) - P_{pi,db2} \cdot e_{g,tr,cg,strands,db2}}{S_{g,tr,bot,db2}} = 2.861 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the beam at debond2 section

$$f_{c,compressive,allow,before,loss} = 3.9 \cdot \text{ksi}$$

$$f_{tensile,allow,debond2} := \begin{cases} f_{tensile,allow,15\%,before,loss} & \text{if } \frac{debond2}{Span} \leq 15\% \\ f_{tensile,allow,70\%,bond,before,loss} & \text{if } \frac{debond2}{Span} > 15\% \end{cases} = -0.588 \cdot \text{ksi}$$

in the pretensioned beams, at the debond2 section, "bonded reinforcements"

$$Check_{top,db2} := \begin{cases} \text{if } f_{c,topbeam,debond2} \geq 0 \text{ksi} \\ \quad \begin{cases} \text{"OK"} & \text{if } f_{c,topbeam,debond2} \leq f_{c,compressive,allow,before,loss} \\ \text{"NG"} & \text{otherwise} \end{cases} \\ \text{if } f_{c,topbeam,debond2} < 0 \text{ksi} \\ \quad \begin{cases} \text{"OK"} & \text{if } -f_{c,topbeam,debond2} \leq -f_{tensile,allow,debond2} \\ \text{"NG"} & \text{otherwise} \end{cases} \end{cases}$$

**Check<sub>top,db2</sub> = "OK"**

Check concrete stress limits at the debond2 section, top fiber, at release

```
Checkbot.db2 := if fc.botbeam.debond2 ≥ 0ksi  
                | "OK" if fc.botbeam.debond2 ≤ fcompressive.allow.before.loss  
                | "NG" otherwise  
                if fc.botbeam.debond2 < 0ksi  
                | "OK" if -fc.botbeam.debond2 ≤ -ftensile.allow.debond2  
                | "NG" otherwise
```

Check<sub>bot.db2</sub> = "OK"

Check concrete stress limits at the debond2 section, bottom fiber, at release

**8.5 Summary**

at midspan midspan = 19.458 ft

Check<sub>top.midspan</sub> = "OK"

Check<sub>bot.midspan</sub> = "OK"

at support L<sub>transfer</sub> = 2.5 ft

Check<sub>sp.top</sub> = "OK"

Check<sub>sp.bot</sub> = "OK"

Check<sub>db</sub> = "No Need to Debond Strands" Need to debond strands?

Check<sub>top.dbs</sub> = "OK"

Check<sub>bot.dbs</sub> = "OK"

at debond1 (if necessary) debond1 = 0

Check<sub>top.db1</sub> = "OK"

Check<sub>bot.db1</sub> = "OK"

at debond2 (if necessary) debond2 = 0

Check<sub>top.db2</sub> = "OK"

Check<sub>bot.db2</sub> = "OK"

**Strand pattern**

without debond	debond number at support	debond number at debond1	debond number at debond2
$n = \begin{pmatrix} 18 \\ 0 \end{pmatrix}$	$n_{dbs} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$n_{db1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$n_{db2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

**9. STRESS LIMITS FOR CONCRETE  
STRESSES AT SERVICE LIMIT STATE (AFTER ALL LOSSES)  
(LRFD 5.9.2.3.2)**

for stresses after all losses

(Compression +, Tension -)

$$P_{pe} := A_{p.total} \cdot f_{pe} = 627.147 \cdot \text{kip}$$

Total prestress force, at service, after all losses

$$f_{compressive.allow.after.loss1.deck} := 0.45 \cdot f_{c.slabs} = 2.475 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$$f_{compressive.allow.after.loss1.beam} := 0.45 \cdot f_{c.beam} = 3.825 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$$f_{compressive.allow.after.loss2.deck} := 0.60 \cdot f_{c.slabs} = 3.3 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$f_{compressive.allow.after.loss2.beam} := 0.60 \cdot f_{c.beam} = 5.1 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$f_{tensile.allow.after.loss.beam} := \begin{cases} \left[ -\min \left[ 0.24 \cdot \left( \frac{f_{ci.beam}}{\text{ksi}} \right)^{0.5} \text{ ksi}, 0.6 \text{ ksi} \right] \right] & \text{if Environment = "Extremely"} \\ \left[ -\min \left[ 0.0948 \cdot \left( \frac{f_{ci.beam}}{\text{ksi}} \right)^{0.5} \text{ ksi}, 0.3 \text{ ksi} \right] \right] & \text{otherwise} \end{cases}$$

$$f_{tensile.allow.after.loss.beam} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete, after all losses (LRFD 5.9.2.3.2b) (Service III)

**9.1 Service I Limit State: under Effective Prestress and Permanent Loads**

Critical section: mid span

for deck, check compression limit

Note : slab under load of utility, wearing surface, and barriers (due to construction stages)

$f_{\text{compressive.allow.after.loss1.deck}} = 2.475 \cdot \text{ksi}$       Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$M_{\text{permanent2}} := M_{\text{utility}}(\text{midspan}) + M_{\text{fws}}(\text{midspan}) + M_{\text{barrier}}(\text{midspan}) = 46.887 \cdot \text{kip} \cdot \text{ft}$

$f_{\text{c.deck.top1}} := \frac{M_{\text{permanent2}}}{S_{\text{g.comp.tr.top}}} = 0.199 \cdot \text{ksi}$       Concrete stress at the top fiber of deck

$\text{Check}_{\text{topdeck1}} := \begin{cases} \text{"OK"} & \text{if } f_{\text{c.deck.top1}} \leq f_{\text{compressive.allow.after.loss1.deck}} \\ \text{"NG"} & \text{otherwise} \end{cases}$

$\text{Check}_{\text{topdeck1}} = \text{"OK"}$       Concrete stress at the top fiber of deck

for beam, check compression limit

$f_{\text{compressive.allow.after.loss1.beam}} = 3.825 \cdot \text{ksi}$       Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$M_{\text{permanent1}} := M_{\text{sf.beam}}(\text{midspan}) + M_{\text{sf.slabs}}(\text{midspan}) + M_{\text{forms}}(\text{midspan}) = 204.056 \cdot \text{kip} \cdot \text{ft}$

$$f_{c.beam.top1} := \left( \frac{P_{pe}}{A_{g.tr}} - \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.top}} \right) + \frac{M_{permanent1}}{S_{g.tr.top}} + \frac{M_{permanent2}}{S_{g.comp.tr.topbeam}}$$

M.permanent1 (self weight of beam, slab, and forms) was applied to non-composite section; M.permanent2 (utility, wearing surface, and barriers) was applied to composite section

$$f_{c.beam.top1} = 1.858 \cdot \text{ksi}$$

Concrete stress at the top fiber of beam

$$\text{Check}_{topbeam1} := \begin{cases} \text{"OK"} & \text{if } f_{c.beam.top1} \leq f_{compressive.allow.after.loss1.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{topbeam1} = \text{"OK"}$$

Concrete stress at the top fiber of beam

## 9.2 Service I Limit State:under Effective Prestress, Permanent Loads, and Transient Loads

*Note: loading during shipping and handling is not included in this example*

Critical section: mid span

for deck, check compression limit

$$f_{compressive.allow.after.loss2.deck} = 3.3 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$M_{LL.IM.mid} = 245.035 \cdot \text{kip} \cdot \text{ft}$$

Moment due to Live Loads (truck and lane)

$$f_{c.deck.top2} := f_{c.deck.top1} + \frac{M_{LL.IM.mid}}{S_{g.comp.tr.top}} = 1.242 \cdot \text{ksi}$$

