



CFRP-PC Training Course

Design Example

FIB-36

For Training Purposes Only, NOT To Be Used For Construction

UNIVERSITY of **HOUSTON**
CULLEN COLLEGE of ENGINEERING
Department of Civil & Environmental Engineering



DESIGN OF DECKED FIB-36 PRETENTIONED (INTERIOR) GIRDER WITH STRAIGHT CFRP CABLES

Geometry from "SR 687/4th Street, NB bridge"

TABLE OF CONTENTS

<u>1. REFERENCES</u>	...4
<u>2. GEOMETRY AND MATERIAL INPUTS</u>	...5
<u>2.1 Bridge Layout and Dimensions</u>	
<u>2.2 FIB-36 Girder</u>	
<u>2.3 Cast in Place Deck</u>	
<u>2.4 GFRP Reinforcing Bar</u>	
<u>2.5 Prestressing CFRP Strand</u>	
<u>2.6 Other Inputs</u>	
<u>3. SECTION PROPERTIES</u>	...13
<u>3.1 Section Properties - Non-Composite Section</u>	
<u>3.2 Section Properties - Composite Section</u>	
<u>4. LOAD AND LOAD DISTRIBUTION</u>	...16
<u>4.1 Dead Load</u>	
<u>4.2 Live Load</u>	
<u>4.3 Live Load Distribution Factors</u>	
<u>4.4 Summary</u>	



<u>5. STRAND PATTERN</u>	...30
<u>6. TRANSFORMED SECTION PROPERTIES</u>	...33
<u>6.1 Transformed Section Properties - Non-Composite Section</u>	
<u>6.2 Transformed Section Properties - Composite Section</u>	
<u>7. PRESTRESS LOSSES</u>	...36
<u>7.1 Prestress Loss Due to Elastic Shortening</u>	
<u>7.2 Long-Term Time-Dependent Prestress losses (Refined Estimation)</u>	
<u>7.3 Summary</u>	
<u>8. STRESS LIMITS FOR CONCRETE -TEMPORARY STRESSES BEFORE LOSSES</u>	...44
<u>8.1 At Mid-Span Section</u>	
<u>8.2 At Support Section</u>	
<u>8.3 At Debond1 Section</u>	
<u>8.4 At Debond2 Section</u>	
<u>8.5 Summary</u>	
<u>9. STRESS LIMITS FOR CONCRETE -STRESSES AT SERVICE LIMIT STATE</u>	...61
<u>9.1 Service I Limit State:under Effective Prestress and Permanent Loads</u>	
<u>9.2 Service I Limit State:under Effective Prestress, Permanent loads, and Transient loads</u>	
<u>9.3 Service III Limit State</u>	
<u>9.4 Summary</u>	
<u>10. FATIGUE LIMIT STATE</u>	...66



<u>11. STRENGTH LIMIT STATE -MOMENT CAPACITY</u>	...67
<u>11.1 Compression-Controlled Section</u>	
<u>11.2 Tension-Controlled Section</u>	
<u>11.3 Summary</u>	
<u>12. MINIMUM REINFORCEMENT -PRESTRESSING CFRP</u>	...77
<u>13. TRANSVERSE SHEAR DESIGN</u>	...79
<u>13.1 Critical Section for Shear</u>	
<u>13.2 Contribution by Prestressing CFRP</u>	
<u>13.3 Contribution by Concrete</u>	
<u>13.4 Required Shear Reinforcement</u>	
<u>13.5 Summary</u>	
<u>14. INTERFACE SHEAR DESIGN</u>	...85
<u>15. MINIMUM LONGITUDINAL REINFORCEMENT</u>	...89
<u>16. DEFLECTION AND CAMBER</u>	...91
<u>17. SUMMARY OF ALL CHECKS</u>	...93



CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FIB-36

1. REFERENCES

LRFD: AASHTO LRFD Bridge Design Specifications, 8th Edition, 2017

AASHTO CFRP: Guide Specifications for the Design of Concrete Bridge Beams Prestressed with Carbon Fiber-Reinforced Polymer (CFRP) Systems, 1st Edition, 2018

AASHTO GFRP: AASHTO LRFD Bridge Design Guide Specifications for GFRP-Reinforced Concrete, 2nd Edition, 2018

SDG: FDOT Structures Manual, Volume 1 -Structures Design Guidelines

FRPG: FDOT Structures Manual, Volume 4 -Fiber Reinforced Polymer Guidelines

FDOT Standard Specifications For Road and Bridge Construction, January 2020

ACI 440.1R: Guide for the Design and Construction of Structural Concrete Reinforced with FRP Bars, 2015