Concrete stress at the top fiber of deck

$$\text{Check}_{topdeck2} := \begin{cases} \text{"OK"} & \text{if } f_{c.deck.top2} \leq f_{compressive.allow.after.loss2.deck} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{topdeck2} = \text{"OK"}$$

Concrete stress at the top fiber of deck

for beam, check compression limit

$$f_{c.compressive.allow.after.loss2.beam} = 5.1 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$f_{c.beam.top2} := f_{c.beam.top1} + \frac{M_{LL.IM.mid}}{S_{g.comp.tr.topbeam}} = 2.222 \cdot \text{ksi}$$

Concrete stress at the top fiber of beam

$$\text{Check}_{topbeam2} := \begin{cases} \text{"OK"} & \text{if } f_{c.beam.top2} \leq f_{c.compressive.allow.after.loss2.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

**Check<sub>topbeam2</sub> = "OK"**

Concrete stress at the top fiber of beam

### 9.3 Service III Limit State

*Note: loading during shipping and handling is not included in this example*

for beam, check tensile limit at the bottom fiber

Critical section: mid span

$$f_{tensile.allow.after.loss.beam} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete, after all losses (LRFD 5.9.2.3.2b) (Service III)

$$f_{c.beam.bot} := \frac{P_{pe}}{A_{g.tr}} + \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.bot}} + \frac{-M_{permanent1}}{S_{g.tr.bot}} + \frac{-M_{permanent2}}{S_{g.comp.tr.bot}} + (0.8) \frac{-M_{LL.IM.mid}}{S_{g.comp.tr.bot}} = -0.567 \cdot \text{ksi}$$

Concrete stress at the bottom fiber of beam

$$\text{Check}_{\text{botbeam}} := \begin{cases} \text{"OK"} & \text{if } -f_{c.\text{beam.bot}} \leq -f_{\text{tensile.allow.after.loss.beam}} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

**Check<sub>botbeam</sub> = "OK"**

Concrete stress at the bottom fiber of beam

**9.4 Summary**

**Check<sub>topdeck1</sub> = "OK"**       $\frac{f_{c.\text{deck.top1}}}{f_{\text{compressive.allow.after.loss1.deck}}} = 0.081$       Service I

**Check<sub>topbeam1</sub> = "OK"**       $\frac{f_{c.\text{beam.top1}}}{f_{\text{compressive.allow.after.loss1.beam}}} = 0.486$       Service I

**Check<sub>topdeck2</sub> = "OK"**       $\frac{f_{c.\text{deck.top2}}}{f_{\text{compressive.allow.after.loss2.deck}}} = 0.376$       Service I

**Check<sub>topbeam2</sub> = "OK"**       $\frac{f_{c.\text{beam.top2}}}{f_{\text{compressive.allow.after.loss2.beam}}} = 0.436$       Service I

**Check<sub>botbeam</sub> = "OK"**       $\frac{-f_{c.\text{beam.bot}}}{-f_{\text{tensile.allow.after.loss.beam}}} = 0.964$       Service III

## 10. FATIGUE LIMIT STATE

According to AASHTO CFRP 1.5.2,

In regions of compressive stress due to unfactored loads and prestress in reinforced concrete components, fatigue shall be checked only if this stress compressive strength is less than the maximum tensile live load stress resulting from the Fatigue I load combination.

Fatigue of the reinforcement need not be checked for prestressed components designed to have extreme fiber tensile stress due to the Service III Limit State within the stress limit specified in LRFD Table 5.9.2.3.2b-1

at midspan

$$f_{c.topbeam} := \frac{P_{pe}}{A_{g.tr}} - \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.top}} + \frac{M_{permanent1}}{S_{g.tr.top}} + \frac{M_{permanent2}}{S_{g.comp.tr.topbeam}} = 1.858 \cdot ksi$$

compressive stress at top fiber of the beam due to unfactored loads and prestress

$$f_{t.bot.fatigue} := \frac{M_{fatigue}(midspan)}{S_{g.comp.tr.bot}} = 0.403 \cdot ksi$$

$$Check_{fatigue} := \begin{cases} \text{"Need to Check Fatigue"} & \text{if } \frac{f_{c.topbeam}}{f_{t.bot.fatigue}} < 1 \\ \text{"No Need to Check Fatigue"} & \text{otherwise} \end{cases}$$

$$Check_{fatigue} = \text{"No Need to Check Fatigue"}$$

Fatigue of the reinforcement need not be checked since the prestressed beam is designed to have extreme fiber tensile stress due to the Service III Limit State within the stress limit specified in LRFD Table 5.9.2.3.2b-1 (shown in section 9.3 of this example)

## 11. STRENGTH LIMIT STATE -MOMENT CAPACITY

*Note: for strength calculations, deck reinforcement is conservatively ignored*

$$P_{pe} = 627.147 \cdot \text{kip}$$

Total prestress force, at service, after all losses

$$M_u := M_{Str1.mid} = 749.92 \cdot \text{kip} \cdot \text{ft}$$

Strength I limit state moment at midspan

$$\phi_{\text{moment}} = 0.75$$

Reduction factor for CFRP prestressed girder - Moment

$$\alpha_{1.beam} := \min \left( \max \left( 0.85 - 0.02 \cdot \frac{f_{c.beam} - 10\text{ksi}}{\text{ksi}}, 0.75 \right), 0.85 \right) = 0.85$$

$$\beta_{1.beam} := \min \left( \max \left( 0.85 - 0.05 \cdot \frac{f_{c.beam} - 4\text{ksi}}{\text{ksi}}, 0.65 \right), 0.85 \right) = 0.65$$

$$\beta_1 := \beta_{1.beam} = 0.65$$

$$\alpha_{1.slab} := \min \left( \max \left( 0.85 - 0.02 \cdot \frac{f_{c.slab} - 10\text{ksi}}{\text{ksi}}, 0.75 \right), 0.85 \right) = 0.85$$

$$\beta_{1.slab} := \min \left( \max \left( 0.85 - 0.05 \cdot \frac{f_{c.slab} - 4\text{ksi}}{\text{ksi}}, 0.65 \right), 0.85 \right) = 0.775$$

Concrete stress block factor  
(LRFD 5.6.2.2)

$$\epsilon_{pu} = 0.015$$

Rupture strain of CFRP strand

$$\epsilon_{pe} := \frac{f_{pe}}{E_p} = 8.659 \times 10^{-3}$$

Effective prestress strain of CFRP

$$\epsilon_{cu} := 0.003$$

Concrete crushing strain

$$A_p = \begin{pmatrix} 3.222 \\ 0 \end{pmatrix} \cdot \text{in}^2$$

Area of strands in each row  
bottom up, 0... rows(n) - 1

$$d_{pf} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 & = \begin{pmatrix} 15 \\ 13 \end{pmatrix} \cdot \text{in} \\ d_{pf_i} \leftarrow d_{p_i} + t_{\text{slab.top}} \\ d_{pf} \end{cases}$$

Distance from top slab section (composite section) to the center of each strand row bottom up, 0... rows(n) - 1