ACI 440.4R: Prestressing Concrete Structures with FRP Tendons, 2011

NCHRP Report 907: Design of Concrete Bridge Beams Prestressed with CFRP systems

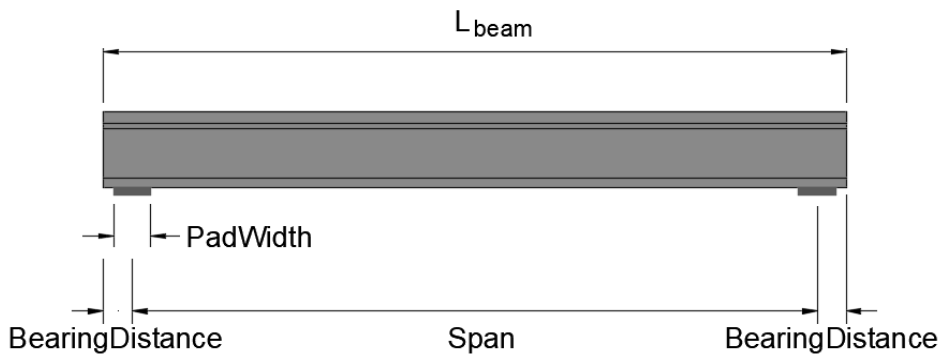
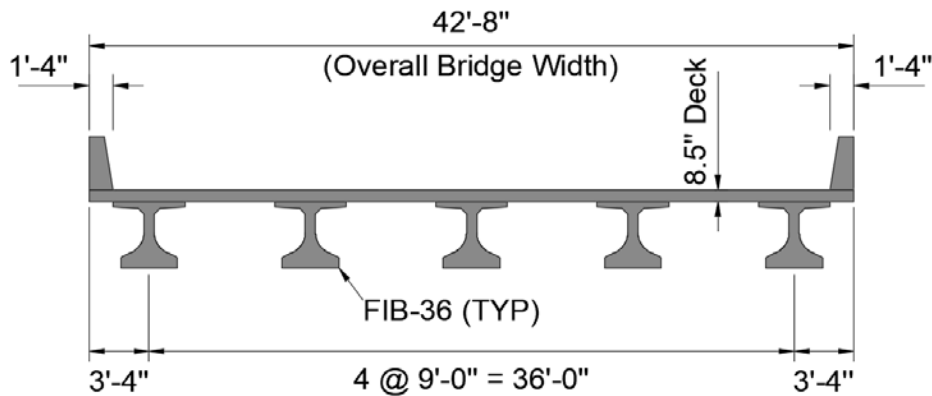
FDOT Design Example: LRFD Design Example #1, Prestressed Precast Concrete Beam Bridge Design

FDOT Design Example: LRFDPBeamV5.2 (MathCAD)

2. GEOMETRY AND MATERIAL INPUTS

2.1 Bridge Layout and Dimensions

$$kcf = \frac{\text{kip}}{\text{ft}^3}$$



(FDOT Design example
LRFDPBeamV5.2-CFRP)

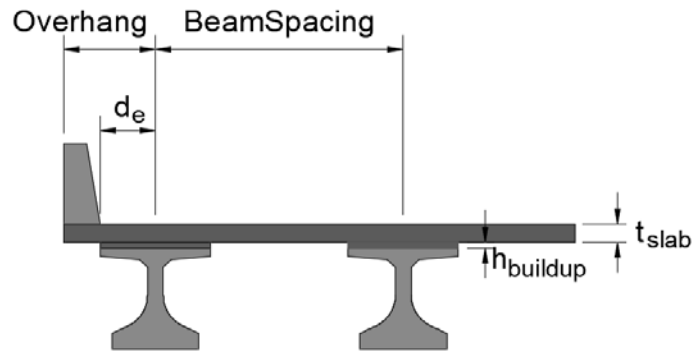
$$\text{Span} := \left(87 + \frac{8}{12} \right) \cdot \text{ft} = 87.667 \cdot \text{ft}$$

Clear span

$$\text{BearingDistance} := 9 \text{in}$$

$$L_{\text{beam}} := \text{Span} + 2 \cdot \text{BearingDistance} = 89.167 \cdot \text{ft}$$

$$\text{PadWidth} := 10 \text{in}$$



$$\text{Overhang} := \left(3 + \frac{4}{12} \right) \text{ft} = 3.333 \cdot \text{ft}$$

$$\text{BeamSpacing} := 9 \text{ft}$$

$$\text{NumberOfBeam} := 5$$

$$\text{TotalBridgeWidth} := \text{BeamSpacing} \cdot (\text{NumberOfBeam} - 1) + 2 \cdot \text{Overhang} = 42.667 \cdot \text{ft}$$

$$d_e := 2 \text{ft}$$



2.2 FIB-36 Girder

$A_{\text{beam}} := 806.58\text{in}^2$	Cross section area
$\text{Perimeter}_{\text{beam}} := 206.57\text{in}$	Perimeter of beam section
$I_{xx.\text{beam}} := 127545\text{in}^4$	Moment of inertia
$I_{yy.\text{beam}} := 81070\text{in}^4$	
$y_{\text{beam.top}} := 19.51\text{in}$	Distance from top of beam to N.A.
$y_{\text{beam.bot}} := 16.49\text{in}$	Distance from bottom of beam to N.A.
$b_{\text{beam.top.flange}} := 48\text{in}$	Top flange width
$h_{\text{beam.top.flange}} := 3.5\text{in}$	Top flange thickness
$b_{\text{beam.bot..flange}} := 38\text{in}$	Bottom flange width
$h_{\text{beam.bot.flange}} := 7\text{in}$	Bottom flange thickness
$\text{Taper}_{\text{beam.bot.flange}} := 15.5\text{in}$	Bottom flange taper
$h_{\text{beam}} := 36\text{in}$	Depth
$b_{\text{web}} := 7\text{in}$	Web width
$h_{\text{web}} := h_{\text{beam}} - h_{\text{beam.top.flange}} - 1.5\text{in} - 7\text{in} - 7.5\text{in} = 16.5\text{in}$	Web depth



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$$f_{c.beam} := 8.5 \text{ksi}$$

Concrete strength

$$f_{ci.beam} := 6 \text{ksi}$$

Concrete strength, at release

$$f_{r.beam} := 0.24 \cdot \left(\frac{f_{c.beam}}{\text{ksi}} \right)^{0.5} \text{ksi} = 0.7 \cdot \text{ksi}$$

Modulus of rupture
(SDG 1.4.1B)

$$\gamma_{concrete} := 0.15 \text{kcf}$$

Unit weight of reinforced concrete

$$\gamma_{beam} := \gamma_{concrete} = 0.15 \cdot \text{kcf}$$

Unit weight of reinforced deck concrete

$$K_1 := 1.0$$

Correction factor for aggregate
(SDG 1.4.1A)

$$w_c := 0.145 \text{kcf}$$

Unit weight of concrete

$$E_{c.beam} := 120000 \cdot K_1 \cdot \left(\frac{w_c}{\text{kcf}} \right)^2 \cdot \left(\frac{f_{c.beam}}{\text{ksi}} \right)^{0.33} \cdot \text{ksi} = 5112 \cdot \text{ksi}$$