### 11.1 Compression-Controlled Section

$$cc_C = 5.588 \cdot \text{in}$$

Concrete compression depth  
This number is defined later to satisfy C=T

$$aa_C := \beta_1 \cdot cc_C = 3.632 \cdot \text{in}$$

$$\varepsilon_{cc.C} := \varepsilon_{cu} = 0.003$$

$$\varepsilon_{f.C} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ \varepsilon_{f.C_i} \leftarrow \varepsilon_{pe} + \varepsilon_{cc.C} \cdot \left( \frac{d_{pf_i} - cc_C}{cc_C} \right) \\ \varepsilon_{f.C} \end{cases}$$

$$\varepsilon_{f.C} = \begin{pmatrix} 0.0137 \\ 0.0126 \end{pmatrix}$$

CFRP strain in each row  
bottom up, 0... rows(n) - 1

$$f_{f.C} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ f_{f.C_i} \leftarrow \varepsilon_{f.C_i} \cdot E_p \\ f_{f.C} \end{cases} = \begin{pmatrix} 308 \\ 284 \end{pmatrix} \cdot \text{ksi}$$

$$f_{f.C} = \begin{pmatrix} 308 \\ 284 \end{pmatrix} \cdot \text{ksi}$$

CFRP stress in each row  
bottom up, 0... rows(n) - 1

$$\text{Check}_{fp.C} := \begin{cases} \text{"CFRP Rupture Before Concrete Crushing, Assumption Not Satisfied"} & \text{if } f_{f.C_0} > f_{pu} \\ \text{"Compression Controlled Section, Assumption Satisfied"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{fp.C} = \text{"Compression Controlled Section, Assumption Satisfied"}$$

$$T_{f,C} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 & = \begin{pmatrix} 993 \\ 0 \end{pmatrix} \cdot \text{kip} \\ T_{f,C_i} \leftarrow f_{f,C_i} \cdot A_{p_i} \\ T_{f,C} \end{cases}$$

$$T_{f,C} = \begin{pmatrix} 993 \\ 0 \end{pmatrix} \cdot \text{kip}$$

CFRP tension force in each row  
bottom up, 0... rows(n) - 1

$$T_{CFRP,C} := \sum_{i=0}^{\text{rows}(n)-1} T_{f,C_i} = 993.136 \cdot \text{kip}$$

Total tension force in all CFRP strands

Define different depth of compression zone

$$tt1 := t_{\text{slab.top}} = 6 \cdot \text{in}$$

$$tt2 := tt1 + t_{\text{slab.bot}} = 14 \cdot \text{in}$$

$$tt3 := tt2 + h_{\text{beam.bot}} = 18 \cdot \text{in}$$

$$C_{\text{concrete},C} := \begin{cases} \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot aa_C \cdot b_{\text{slab.top}} & \text{if } aa_C \leq tt1 \\ \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot tt1 \cdot b_{\text{slab.top}} \dots & \text{if } tt1 < aa_C \leq tt2 \\ + \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (aa_C - tt1) \cdot b_{\text{slab.bot}} \dots \\ + \alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_C - tt1) \cdot b_{\text{beam.top}} \\ \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot tt1 \cdot b_{\text{slab.top}} \dots & \text{if } tt2 < aa_C \leq tt3 \\ + \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (tt2 - tt1) \cdot b_{\text{slab.bot}} \dots \\ + \alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (tt2 - tt1) \cdot b_{\text{beam.top}} \dots \\ + \alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_C - tt2) \cdot b_{\text{beam.bot}} \end{cases}$$

Compression force from concrete

$$C_{\text{concrete},C} = 993.361 \cdot \text{kip}$$

$$M_{n,C} := \begin{cases} \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } aa_C \leq tt1 \\ + (-1) \cdot \left[ (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot aa_C \cdot b_{\text{slab,top}}) \cdot \left( \frac{aa_C}{2} \right) \right] \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } tt1 < aa_C \leq tt2 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot tt1 \cdot b_{\text{slab,top}}) \cdot \left( \frac{tt1}{2} \right) \dots \\ + (-1) \cdot \left[ \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (aa_C - tt1) \cdot b_{\text{slab,bot}} \right] \cdot \left( tt1 + \frac{aa_C - tt1}{2} \right) \dots \\ + (-1) \cdot \left[ \alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_C - tt1) \cdot b_{\text{beam,top}} \right] \cdot \left( tt1 + \frac{aa_C - tt1}{2} \right) \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } tt2 < aa_C \leq tt3 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot tt1 \cdot b_{\text{slab,top}}) \cdot \left( \frac{tt1}{2} \right) \dots \\ + (-1) \cdot \left[ \alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (tt2 - tt1) \cdot b_{\text{slab,bot}} \right] \cdot \left( tt1 + \frac{tt2 - tt1}{2} \right) \dots \\ + (-1) \cdot \left[ \alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (tt2 - tt1) \cdot b_{\text{beam,top}} \right] \cdot \left( tt1 + \frac{tt2 - tt1}{2} \right) \dots \\ + (-1) \cdot \left[ \alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_C - tt2) \cdot b_{\text{beam,bot}} \right] \cdot \left( tt2 + \frac{aa_C - tt2}{2} \right) \end{cases}$$

$$M_{n,C} = 1.091 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment capacity (with respect to top fiber of the composite section)

$$\text{Check}_{CT,C} := C_{\text{concrete},C} - T_{\text{CFRP},C} = 0 \cdot \text{kip}$$

Tension = Compression

$$\text{Check}_{fp,C} = \text{"Compression Controlled Section, Assumption Satisfied"}$$

$$cc_C \equiv 5.588 \text{ in}$$

Adjust cc until C-T~0

$$\text{SectionControl} := \begin{cases} \text{"Tension Controlled Section"} & \text{if } f_{f,C_0} > f_{pu} \\ \text{"Compression Controlled Section"} & \text{otherwise} \end{cases}$$

$$\text{SectionControl} = \text{"Compression Controlled Section"}$$

**11.2 Tension-Controlled Section**

$$cc_T = 6.178 \cdot \text{in}$$

Concrete compression depth  
This number is defined later to satisfy C=T

$$aa_T := \beta_1 \cdot cc_T = 4.016 \cdot \text{in}$$

$$\epsilon_{pf.T} := \epsilon_{pu} = 0.0152$$

CFRP strain in bottom row is rupture strain

$$\epsilon_{cc.T} := (\epsilon_{pu} - \epsilon_{pe}) \cdot \left( \frac{cc_T}{d_{pf_0} - cc_T} \right) = 4.552 \times 10^{-3}$$

Concrete strain at the top fiber of the section (deck)

$$\text{Check}_{cc.T} := \begin{cases} \text{"OK"} & \text{if } \epsilon_{cc.T} \leq \epsilon_{cu} \\ \text{"NG, Concrete Strain Exceeds } \epsilon_{cu}. \text{ Compression-Controlled Section"} & \text{otherwise} \end{cases}$$

**Check<sub>cc.T</sub> = "NG, Concrete Strain Exceeds ε<sub>cu</sub>. Compression-Controlled Section"**

$$\epsilon_{f.T} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ \epsilon_{f.T_i} \leftarrow \epsilon_{pe} + \epsilon_{cc.T} \cdot \left( \frac{d_{pf_i} - cc_T}{cc_T} \right) \\ \epsilon_{f.T} \end{cases}$$

$$\epsilon_{f.T} = \begin{pmatrix} 0.0152 \\ 0.0137 \end{pmatrix}$$

CFRP strain in each row  
bottom up, 0... rows(n) - 1

$$f_{f.T} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ f_{f.T_i} \leftarrow \epsilon_{f.T_i} \cdot E_p \\ f_{f.T} \end{cases} = \begin{pmatrix} 341 \\ 308 \end{pmatrix} \cdot \text{ksi}$$

$$f_{f.T} = \begin{pmatrix} 341 \\ 308 \end{pmatrix} \cdot \text{ksi}$$

CFRP stress in each row  
bottom up, 0... rows(n) - 1

$$\text{Check}_{f_p.T} := \begin{cases} \text{"CFRP Rupture, Assumption Not Satisfied"} & \text{if } f_{f.T_0} > f_{pu} \\ \text{"OK"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{f_p.T} = \text{"OK"}$$