Elastic modulus of beam concrete
(LRFD 5.4.2.4-1)

$$E_{ci.beam} := 120000 \cdot K_1 \cdot \left(\frac{w_c}{\text{kcf}} \right)^2 \cdot \left(\frac{f_{ci.beam}}{\text{ksi}} \right)^{0.33} \cdot \text{ksi} = 4557 \cdot \text{ksi}$$

Elastic modulus of beam concrete, at
release
(LRFD 5.4.2.4-1)

2.3 Cast in Place Deck

$$t_{\text{slab}} := 8.5\text{in}$$

$$h_{\text{buildup}} := 1\text{in}$$

$$b_e := \text{BeamSpacing} = 9\text{-ft}$$

Effective slab width, for interior beam
(LRFD 4.6.2.6)

$$A_{\text{slab}} := b_e \cdot t_{\text{slab}} = 918 \cdot \text{in}^2$$

Area of slab

$$A_{\text{buildup}} := b_{\text{beam.bot..flange}} \cdot h_{\text{buildup}} = 38 \cdot \text{in}^2$$

Area of buildup

$$f_{c.\text{slab}} := 5.5\text{ksi}$$

Concrete strength

$$\gamma_{\text{slab}} := \gamma_{\text{concrete}} = 0.15 \cdot \text{kcf}$$

Unit weight of reinforced deck concrete

$$E_{c.\text{slab}} := 120000 \cdot K_1 \cdot \left(\frac{w_c}{\text{kcf}} \right)^2 \cdot \left(\frac{f_{c.\text{slab}}}{\text{ksi}} \right)^{0.33} \cdot \text{ksi} = 4428 \cdot \text{ksi}$$

Elastic modulus of deck concrete
(LRFD 5.4.2.4-1)

$$n_c := \frac{E_{c.\text{beam}}}{E_{c.\text{slab}}} = 1.154$$

$$b_{e.\text{tr}} := b_e \cdot \frac{E_{c.\text{slab}}}{E_{c.\text{beam}}} = 7.796 \cdot \text{ft}$$

Effective slab width, transformed , for
interior beam

(LRFD 4.6.2.6)

2.4 GFRP Reinforcing Bar

$GFRPBarSize := 5$	Size of GFRP bar
$d_{GFRP} := \frac{GFRPBarSize}{8} \text{ in} = 0.625 \cdot \text{in}$	Nominal diameter of GFRP bar
$A_{GFRP} := \frac{1}{4} \cdot \pi \cdot d_{GFRP}^2 = 0.307 \cdot \text{in}^2$	Area of one GFRP bar
$f_{fu,GFRP}^* := \frac{29.1 \text{ kip}}{A_{GFRP}} = 94.851 \cdot \text{ksi}$	Tensile strength of GFRP bar (FDOT Standard Specifications, Section 932, Table 3-1)
$C_{E,GFRP} := 0.7$	Environmental reduction factor for GFRP (AASHTO-GFRP Table 2.6.1.2-1) Here assume "concrete exposed to earth and weather"
$f_{fu,GFRP} := C_{E,GFRP} \cdot f_{fu,GFRP}^* = 66.396 \cdot \text{ksi}$	Design tensile strength
$E_{GFRP} := 6500 \text{ ksi}$	Elastic modulus of GFRP bar (No less than 6500 ksi per FDOT Standard Specifications, Section 932)
$\phi_{bend} = f_{fu} \cdot \left(0.11 + 0.05 \frac{r}{d_b} \right)$	Strength reduction for bent FRP stirrup (ACI 440.4R 5.4)
$\phi_{bend,GFRP} := 0.6$	take 0.6 in this design example
$n_b := \frac{E_{GFRP}}{E_{c,beam}} = 1.271$	

2.5 Prestressing CFRP Strand

[Assume 0.6in diameter CFRP Cable; Properties refer to FDOT Standard Specification Section 933 Table 1-2, 7-strand-15.2mm CFRP prestressing strand]

$StrandType_{CFRP} := 1$	CFRP strand type: 1 for Cable; 2 for Bar
$D_p := 0.6in$	Diameter of CFRP prestressing tendon
$A_{pf} := 0.179in^2$	Effective cross-sectional area of each tendon
$E_p := 22480ksi$	Elastic modulus of prestressing tendon
$P_u := 61kip$	Ultimate tensile force for each tendon
$f_{pu} := \frac{P_u}{A_{pf}} = 341 \cdot ksi$	Ultimate tensile strength Design tensile strength , f_{pu} , is defined later in "other inputs"
$C_{E.CFRP} := 1$	Environmental reduction factor for CFRP (AASHTO CFRP Table 1.4.1.2-1)
$f_{pu} := C_{E.CFRP} \cdot f_{pu} = 341 \cdot ksi$	Design tensile strength of CFRP tendon
$\epsilon_{pu} := \frac{f_{pu}}{E_p} = 0.015$	Design tensile strain
$f_{pi.limit} := \begin{cases} 0.70 \cdot f_{pu} & \text{if } StrandType_{CFRP} = 1 \\ 0.65 \cdot f_{pu} & \text{if } StrandType_{CFRP} = 2 \end{cases} = 238.5 \cdot ksi$	Jacking stress limit (AASHTO CFRP Table 1.9.1.1)
$f_{pi} := f_{pi.limit} = 239 \cdot ksi$	Jacking stress
$P_i := f_{pi} \cdot A_{pf} = 42.7 \cdot kip$	Jacking force