CFRP not rupture

$$T_{f.T} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ T_{f.T_i} \leftarrow f_{f.T_i} \cdot A_{p_i} \\ T_{f.T} \end{cases}$$

$$T_{f.T} = \begin{pmatrix} 1098 \\ 0 \end{pmatrix} \cdot \text{kip}$$

CFRP tension force in each row  
bottom up, 0... rows(n) - 1

$$T_{CFRP.T} := \sum_{i=0}^{\text{rows}(n)-1} T_{f.T_i} = 1.098 \times 10^3 \cdot \text{kip}$$

Total tension force in all CFRP strands

$$C_{\text{concrete}.T} := \begin{cases} \alpha_1 \cdot \text{slab} \cdot f_{c.\text{slab}} \cdot aa_T \cdot b_{\text{slab.top}} & \text{if } aa_T \leq tt1 \\ \alpha_1 \cdot \text{slab} \cdot f_{c.\text{slab}} \cdot tt1 \cdot b_{\text{slab.top}} \dots & \text{if } tt1 < aa_T \leq tt2 \\ + \alpha_1 \cdot \text{slab} \cdot f_{c.\text{slab}} \cdot (aa_T - tt1) \cdot b_{\text{slab.bot}} \dots \\ + \alpha_1 \cdot \text{beam} \cdot f_{c.\text{beam}} \cdot (aa_T - tt1) \cdot b_{\text{beam.top}} \\ \alpha_1 \cdot \text{slab} \cdot f_{c.\text{slab}} \cdot tt1 \cdot b_{\text{slab.top}} \dots & \text{if } tt2 < aa_T \leq tt3 \\ + \alpha_1 \cdot \text{slab} \cdot f_{c.\text{slab}} \cdot (tt2 - tt1) \cdot b_{\text{slab.bot}} \dots \\ + \alpha_1 \cdot \text{beam} \cdot f_{c.\text{beam}} \cdot (tt2 - tt1) \cdot b_{\text{beam.top}} \dots \\ + \alpha_1 \cdot \text{beam} \cdot f_{c.\text{beam}} \cdot (aa_T - tt2) \cdot b_{\text{beam.bot}} \end{cases}$$

Compression force from concrete

$$C_{\text{concrete}.T} = 1.098 \times 10^3 \cdot \text{kip}$$

$$M_{n,T} := \begin{cases} \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } aa_T \leq tt1 \\ + (-1) \cdot \left[ (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot aa_T \cdot b_{\text{slab.top}}) \cdot \left( \frac{aa_T}{2} \right) \right] \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } tt1 < aa_T \leq tt2 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot tt1 \cdot b_{\text{slab.top}}) \cdot \left( \frac{tt1}{2} \right) \dots \\ + (-1) \cdot [\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (aa_T - tt1) \cdot b_{\text{slab.bot}}] \cdot \left( tt1 + \frac{aa_T - tt1}{2} \right) \dots \\ + (-1) \cdot [\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_T - tt1) \cdot b_{\text{beam.top}}] \cdot \left( tt1 + \frac{aa_T - tt1}{2} \right) \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } tt2 < aa_T \leq tt3 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot tt1 \cdot b_{\text{slab.top}}) \cdot \left( \frac{tt1}{2} \right) \dots \\ + (-1) \cdot [\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (tt2 - tt1) \cdot b_{\text{slab.bot}}] \cdot \left( tt1 + \frac{tt2 - tt1}{2} \right) \dots \\ + (-1) \cdot [\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (tt2 - tt1) \cdot b_{\text{beam.top}}] \cdot \left( tt1 + \frac{tt2 - tt1}{2} \right) \dots \\ + (-1) \cdot [\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_T - tt2) \cdot b_{\text{beam.bot}}] \cdot \left( tt2 + \frac{aa_T - tt2}{2} \right) \end{cases}$$

$$M_{n,T} = 1.189 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment capacity (with respect to top fiber of the composite section)

$$\text{Check}_{CT,T} := C_{\text{concrete},T} - T_{\text{CFRP},T} = 0 \cdot \text{kip}$$

Equilibrium check

$$\text{Check}_{fp,T} = \text{"OK"}$$

CFRP strain limits check

$$\text{Check}_{cc,T} = \text{"NG, Concrete Strain Exceeds } \epsilon_{cu} \cdot \text{Compression-Controlled Section"}$$

Concrete strain limits check

$$cc_T = 6.178 \text{ in}$$

Adjust cc until C-T=0

**11.3 Summary**

SectionControl = "Compression Controlled Section"

$$cc := \begin{cases} cc_T & \text{if SectionControl = "Tension Controlled Section"} \\ cc_C & \text{if SectionControl = "Compression Controlled Section"} \end{cases} = 5.588 \cdot \text{in}$$

Compression Depth

$$M_n := \begin{cases} M_{n.T} & \text{if SectionControl = "Tension Controlled Section"} \\ M_{n.C} & \text{if SectionControl = "Compression Controlled Section"} \end{cases} = 1.091 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment capacity

$$M_r := \phi_{\text{moment}} \cdot M_n = 818 \cdot \text{kip} \cdot \text{ft}$$

$$M_u = 750 \cdot \text{kip} \cdot \text{ft}$$

$$\text{Check}_M := \begin{cases} \text{"OK"} & \text{if } M_r \geq M_u \\ \text{"NG"} & \text{if } M_r < M_u \end{cases}$$

$$\frac{\phi_{\text{moment}} \cdot M_n}{M_u} = 1.091$$

Check<sub>M</sub> = "OK"

Strength I, moment capacity

## 12. MINIMUM REINFORCEMENT -PRESTRESSING CFRP

For tension-controlled beams, the minimum reinforcement requirements ensure the factored moment capacity provided is at least equal to the lesser of the cracking moment  $M_{cr}$  and 1.33 times the factored moment required by the applicable strength load combinations ( $M_u$ ).

(AASHTO CFRP 1.7.3.3.1)

$$M_{cr} = \gamma_3 \cdot \left[ (\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}) \cdot S_c - M_{dnc} \cdot \left( \frac{S_c}{S_{nc}} - 1 \right) \right]$$

Cracking moment  
(AASHTO CFRP 1.7.3.3.1-1)

$$f_{r.beam} = 0.7 \cdot \text{ksi}$$

Modulus of rupture of beam concrete  
(SDG 1.4.1B)

$$\gamma_1 := 1.6$$

flexural cracking variability factor

$$\gamma_2 := 1.1$$

Prestress variability factor  
1.1 for bonded prestressing CFRP  
1.0 for unbonded prestressing CFRP  
(AASHTO CFRP 1.7.3.3.1)

$$\gamma_3 := 1.0$$

$$f_{cpe} := \frac{P_{pe}}{A_{g.tr}} + \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.bot}} = 2.334 \cdot \text{ksi}$$

Compressive stress in concrete due to effective prestress forces only (after all losses) at extreme fiber of section where tensile stress is caused by externally applied loads

$$M_{dnc} := M_{sf.beam}(\text{midspan}) + M_{sf.slab}(\text{midspan}) \dots = 204.056 \cdot \text{kip} \cdot \text{ft} \\ + M_{forms}(\text{midspan})$$

Total unfactored dead load moment acting on the noncomposite section

$$M_{cr} := \gamma_3 \cdot \left[ (\gamma_1 \cdot f_{r.beam} + \gamma_2 \cdot f_{cpe}) \cdot S_{g.comp.tr.bot} - M_{dnc} \cdot \left( \frac{S_{g.comp.tr.bot}}{S_{g.tr.bot}} - 1 \right) \right] = 640.881 \cdot \text{kip} \cdot \text{ft}$$

Cracking moment  
(AASHTO CFRP 1.7.3.3.1-1)



## CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FSB 12x57

$$M_{req} := \min(M_{cr}, 1.33M_u) = 640.881 \cdot \text{kip} \cdot \text{ft}$$

Required minimum amount of CFRP shall develop the moment capacity Mreq

$$\text{SectionCT} := \begin{cases} \text{"C"} & \text{if SectionControl} = \text{"Compression Controlled Section"} \\ \text{"T"} & \text{if SectionControl} = \text{"Tension Controlled Section"} \end{cases}$$

$$\text{Check}_{\min\text{CFRP}} := \begin{cases} \text{"Compression Controlled Section, Not Applicable"} & \text{if SectionCT} = \text{"C"} \\ \text{"OK"} & \text{if SectionCT} = \text{"T"} \wedge M_n \geq M_{req} \\ \text{"NG"} & \text{if SectionCT} = \text{"T"} \wedge M_n < M_{req} \end{cases}$$

$$\text{Check}_{\min\text{CFRP}} = \text{"Compression Controlled Section, Not Applicable"}$$

minimum amount of prestressing CFRP

### **13. TRANSVERSE SHEAR DESIGN**

$$V_u \leq V_r = \phi_{\text{shear}} \cdot V_n$$

(AASHTO CFRP 1.8.2.1)

$$\phi_{\text{shear}} = 0.9$$

Reduction factor for CFRP prestressed girder - Shear  
(AASHTO CFRP 1.8.2.1, LRFD 5.5.4.2)

$$V_n = \min(V_{n1}, V_{n2})$$

$$V_{n1} = V_c + V_f + V_p$$

$V_c, V_f, V_p$  are shear resistance contribution by concrete, FRP stirrups, and CFRP prestressing strands

$$V_{n2} = 0.25f_c \cdot b_v \cdot d_v + V_p$$

$$V_u > 0.5 \cdot \phi_{\text{shear}} \cdot (V_c + V_p)$$

Transverse reinforcement should be provided in this case (LRFD 5.7.3.2-1)

#### **13.1 Critical Section for Shear**

critical section for shear shall be taken as  $d_v$  from the internal face of support

$$d_{e.\text{shear}} := h_{g.\text{comp}} - d_{p.\text{strands.c.g.bot}} = 15 \cdot \text{in}$$

Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcements

$$d_v := \max\left(d_{e.\text{shear}} - \frac{\beta_1 \cdot cc}{2}, 0.9 \cdot d_{e.\text{shear}}, 0.72 \cdot h_{g.\text{comp}}\right) = 13.5 \cdot \text{in}$$

Effective shear depth  
(LRFD 5.7.2.8)

$$\text{shearcheck} := d_v + \frac{\text{PadWidth}}{2} = 1.458 \text{ ft}$$

Critical shear section, from left support

$$b_v := b_{\text{beam.top}} = 45 \cdot \text{in}$$

Width of shear web, take  $b_{\text{beam.top}}$  conservatively  
(LRFD 5.7.2.7)

**13.2 Contribution by Prestressing CFRP**

$V_p := 0$  Straight strands, no draped,  $V_p=0$

**13.3 Contribution by Concrete,  $V_c$**

$V_c = 0.0316 \cdot \beta \cdot \sqrt{f_{c.beam}} \cdot b_v \cdot d_v$  (LRFD 5.7.3.3-3)

**Determine parameters  $\beta$  and  $\theta$**

*Note: assume the section containing at least the minimum amount of transverse reinforcement*

$$\epsilon_{f.shear} = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{pf} \cdot f_{p0}}{E_f \cdot A_{pf}}$$

Net longitudinal tensile strain  
(AASHTO CFRP 1.8.3.2)

$N_{u.shear} := 0$  Factored axial force at shear critical section

$V_{u.shear} := V_{Str1}(shearcheck) = 102.651 \cdot kip$  Factored shear at shear critical section

$M_{Str1}(shearcheck) = 120.948 \cdot kip \cdot ft$

$M_{u.shear} := \max(M_{Str1}(shearcheck), |V_{u.shear} - V_p| \cdot d_v) = 120.948 \cdot kip \cdot ft$

Factored moment at shear critical section  
(AASHTO CFRP 1.8.3.2)

$f_{p0.shear} := 0.6 \cdot f_{pu} = 204.469 \cdot ksi$  (AASHTO CFRP 1.8.3.2)

$$A_{pf.shear} := \begin{cases} A_{p.total.db1} & \text{if } shearcheck < debond1 \\ A_{p.total.db2} & \text{if } debond1 \leq shearcheck < debond2 \\ A_{p.total.db3} & \text{if } shearcheck \geq debond2 \end{cases}$$

$$A_{pf.shear} = 3.222 \cdot \text{in}^2$$

Area of prestressing CFRP on the flexural tension side of the member, exclude debonded strands

$$\epsilon_{f.shear1} := \min \left( \epsilon_{pu}, \frac{\left( \frac{|M_{u.shear}|}{d_v} + 0.5N_{u.shear} + |V_{u.shear} - V_p| - A_{pf.shear} \cdot f_{p0.shear} \right)}{E_p \cdot A_{pf.shear}} \right)$$

$$\epsilon_{f.shear1} = -6.194 \times 10^{-3}$$

$$A_{ct.shear} := b_{beam.top} \cdot \frac{h_{beam}}{2} = 270 \cdot \text{in}^2$$

Area of concrete on the flexural tension side of the member (LRFD B5.2)  
Approximate

$$\epsilon_{f.shear2} := \min \left( \epsilon_{pu}, \frac{\left( \frac{|M_{u.shear}|}{d_v} + 0.5N_{u.shear} + |V_{u.shear} - V_p| - A_{pf.shear} \cdot f_{p0.shear} \right)}{A_{ct.shear} \cdot E_{c.beam} + E_p \cdot A_{pf.shear}} \right)$$

$$\epsilon_{f.shear2} = -3.088 \times 10^{-4}$$

$$\epsilon_{f.shear} := \begin{cases} \epsilon_{f.shear1} & \text{if } \epsilon_{f.shear1} \geq 0 \\ \max(-0.0004, \epsilon_{f.shear2}) & \text{if } \epsilon_{f.shear1} < 0 \end{cases} = -3.088 \times 10^{-4}$$

$$\beta := \frac{4.8 \cdot \text{deg}}{1 + 750 \cdot \epsilon_{f.shear}} = 6.2 \cdot \text{deg} \quad (\text{LRFD 5.7.3.4.2-1})$$

$$\theta := (29 + 3500 \cdot \epsilon_{f.shear}) \cdot \text{deg} = 27.9 \cdot \text{deg} \quad (\text{LRFD 5.7.3.4.2-3})$$

$$V_c := 0.0316 \cdot \frac{\beta}{\text{deg}} \cdot \sqrt{\frac{f_{c.beam}}{\text{ksi}}} \cdot \frac{b_v}{\text{in}} \cdot \frac{d_v}{\text{in}} \cdot \text{kip} = 349.625 \cdot \text{kip} \quad (\text{LRFD 5.7.3.3-3})$$