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$$f_{pe.limit} := \begin{cases} 0.65 \cdot f_{pu} & \text{if StrandType}_{CFRP} = 1 \\ 0.60 \cdot f_{pu} & \text{if StrandType}_{CFRP} = 2 \end{cases} = 221.5 \cdot \text{ksi}$$

Service stress limit (after all losses)
(AASHTO CFRP Table 1.9.1.1)

$$n_p := \frac{E_p}{E_{c.beam}} = 4.397$$

$$L_{transfer} := 50 \cdot D_p = 30 \cdot \text{in}$$

Transfer length
(AASHTO CFRP 1.9.3.2)

2.6 Other Inputs

Environment := "Extremely"

RH := 75
Ambient relative humidity
(SDG 4.6.6)

$\Delta f_{pTH} := 0$
Prestress loss due to temperature change. Not considered in this example

$t_i := 1$
Age at transfer (day)

$t_d := 120$
Age at deck placement (day)

$t_f := 10000$
Final time (day)

$\phi_{moment} := 0.75$
Reduction factor for CFRP prestressed girder - Moment
(AASHTO CFRP 1.5.3.2)

$\phi_{shear} := 0.9$
Reduction factor for CFRP prestressed girder - Shear
(AASHTO CFRP 1.8.2.1, LRFD 5.5.4.2)

3. SECTION PROPERTIES

3.1 Section Property - Non-Composite Section (I-girder)

$$h_g := h_{\text{beam}} = 36 \cdot \text{in}$$

Depth

$$b_w := b_{\text{web}} = 7 \cdot \text{in}$$

Web thickness

$$A_g := A_{\text{beam}} = 806.58 \cdot \text{in}^2$$

Area

$$I_g := I_{xx,\text{beam}} = 1.275 \times 10^5 \cdot \text{in}^4$$

Moment inertia

$$y_{g,\text{top}} := y_{\text{beam,top}} = 19.51 \cdot \text{in}$$

Distance from top fiber (beam) to N.A

$$y_{g,\text{bot}} := y_{\text{beam,bot}} = 16.49 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A

$$S_{g,\text{top}} := \frac{I_g}{y_{g,\text{top}}} = 6.537 \times 10^3 \cdot \text{in}^3$$

$$S_{g,\text{bot}} := \frac{I_g}{y_{g,\text{bot}}} = 7.735 \times 10^3 \cdot \text{in}^3$$

3.2 Section Property - Composite Section (I-girder+Deck)

$$h_{g.comp} := t_{slab} + h_{buildup} + h_{beam} = 45.5 \cdot \text{in} \quad \text{Overall height of the section}$$

$$A_{deck.comp} := t_{slab} \cdot b_{e.tr} = 795.159 \cdot \text{in}^2$$

$$A_{buildup.comp} := h_{buildup} \cdot b_{beam.top.flange} \cdot \frac{E_{c.slab}}{E_{c.beam}} = 41.577 \cdot \text{in}^2$$

$$A_{g.comp} := A_{beam} + A_{deck.comp} + A_{buildup.comp} = 1.643 \times 10^3 \cdot \text{in}^2$$

Area

$$y_{g.comp.bot} := \frac{A_{deck.comp} \cdot \left(h_{g.comp} - \frac{t_{slab}}{2} \right) \dots + A_{buildup.comp} \cdot \left(h_{g.comp} - t_{slab} - \frac{h_{buildup}}{2} \right) \dots + A_{beam} \cdot y_{g.bot}}{A_{g.comp}} = 28.977 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A

$$y_{g.comp.top} := h_{g.comp} - y_{g.comp.bot} = 16.523 \cdot \text{in} \quad \text{Distance from top fiber (slab) to N.A}$$

$$y_{g.comp.topbeam} := h_{beam} - y_{g.comp.bot} = 7.023 \cdot \text{in} \quad \text{Distance from top fiber (beam) to N.A}$$

$$I_{g.comp} := \frac{1}{12} \cdot \left(b_e \cdot \frac{E_{c.slab}}{E_{c.beam}} \right) \cdot t_{slab}^3 + A_{deck.comp} \cdot \left(y_{g.comp.top} - \frac{t_{slab}}{2} \right)^2 \dots = 4.333 \times 10^5 \cdot \text{in}^4$$

$$+ \frac{1}{12} \cdot \left(b_{beam.top.flange} \cdot \frac{E_{c.slab}}{E_{c.beam}} \right) \cdot h_{buildup}^3 \dots$$

$$+ A_{buildup.comp} \cdot \left(h_{g.comp} - t_{slab} - \frac{h_{buildup}}{2} \right)^2 \dots$$

$$+ I_g + A_g \cdot (y_{g.comp.bot} - y_{g.bot})^2 \quad \text{Moment inertia}$$

$$S_{g.comp.top} := \frac{I_{g.comp}}{y_{g.comp.top}} = 2.622 \times 10^4 \cdot \text{in}^3$$

$$S_{g.comp.bot} := \frac{I_{g.comp}}{y_{g.comp.bot}} = 1.495 \times 10^4 \cdot \text{in}^3$$

$$S_{g.comp.topbeam} := \frac{I_{g.comp}}{y_{g.comp.topbeam}} = 6.169 \times 10^4 \cdot \text{in}^3$$