**13.4 Required Shear Reinforcement**

$$\text{CheckVf} := \begin{cases} \text{"Need Shear Reinforcement"} & \text{if } V_{u,\text{shear}} \geq 0.5 \cdot \phi_{\text{shear}} \cdot (V_c + V_p) \\ \text{"Shear Reinforcement Not Needed"} & \text{if } V_{u,\text{shear}} < 0.5 \cdot \phi_{\text{shear}} \cdot (V_c + V_p) \end{cases}$$

CheckVf = "Shear Reinforcement Not Needed"

$$d_{v,\text{shear}} := d_{\text{GFRP}} = 0.5 \cdot \text{in} \quad \text{Diameter of GFRP stirrup}$$

$$A_{v,\text{shear}} := A_{\text{GFRP}} = 0.196 \cdot \text{in}^2 \quad \text{Area of each GFRP bar}$$

$$E_{v,\text{shear}} := E_{\text{GFRP}} = 6.5 \times 10^3 \cdot \text{ksi} \quad \text{Elastic modulus of GFRP stirrup}$$

$$f_{v,\text{shear}} := f_{t_u,\text{GFRP}} = 77.006 \cdot \text{ksi} \quad \text{Tensile strength of GFRP stirrup}$$

$$\phi_{\text{bend.GFRP}} = 0.6 \quad \text{take 0.6 in this design example}$$

$$V_n := \min\left(\frac{V_{u,\text{shear}}}{\phi_{\text{shear}}}, 0.25 \cdot f_{c,\text{beam}} \cdot b_v \cdot d_v + V_p\right) = 114.057 \cdot \text{kip}$$

$$V_{\text{FRP.req}} := \max(0, V_n - V_c - V_p) = 0 \cdot \text{kip} \quad \text{Required shear provided by FRP stirrup}$$

$$V_{\text{FRP.req}} = \frac{f_{fV} \cdot A_{v,\text{shear.total}} \cdot d_v}{s_{\text{shear}}} \quad \text{(AASHTO GFRP 2.10.3.2.2)}$$

$$f_{fV} := \min(0.004 \cdot E_{v,\text{shear}} \cdot \phi_{\text{bend.GFRP}} \cdot f_{v,\text{shear}}, 22 \text{ksi}) = 22 \cdot \text{ksi}$$

Max bended FRP tensile stress  
(AASHTO GFRP 2.10.3.2.2), modified with  
FDOT Material Spec 932-3, which limits  
the shear strength to 22 ksi

$$\text{leg}_{\text{stirrup}} := 4 \quad \text{Number of legs of stirrup}$$

$$A_{v,\text{shear.total}} := \text{leg}_{\text{stirrup}} \cdot A_{v,\text{shear}} = 0.785 \cdot \text{in}^2$$



## CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FSB 12x57

$$s_{\text{shear.req}} := \begin{cases} 1000\text{in} & \text{if } V_{\text{FRP.req}} = 0 \\ \frac{f_{\text{fv}} \cdot A_{\text{v.shear.total}} \cdot d_{\text{v}}}{V_{\text{FRP.req}}} & \text{otherwise} \end{cases} = 1000\text{-in}$$

Required FRP stirrup spacing from calculation  
(if not needed, output 1000 inch spacing)

$$s_{\text{shear.max}} := \begin{cases} \min(0.75 \cdot h_{\text{g}}, 24\text{in}) & \text{if } V_{\text{FRP.req}} \leq 4.0 \cdot \sqrt{\frac{f_{\text{c.beam}}}{\text{ksi}}} \cdot b_{\text{v}} \cdot d_{\text{v}} \cdot \text{ksi} \\ \min(0.375 \cdot h_{\text{g}}, 12\text{in}) & \text{if } V_{\text{FRP.req}} > 4.0 \cdot \sqrt{\frac{f_{\text{c.beam}}}{\text{ksi}}} \cdot b_{\text{v}} \cdot d_{\text{v}} \cdot \text{ksi} \end{cases} = 9\text{-in}$$

Maximum FRP stirrup spacing  
(ACI 440.4R 5.3)

$$A_{\text{v.min}} = 0.05 \cdot \frac{s \cdot b_{\text{w}}}{f_{\text{fv}}}$$

Minimum GFRP transverse reinforcement  
(AASHTO GFRP 2.10.2.2.1)

$$s_{\text{shear.minAf}} := \frac{A_{\text{v.shear.total}} \cdot \frac{f_{\text{fv}}}{\text{ksi}}}{0.05 \cdot b_{\text{beam.top}}} = 7.679\text{-in}$$

Max spacing corresponding to minimum GFRP transverse reinforcement

$$s_{\text{shear}} := \min(s_{\text{shear.req}}, s_{\text{shear.max}}, s_{\text{shear.minAf}}) = 7.679\text{-in}$$

$$s_{\text{shear.provide}} := \text{floor}\left(\frac{s_{\text{shear}}}{\text{in}}\right) \cdot \text{in} = 7\text{-in}$$

Provided transverse reinforcement spacing

$$V_{\text{FRP}} := \frac{f_{\text{fv}} \cdot A_{\text{v.shear.total}} \cdot d_{\text{v}}}{s_{\text{shear.provide}}} = 33.323\text{-kip}$$

Provided shear resistance from FRP transverse reinforcement

**13.5 Summary**

$$V_{u,\text{shear}} = 103 \cdot \text{kip}$$

$$V_c = 350 \cdot \text{kip}$$

$$V_p = 0$$

$$V_{\text{FRP}} = 33 \cdot \text{kip}$$

$$d_{v,\text{shear}} = 0.5 \cdot \text{in}$$

Diameter of GFRP stirrup

$$\text{leg}_{\text{stirrup}} = 4$$

Number of legs of GFRP stirrup

$$s_{\text{shear,provide}} = 7 \cdot \text{in}$$

Spacing of GFRP stirrup

$$V_{n,\text{provide}} := V_c + V_{\text{FRP}} + V_p = 383 \cdot \text{kip}$$

$$\text{CheckShearCapacity} := \begin{cases} \text{"OK"} & \text{if } V_{n,\text{provide}} \geq \min \left( \frac{V_{u,\text{shear}}}{\phi_{\text{shear}}}, 0.25 \cdot f_{c,\text{beam}} \cdot b_v \cdot d_v + V_p \right) \\ \text{"NG"} & \text{otherwise} \end{cases}$$

**CheckShearCapacity = "OK"**

Transverse shear capacity

## 14. INTERFACE SHEAR DESIGN

$$V_{u1} := V_{Str1}(\text{shearcheck}) = 102.651 \cdot \text{kip}$$

Factored interface shear force

$$V_{ri} = \phi_{\text{shear}} \cdot V_{ni} \geq V_{ui}$$

(LRFD 5.7.4.3)

$$\phi_{\text{shear}} = 0.9$$

Assumed numbers:

$$c_{vi} := 0.28 \text{ksi}$$

Cohesion factor

$$\mu_{vi} := 1$$

Friction factor

$$K_{1,vi} := 0.3$$

Fraction of concrete strength available to resist interface shear

$$K_{2,vi} := 1.8 \text{ksi}$$

Limiting interface shear resistance

### Factored interface shear force, per ft

$$d_{vi} := \begin{cases} h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot.dbs}} - \frac{t_{\text{slab}}}{2} & \text{if shearcheck} < \text{debond1} \\ h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot.db1}} - \frac{t_{\text{slab}}}{2} & \text{if debond1} \leq \text{shearcheck} < \text{debond2} \\ h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot.db2}} - \frac{t_{\text{slab}}}{2} & \text{if shearcheck} \geq \text{debond2} \end{cases}$$

$$d_{vi} = 12 \cdot \text{in}$$

Distance between the centroid of the tension strands and the mid-thickness of the slab

$$V_{hi} := \frac{V_{u1}}{d_{vi}} = 102.651 \cdot \frac{\text{kip}}{\text{ft}}$$

Factored interface shear force, per ft

**Required nominal interface shear resistance, per ft**

$$V_{ni} := \frac{V_{hi}}{\phi_{shear}} = 114.057 \cdot \frac{\text{kip}}{\text{ft}} \quad (\text{LRFD 5.7.4.3})$$