$$e_g := y_{g.top} + h_{buildup} + \frac{t_{slab}}{2} = 24.76 \cdot \text{in}$$

Distance between the c.g. of the basic beam and deck

4. LOAD AND LOAD DISTRIBUTION

4.1 Dead Load

$w_{\text{beam}} := A_g \cdot \gamma_{\text{beam}} = 0.840 \cdot \frac{\text{kip}}{\text{ft}}$	Self weight of beam
$w_{\text{slab}} := A_{\text{slab}} \cdot \gamma_{\text{slab}} = 0.956 \cdot \frac{\text{kip}}{\text{ft}}$	Self weight of slab
$w_{\text{buildup}} := A_{\text{buildup}} \cdot \gamma_{\text{slab}} = 0.040 \cdot \frac{\text{kip}}{\text{ft}}$	Self weight of buildup
$w_{\text{slab.buildup}} := w_{\text{slab}} + w_{\text{buildup}} = 0.996 \cdot \frac{\text{kip}}{\text{ft}}$	Sum of self weight of slab and buildup
$w_{\text{barrier.each}} := 0.430 \frac{\text{kip}}{\text{ft}}$	Traffic railing barrier DL, "SR 687/4th Street, NB bridge"
$w_{\text{barrier}} := \frac{2 \cdot w_{\text{barrier.each}}}{\text{NumberOfBeam}} = 0.172 \cdot \frac{\text{kip}}{\text{ft}}$	For purpose of this design example, all barrier loads are equally distributed to the total number of beams
$w_{\text{forms}} := 20\text{psf} \cdot (\text{BeamSpacing} - b_{\text{beam.top.flange}}) = 0.100 \cdot \frac{\text{kip}}{\text{ft}}$	Stay-in-place forms, 20 psf "SR 687/4th Street, NB bridge"
$w_{\text{fws}} := 0 \frac{\text{kip}}{\text{ft}}$	Future wearing surface
$w_{\text{utility}} := 0 \frac{\text{kip}}{\text{ft}}$	Utility dead load
<u>for interior beams</u>	DC Dead load (LRFD 3.3.2)

$$M_{sf.beam}(x) := \frac{w_{beam} \cdot Span}{2} \cdot x - \frac{w_{beam} \cdot x^2}{2}$$

Self weight of beam
(x distance from support)

$$V_{sf.beam}(x) := \frac{w_{beam} \cdot Span}{2} - w_{beam} \cdot x$$

$$M_{sf.slab}(x) := \frac{w_{slab.buildup} \cdot Span}{2} \cdot x - \frac{w_{slab.buildup} \cdot x^2}{2}$$

Self weight of deck and buildup

$$V_{sf.slab}(x) := \frac{w_{slab.buildup} \cdot Span}{2} - w_{slab.buildup} \cdot x$$

$$M_{forms}(x) := \frac{w_{forms} \cdot Span}{2} \cdot x - \frac{w_{forms} \cdot x^2}{2}$$

Stay-in-place forms

$$V_{forms}(x) := \frac{w_{forms} \cdot Span}{2} - w_{forms} \cdot x$$

$$M_{barrier}(x) := \frac{w_{barrier} \cdot Span}{2} \cdot x - \frac{w_{barrier} \cdot x^2}{2}$$

Barriers

$$V_{barrier}(x) := \frac{w_{barrier} \cdot Span}{2} - w_{barrier} \cdot x$$

Total DC dead load

$$w_{DC} := w_{beam} + w_{slab.buildup} + w_{forms} + w_{barrier} = 2.108 \cdot \frac{\text{kip}}{\text{ft}}$$

$$M_{DC}(x) := M_{sf.beam}(x) + M_{sf.slab}(x) + M_{forms}(x) + M_{barrier}(x)$$

$$V_{DC}(x) := V_{sf.beam}(x) + V_{sf.slab}(x) + V_{forms}(x) + V_{barrier}(x)$$

DW Dead load

(LRFD 3.3.2)

$$M_{fws}(x) := \frac{w_{fws} \cdot \text{Span}}{2} \cdot x - \frac{w_{fws} \cdot x^2}{2}$$

Future wearing surface
(x distance from support)

$$V_{fws}(x) := \frac{w_{fws} \cdot \text{Span}}{2} - w_{fws} \cdot x$$

$$M_{utility}(x) := \frac{w_{utility} \cdot \text{Span}}{2} \cdot x - \frac{w_{utility} \cdot x^2}{2}$$

Utility

$$V_{utility}(x) := \frac{w_{utility} \cdot \text{Span}}{2} - w_{utility} \cdot x$$

$$w_{DW} := w_{fws} + w_{utility} = 0 \cdot \frac{\text{kip}}{\text{ft}}$$

Total DW dead load

$$M_{DW}(x) := M_{fws}(x) + M_{utility}(x)$$

$$V_{DW}(x) := V_{fws}(x) + V_{utility}(x)$$

4.2 Live Load

$$IM_{\text{fatigue}} := 1.15$$

Impact factor (dynamic load effects) for fatigue limit states

(LRFD Table 3.6.2.1-1)

$$IM := 1.33$$

Impact factor (dynamic load effects) for limit states other than fatigue and fracture

(LRFD Table 3.6.2.1-1)

$$w_{\text{lane}} := 0.64 \frac{\text{kip}}{\text{ft}}$$

Uniformly distributed design lane load

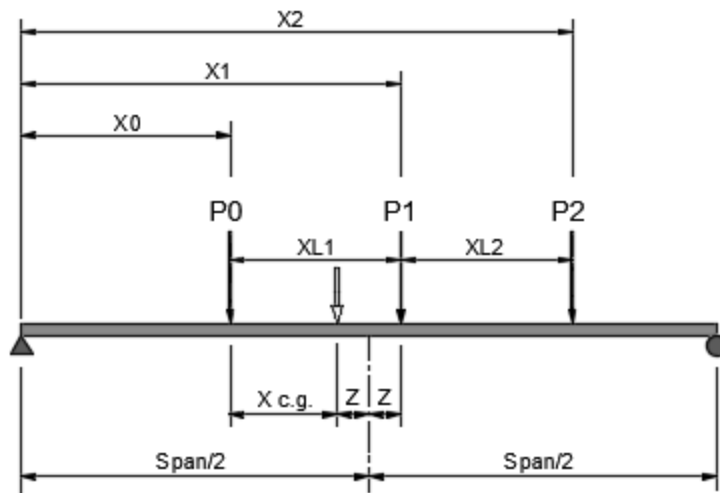
(LRFD 3.6.1.2.4)

$$M_{\text{lane}}(x) := \frac{w_{\text{lane}} \cdot \text{Span}}{2} \cdot x - \frac{w_{\text{lane}} \cdot x^2}{2}$$