**Required interface shear reinforcement, per ft**

$$V_{ni} = c \cdot A_{cv} + \mu \cdot (A_{vf} \cdot f_y + P_c) \quad \begin{array}{l} \text{Nominal interface shear resistance} \\ (\text{LRFD 5.7.4.3}) \end{array}$$

$$b_{vi} := b_{slab.top} = 58.5 \cdot \text{in} \quad \begin{array}{l} \text{Interface width considered to be engaged in} \\ \text{shear transfer} \end{array}$$

$$A_{cv} := b_{vi} = 702 \cdot \frac{\text{in}^2}{\text{ft}} \quad \begin{array}{l} \text{Area of concrete considered to be engaged} \\ \text{in shear transfer, per 1-ft length} \end{array}$$

$$P_{c.vi} := 0 \quad \begin{array}{l} \text{Permanent net compression force normal} \\ \text{to the shear plane} \end{array}$$

$$A_{vf.req} := \frac{\frac{V_{ni} - c_{vi} \cdot A_{cv}}{\mu_{vi}} - P_{c.vi}}{f_{fv}} = -3.75 \cdot \frac{\text{in}^2}{\text{ft}} \quad \begin{array}{l} \text{Required interface shear reinforcement} \end{array}$$

**Minimum interface shear reinforcement, per ft**

$$A_{vf.min1} := 0.05 \cdot \frac{A_{cv}}{\frac{f_{fv}}{\text{ksi}}} = 1.595 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$A_{vf.min2} := \frac{\frac{1.33V_{ni} - c_{vi} \cdot A_{cv}}{\mu_{vi}} - P_{c.vi}}{f_{fv}} = -2.039 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$A_{vf.min} := \begin{cases} \min(A_{vf.min1}, A_{vf.min2}) & \text{if } \min(A_{vf.min1}, A_{vf.min2}) \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0 \cdot \text{in}^2$$

Minimum interface shear reinforcement  
(LRFD 5.7.4.2)

**Design interface shear reinforcement, per ft**

$$A_{vf,design,need} := \max(A_{vf,req}, A_{vf,min}, 0) = 0 \cdot \frac{\text{in}^2}{\text{ft}} \quad \text{Design interface reinforcement need to provide}$$

$$A_{vf,provide} := \frac{A_{v, shear, total}}{s_{shear, provide}} = 1.346 \cdot \frac{\text{in}^2}{\text{ft}} \quad \text{Interface shear reinforcement already provided by GFRP stirrup}$$

$$\text{Check}_{iv0} := \begin{cases} \text{"OK"} & \text{if } A_{vf,provide} \geq A_{vf,design,need} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

**Check<sub>iv0</sub> = "OK"**

Interface shear reinforcement,  
Provided > Required

**Maximum interface shear resistance, per ft**

$$V_{ni,provide} := c_{vi} \cdot A_{cv} + \mu_{vi} (A_{vf,provide} \cdot f_{fv} + P_{c,vi}) = 226 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Provided nominal interface shear resistance (LRFD 5.7.4.3)}$$

$$V_{ni,max} := \min(K_{1,vi} \cdot f_{c,beam} \cdot A_{cv}, K_{2,vi} \cdot A_{cv}) = 1264 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Design interface shear resistance limitation (LRFD 5.7.4.3)}$$

$$\text{Check}_{iv,max} := \begin{cases} \text{"OK"} & \text{if } V_{ni,provide} \leq \min(K_{1,vi} \cdot f_{c,beam} \cdot A_{cv}, K_{2,vi} \cdot A_{cv}) \\ \text{"NG"} & \text{otherwise} \end{cases}$$

**Check<sub>iv,max</sub> = "OK"**

Maximum interface shear resistance

**Summary**

$$V_{ni.provide} = 226.181 \cdot \frac{\text{kip}}{\text{ft}}$$

Nominal interface shear resistance

$$V_{hi} = 102.651 \cdot \frac{\text{kip}}{\text{ft}}$$

Factored interface shear force, per ft

$$\text{Check}_{vi.capacity} := \begin{cases} \text{"OK"} & \text{if } \phi_{shear} \cdot V_{ni.provide} \geq V_{hi} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{vi.capacity} = \text{"OK"}$$

Check capacity

$$\text{Check}_{iv.max} = \text{"OK"}$$

Check max nominal resistance

### **15. MINIMUM LONGITUDINAL REINFORCEMENT**

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy

$$\sum_{x=1}^n (A_{px} \cdot f_{px}) \geq \frac{|M_u|}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u}{\phi_c} + \left( \left| \frac{V_u}{\phi_v} - V_p \right| - 0.5 \cdot V_f \right) \cdot \frac{1}{\tan(\theta)}$$

(AASHTO CFRP 1.8.3.3)

$$N_u := 0$$

Axial load

$$\phi_{\text{moment}} = 0.75$$

Resistance factor for bending

$$\phi_{\text{shear}} = 0.9$$

Resistance factor for shear

$$\theta = 27.919 \cdot \text{deg}$$

Angle of inclination of diagonal compressive stresses

$$d_v = 13.5 \cdot \text{in}$$

Effective shear depth

$$V_p = 0$$

Shear capacity contribution by prestressing CFRP

$$f_{pe} = 195 \cdot \text{ksi}$$

Effective prestressing stress

#### **At the support location**

$$\text{support} = 0$$

$$V_{f,\text{support}} := \min \left( V_{\text{FRP}}, \frac{V_{\text{Str1}}(\text{support})}{\phi_{\text{shear}}} \right) = 33.323 \cdot \text{kip} \quad (\text{AASHTO CFRP 1.8.3.3})$$

$$\text{right} := \frac{|M_{\text{Str1}}(\text{support})|}{d_v \cdot \phi_{\text{moment}}} + \left( \left| \frac{V_{\text{Str1}}(\text{support})}{\phi_{\text{shear}}} - V_p \right| - 0.5 \cdot V_{f,\text{support}} \right) \cdot \frac{1}{\tan(\theta)} = 197 \cdot \text{kip}$$

the crack plane crosses the c.g of the strand group at a distance

$$x_{\text{crackplane}} := \text{BearingDistance} + d_{p,\text{strands.c.g.bot.dbs}} \cdot \frac{1}{\tan(\theta)} = 12.161 \cdot \text{in}$$

$$L_{\text{transfer}} = 30 \cdot \text{in}$$

Transfer length of prestressing CFRP  
 (AASHTO CFRP 1.9.3.2)

$$f_{\text{px}} := f_{\text{pe}} \cdot \frac{x_{\text{crackplane}}}{L_{\text{transfer}}}$$

Approximate prestressing stress at the  
 crack plane

$$\text{left} := \sum_{i=0}^{\text{rows}(n)-1} (A_{\text{p.db}})_i \cdot f_{\text{px}} = 254 \cdot \text{kip}$$

$$\text{Check}_{\text{long.rebar}} := \begin{cases} \text{"OK, No Additional Longitudinal Reinforcement is Required"} & \text{if } \text{left} \geq \text{right} \\ \text{"NG, Additional Longitudinal Reinforcement is Required"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{\text{long.rebar}} = \text{"OK, No Additional Longitudinal Reinforcement is Required"}$$