Moment and shear introduced by design lane load

x distance from left support

$$V_{\text{lane}}(x) := \frac{w_{\text{lane}} \cdot \text{Span}}{2} - w_{\text{lane}} \cdot x$$



Maximum Live Load Moment

(FDOT design examples)

----- Live Load (except for fatigue limit state) -----

$$P := \begin{pmatrix} 32 \\ 32 \\ 8 \end{pmatrix} \cdot \text{kip}$$

HL-93 Truck load, axle loads and spacing
(LRFD 3.6.1.2.2)

$$XL := \begin{pmatrix} 0 \\ 14 \\ 14 \end{pmatrix} \cdot \text{ft}$$

$$x_{cg} := \frac{P_0 \cdot XL_0 + P_1 \cdot (XL_0 + XL_1) + P_2 \cdot (XL_0 + XL_1 + XL_2)}{P_0 + P_1 + P_2} = 9.333 \text{ ft}$$

Center of gravity for axle loads

$$z := \frac{XL_1 - x_{cg}}{2} = 2.333 \cdot \text{ft}$$

Distance from c.g. of axle loads to centerline of span

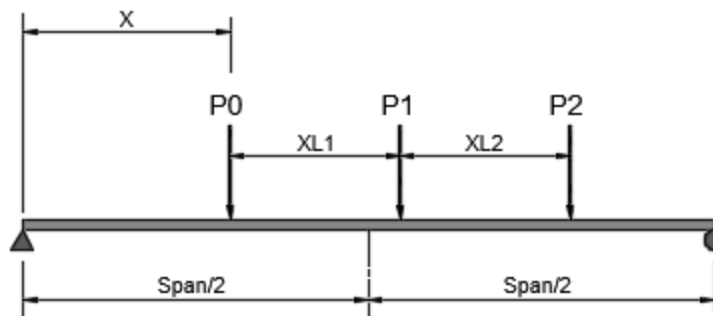
$$X_{left} := \frac{\text{Span}}{2} - z - x_{cg} = 32.167 \text{ ft}$$

$$X := \begin{pmatrix} X_{left} \\ X_{left} + XL_1 \\ X_{left} + XL_1 + XL_2 \end{pmatrix} = \begin{pmatrix} 32.167 \\ 46.167 \\ 60.167 \end{pmatrix} \text{ ft}$$

Distance from left support to axle loads

Live load moments and shear - One HL-93 truck

Case 1



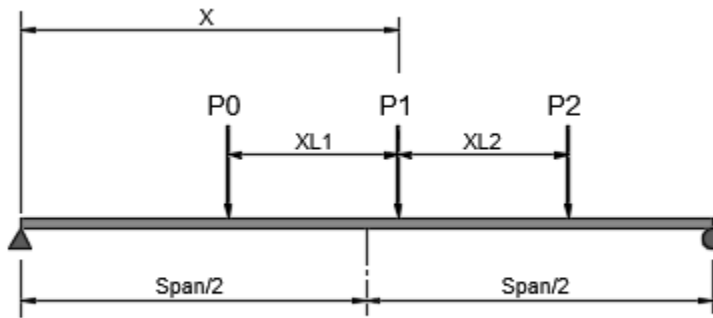
CASE 1

(FDOT design examples)

$$M_{\text{truck1}}(x) := P_0 \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x - XL_1) \cdot x}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_1 - XL_2) \cdot x}{\text{Span}}$$

$$V_{\text{truck1}}(x) := P_0 \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x - XL_1)}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_1 - XL_2)}{\text{Span}}$$

Case 2



CASE 2

(FDOT design examples)

$$M_{\text{truck2}}(x) := P_0 \cdot \frac{(\text{Span} - x) \cdot (x - XL_1)}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_2) \cdot x}{\text{Span}}$$

$$V_{\text{truck2}}(x) := P_0 \cdot \frac{-(x - XL_1)}{\text{Span}} + P_1 \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_2 \cdot \frac{(\text{Span} - x - XL_2)}{\text{Span}}$$

$$M_{\text{truck}}(x) := \max(M_{\text{truck1}}(x), M_{\text{truck2}}(x))$$

Moment and shear introduced by Truck HL-93

$$V_{\text{truck}}(x) := \max(V_{\text{truck1}}(x), V_{\text{truck2}}(x))$$

$$M_{\text{LL.IM}}(x) := M_{\text{truck}}(x) \cdot \text{IM} + M_{\text{lane}}(x)$$

Live load (truck and lane) moment and shear, including impact (IM is for limit states other than fatigue)

$$V_{\text{LL.IM}}(x) := V_{\text{truck}}(x) \cdot \text{IM} + V_{\text{lane}}(x)$$



CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FIB-36

$$M_{LL}(x) := M_{truck}(x) + M_{lane}(x)$$

Live load (truck and lane) moment and shear, excluding impact

$$V_{LL}(x) := V_{truck}(x) + V_{lane}(x)$$

----- Live Load for fatigue limit state -----

$$P_f := \begin{pmatrix} 32 \\ 32 \\ 8 \end{pmatrix} \cdot \text{kip}$$

$$XL_f := \begin{pmatrix} 0 \\ 30 \\ 14 \end{pmatrix} \cdot \text{ft}$$

HL-93 Truck load, axle loads and spacing
(LRFD 3.6.1.2.2)

$$x_{cg.f} := \frac{P_{f_0} \cdot XL_{f_0} + P_{f_1} \cdot (XL_{f_0} + XL_{f_1}) + P_{f_2} \cdot (XL_{f_0} + XL_{f_1} + XL_{f_2})}{P_{f_0} + P_{f_1} + P_{f_2}} = 18.222 \text{ ft}$$

Center of gravity for axle loads

$$z_f := \frac{XL_{f_1} - x_{cg.f}}{2} = 5.889 \cdot \text{ft}$$

Distance from c.g. of axle loads to centerline of span

$$X_{left.f} := \frac{\text{Span}}{2} - z_f - x_{cg.f} = 19.722 \text{ ft}$$

$$X_f := \begin{pmatrix} X_{left.f} \\ X_{left.f} + XL_{f_1} \\ X_{left.f} + XL_{f_1} + XL_{f_2} \end{pmatrix} = \begin{pmatrix} 19.722 \\ 49.722 \\ 63.722 \end{pmatrix} \text{ ft}$$

Distance from left support to axle loads

Live load moments and shear - One HL-93 truck

Case 1

$$M_{truck1.f}(x) := P_{f_0} \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x - XL_{f_1}) \cdot x}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - XL_{f_1} - XL_{f_2}) \cdot x}{\text{Span}}$$

$$V_{truck1.f}(x) := P_{f_0} \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x - XL_{f_1})}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - XL_{f_1} - XL_{f_2})}{\text{Span}}$$

Case 2

$$M_{\text{truck2.f}(x)} := P_{f_0} \cdot \frac{(\text{Span} - x) \cdot (x - XL_{f_1})}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x) \cdot x}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - XL_{f_2}) \cdot x}{\text{Span}}$$

$$V_{\text{truck2.f}(x)} := P_{f_0} \cdot \frac{-(x - XL_{f_1})}{\text{Span}} + P_{f_1} \cdot \frac{(\text{Span} - x)}{\text{Span}} + P_{f_2} \cdot \frac{(\text{Span} - x - XL_{f_2})}{\text{Span}}$$

$$M_{\text{truck.f}(x)} := \max(M_{\text{truck1.f}(x)}, M_{\text{truck2.f}(x)})$$

Moment and shear introduced by Truck HL-93, for fatigue limit state

$$V_{\text{truck.f}(x)} := \max(V_{\text{truck1.f}(x)}, V_{\text{truck2.f}(x)})$$

$$M_{\text{LL.IM.f}(x)} := M_{\text{truck.f}(x)} \cdot IM_{\text{fatigue}}$$

Fatigue Live load (truck) moment and shear

$$V_{\text{LL.IM.f}(x)} := V_{\text{truck.f}(x)} \cdot IM_{\text{fatigue}}$$

at midspan

$$M_{\text{LL.IM.f}}\left(\frac{\text{Span}}{2}\right) = 1198 \cdot \text{kip} \cdot \text{ft}$$

$$V_{\text{LL.IM.f}}\left(\frac{\text{Span}}{2}\right) = 24 \cdot \text{kip}$$

4.3 Live Load Distribution Factors

(LRFD 4.6.2.2)

Live load on the deck must be distributed to the precast prestressed beams. AASHTO provides factors for the distribution of live load into the beams. The factors can be used if the following criteria is met:

- (1) Width of deck is constant
- (2) Number of beams is not less than four
- (3) Beams are parallel and have approximately the same stiffness
- (4) The overhang minus the barrier width does not exceed 3.0 feet
- (5) Curvature in plan is less than the limit specified in LRFD 4.6.1.2.4
- (6) Cross-section is consistent with one of the cross-sections shown in Table 4.6.2.2.1-1

In this example, all the conditions are satisfied

$$K_g := \frac{E_{c.beam}}{E_{c.slub}} \cdot (I_g + A_g \cdot e_g^2) = 7.181 \times 10^5 \cdot \text{in}^4$$

Longitudinal stiffness parameter
(LRFD 4.6.2.2.1)

Type k cross-section in LRFD Table 4.6.2.2.1

Moment distribution factor for interior beams

When one design lane loaded

$$g_{m.1} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot t_s^3}\right)^{0.1}$$

(LRFD Table 4.6.2.2.2b-1)

$$g_{m.1} := 0.06 + \left(\frac{\text{BeamSpacing}}{14 \text{ ft}}\right)^{0.4} \cdot \left(\frac{\text{BeamSpacing}}{\text{Span}}\right)^{0.3} \cdot \left(\frac{K_g}{12.0 \cdot \frac{\text{in}}{\text{ft}} \cdot \text{Span} \cdot t_{\text{slab}}^3}\right)^{0.1} = 0.488$$

Distribution factor for moment in interior
beams when one lane is loaded

When two or more design lane loaded

$$g_{m.2} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot t_s^3}\right)^{0.1}$$

(LRFD Table 4.6.2.2.2b-1)

$$g_{m.2} := 0.075 + \left(\frac{\text{BeamSpacing}}{9.5\text{-ft}}\right)^{0.6} \cdot \left(\frac{\text{BeamSpacing}}{\text{Span}}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot \frac{\text{in}}{\text{ft}} \cdot \text{Span} \cdot t_{\text{slab}}^3}\right)^{0.1} = 0.696$$