## 16. DEFLECTION AND CAMBER

### Immediate deflection, due to prestressing force and DL

downward -> positive

Deflection due to prestressing force at transfer

$$\Delta = \frac{M \cdot L^2}{8 \cdot E \cdot I}$$

$$\Delta_{p1} := \frac{-(A_{p,\text{total.dbs}} \cdot f_{pt}) \cdot e_{g,\text{tr.cg.strands}} \cdot (\text{Span})^2}{8 \cdot E_{\text{ci.beam}} \cdot I_g} = -1.635 \cdot \text{in}$$

$$\Delta_{p2} := \frac{-[(A_{p,\text{total.db1}} - A_{p,\text{total.dbs}}) \cdot f_{pt}] \cdot e_{g,\text{tr.cg.strands}} \cdot (\text{Span} - 2 \cdot \text{debond1})^2}{8 \cdot E_{\text{ci.beam}} \cdot I_g} = 0 \cdot \text{in}$$

$$\Delta_{p3} := \frac{-[(A_{p,\text{total.db2}} - A_{p,\text{total.db1}}) \cdot f_{pt}] \cdot e_{g,\text{tr.cg.strands}} \cdot (\text{Span} - 2 \cdot \text{debond2})^2}{8 \cdot E_{\text{ci.beam}} \cdot I_g} = 0 \cdot \text{in}$$

$$\Delta_p := \Delta_{p1} + \Delta_{p2} + \Delta_{p3} = -1.635 \cdot \text{in}$$

Deflection due to self weight of beam

$$\Delta = \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I}$$

$$\Delta_{\text{sf.beam}} := \frac{5 \cdot w_{\text{beam}} \cdot \text{Span}^4}{384 \cdot E_{\text{ci.beam}} \cdot I_g} = 0.969 \cdot \text{in}$$

Deflection due to self weight of slab, buildup, and permanent forms

$$\Delta_{\text{permanent1}} := \frac{5 \cdot (w_{\text{slab}} + w_{\text{forms}}) \cdot \text{Span}^4}{384 \cdot E_{\text{c.beam}} \cdot I_{\text{g}}} = 0.672 \cdot \text{in}$$

Deflection due to barrier, future wearing surface, and utility

$$\Delta_{\text{permanent2}} := \frac{5 \cdot (w_{\text{barrier}} + w_{\text{fws}} + w_{\text{utility}}) \cdot \text{Span}^4}{384 \cdot E_{\text{c.beam}} \cdot I_{\text{g.comp}}} = 0.096 \cdot \text{in}$$

## **17. SUMMARY OF ALL CHECKS**

All "Check"

## **7. PRESTRESS LOSSES**

### **7.3 Summary**

Check<sub>pe.limit</sub> = "OK"

Service stress limit (after all losses)

## **8. STRESS LIMITS FOR CONCRETE -TEMPORARY STRESSES BEFORE LOSSES**

### **8.1 At Mid-Span Section**

Check<sub>top.midspan</sub> = "OK"

Check concrete stress limits at the midspan, at release

Check<sub>bot.midspan</sub> = "OK"

Check concrete stress limits at the midspan, at release

### **8.2 At Support Section**

Check<sub>sp.top</sub> = "OK"

Check concrete stress limits at the support, top fiber, at release

Check<sub>sp.bot</sub> = "OK"

Check concrete stress limits at the support, bottom fiber, at release

Check<sub>db</sub> = "No Need to Debond Strands"

Need to debond strands?

Check<sub>30%</sub> = "OK"

(Not necessary in this example)

Total debonded strands shall not exceed 30% of total strands (SDG 4.3.1)

Check<sub>40%</sub> = "OK"

(Not necessary in this example)

Debonded strands in each row shall not exceed 40% of total strands in that row (LRFD 5.9.4.3.3)

Check<sub>top.dbs</sub> = "OK"

(Not necessary in this example)

Check concrete stress limits at the support, top fiber, at release

Check<sub>bot.dbs</sub> = "OK"

(Not necessary in this example)

Check concrete stress limits at the support, bottom fiber, at release



**8.3 At Debond1 Section (if necessary)**

Check<sub>top.db1</sub> = "OK"

(Not necessary  
in this exmaple)

Check concrete stress limits at the  
debond1 section, top fiber, at release

Check<sub>bot.db1</sub> = "OK"

(Not necessary  
in this exmaple)

Check concrete stress limits at the  
debond1 section, bottom fiber, at release

**8.4 At Debond2 Section (if necessary)**

Check<sub>top.db2</sub> = "OK"

(Not necessary  
in this exmaple)

Check concrete stress limits at the  
debond2 section, top fiber, at release

Check<sub>bot.db2</sub> = "OK"

(Not necessary  
in this exmaple)

Check concrete stress limits at the  
debond2 section, bottom fiber, at release

**9. STRESS LIMITS FOR CONCRETE  
STRESSES AT SERVICE LIMIT STATE**

**9.1 Service I Limit State:under Effective Prestress and Permanent Loads**

Check<sub>topdeck1</sub> = "OK"

Concrete stress at the top fiber of deck

Check<sub>topbeam1</sub> = "OK"

Concrete stress at the top fiber of beam

**9.2 Service I Limit State:under Effective Prestress, Permanent loads, and  
Transient loads**

Check<sub>topdeck2</sub> = "OK"

Concrete stress at the top fiber of deck

Check<sub>topbeam2</sub> = "OK"

Concrete stress at the top fiber of beam

**9.3 Service III Limit State**

Check<sub>botbeam</sub> = "OK"

Concrete stress at the bottom fiber of  
beam

**10. FATIGUE LIMIT STATE**

Check<sub>fatigue</sub> = "No Need to Check Fatigue"

## **11. STRENGTH LIMIT STATE -MOMENT CAPACITY**

### **11.1 Compression-Controlled Section**

Check<sub>fp.C</sub> = "Compression Controlled Section, Assumption Satisfied"

Check<sub>CT.C</sub> = 0·kip Equilibrium

### **11.2 Tension-Controlled Section**

Check<sub>fp.T</sub> = "OK" CFRP not rupture

Check<sub>cc.T</sub> = "NG, Concrete Strain Exceeds  $\epsilon_{cu}$ . Compression-Controlled Section"

Check<sub>CT.T</sub> = 0·kip Equilibrium

### **11.3 Summary**

SectionControl = "Compression Controlled Section"

Check<sub>M</sub> = "OK" Strength I, moment capacity

## **12. MINIMUM REINFORCEMENT -PRESTRESSING CFRP**

Check<sub>minCFRP</sub> = "Compression Controlled Section, Not Applicable"

Minimum amount of prestressing CFRP

## **13. TRANSVERSE SHEAR DESIGN**

### **13.5 Summary**

CheckShearCapacity = "OK" Transverse shear capacity

## **14. INTERFACE SHEAR DESIGN**

Check<sub>iv0</sub> = "OK" Interface shear reinforcement,  
Provided > Required

Check<sub>iv.max</sub> = "OK" Maximum interface shear resistance

Check<sub>vi.capacity</sub> = "OK" Check interface shear capacity

## **15. MINIMUM LONGITUDINAL REINFORCEMENT**

Check<sub>long.rebar</sub> = "OK, No Additional Longitudinal Reinforcement is Required"