Distribution factor for moment in interior beams when two or more lanes are loaded

$$g_m := \max(g_{m.1}, g_{m.2}) = 0.696$$

Distribution factor for moment in interior beams

Verify if the distribution factor satisfies LRFD criteria for "Range of Applicability"

$$g_{m.\text{check}} := \begin{cases} S \leftarrow 3.5\text{ft} \leq \text{BeamSpacing} \leq 16\text{ft} & = \text{"OK"} \\ t_s \leftarrow 4.5\text{in} \leq t_{\text{slab}} \leq 12\text{in} \\ L \leftarrow 20\text{ft} \leq \text{Span} \leq 240\text{ft} \\ N_b \leftarrow \text{NumberOfBeam} \geq 4 \\ K_g \leftarrow 10000\text{in}^4 \leq K_g \leq 7000000\text{in}^4 \\ \text{"OK"} \text{ if } (S \cdot t_s \cdot L \cdot N_b \cdot K_g) = 1 \\ \text{"NG"} \text{ otherwise} \end{cases}$$

$$g_{m.\text{check}} = \text{"OK"}$$

Shear distribution factor for interior beams

When one design lane loaded

$$g_{v.1} = 0.36 + \frac{S}{25.0} \quad (\text{LRFD Table 4.6.2.2.3a-1})$$

$$g_{v.1} := 0.36 + \frac{\text{BeamSpacing}}{25.0\text{-ft}} = 0.72 \quad \text{Distribution factor for shear in interior beams when one lane is loaded}$$

When two or more design lane loaded

$$g_{m.2} = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0} \quad (\text{LRFD Table 4.6.2.2.3a-1})$$



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$$g_{v,2} := 0.2 + \frac{\text{BeamSpacing}}{12 \cdot \text{ft}} - \left(\frac{\text{BeamSpacing}}{35 \cdot \text{ft}} \right)^{2.0} = 0.884$$

Distribution factor for shear in interior beams when two or more lanes are loaded

$$g_v := \max(g_{v,1}, g_{v,2}) = 0.884$$

Distribution factor for shear in interior beams

Verify if the distribution factor satisfies LRFD criteria for "Range of Applicability"

$$g_{v,\text{check}} := \begin{cases} S \leftarrow 3.5\text{ft} \leq \text{BeamSpacing} \leq 16\text{ft} = \text{"OK"} \\ t_s \leftarrow 4.5\text{in} \leq t_{\text{slab}} \leq 12\text{in} \\ L \leftarrow 20\text{ft} \leq \text{Span} \leq 240\text{ft} \\ N_b \leftarrow \text{NumberOfBeam} \geq 4 \\ \text{"OK"} \quad \text{if } (S \cdot t_s \cdot L \cdot N_b) = 1 \\ \text{"NG"} \quad \text{otherwise} \end{cases}$$

$$g_{v,\text{check}} = \text{"OK"}$$

Summary of live load distribution factors, interior beams

$$g_m = 0.696$$

Distribution factor for moment in interior beams

$$g_v = 0.884$$

Distribution factor for shear in interior beams

$$\underline{M_{LL,IM}(x)} := g_m \cdot M_{LL,IM}(x)$$

Re-define
Live load (truck and lane) moment and shear, including impact (IM is for limit states other than fatigue)
Consider LL distribution factor

$$\underline{V_{LL,IM}(x)} := g_v \cdot V_{LL,IM}(x)$$

$$\underline{M_{LL}(x)} := g_m \cdot M_{LL}(x)$$

Re-define
Live load (truck and lane) moment and shear, excluding impact
Consider LL distribution factor

$$\underline{V_{LL}(x)} := g_v \cdot V_{LL}(x)$$



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$$M_{LL.IM.f}(x) := \frac{g_m}{1.2} \cdot M_{LL.IM.f}(x)$$

$$V_{LL.IM.f}(x) := \frac{g_v}{1.2} \cdot V_{LL.IM.f}(x)$$

Re-define
Live load (truck) moment and shear,
including impact (IM is for fatigue)
Consider LL distribution factor

Noted that the distribution factor calculated in LRFD 4.6.2.2 for a single lane loaded already includes the 1.2 "multiple presence factor" for a single lane, therefore, this value may be used for the service and strength limit states. However, multiple presence factors should not be used for the fatigue limit state. Therefore, the multiple presence factor of 1.2 for the single lane is required to be removed from the value calculated above to determine the factor used for the fatigue limit state

4.4 Summary of Moments and Shear Forces

Strength1 = 1.25DC + 1.5DW + 1.75LL Strength I limit state

Service1 = 1.0DC + 1.0DW + 1.0LL Service I limit state

Service3 = 1.0DC + 1.0DW + 0.8LL Service III limit state

Fatigue1 = 1.75LL_{fatigue} Fatigue I limit state

$M_{Str1}(x) := 1.25 \cdot M_{DC}(x) + 1.5 \cdot M_{DW}(x) + 1.75 \cdot M_{LL.IM}(x)$ Strength I limit state

$M_{Srv1}(x) := 1.0 \cdot M_{DC}(x) + 1.0 \cdot M_{DW}(x) + 1.0 \cdot M_{LL.IM}(x)$ Service I limit state

$M_{Srv3}(x) := 1.0 \cdot M_{DC}(x) + 1.0 \cdot M_{DW}(x) + 0.8 \cdot M_{LL.IM}(x)$ Service III limit state

$M_{fatigue}(x) := 1.75 \cdot M_{LL.IM.f}(x)$ Fatigue I limit state

$V_{Str1}(x) := 1.25 \cdot V_{DC}(x) + 1.5 \cdot V_{DW}(x) + 1.75 \cdot V_{LL.IM}(x)$ Strength I limit state

$V_{Srv1}(x) := 1.0 \cdot V_{DC}(x) + 1.0 \cdot V_{DW}(x) + 1.0 \cdot V_{LL.IM}(x)$ Service I limit state

$V_{Srv3}(x) := 1.0 \cdot V_{DC}(x) + 1.0 \cdot V_{DW}(x) + 0.8 \cdot V_{LL.IM}(x)$ Service III limit state

$V_{fatigue}(x) := 1.75 \cdot V_{LL.IM.f}(x)$ Fatigue I limit state

at mispan

$M_{DC.mid} := M_{DC} \left(\frac{Span}{2} \right) = 2025 \cdot \text{kip} \cdot \text{ft}$ Due to DC

$M_{DW.mid} := M_{DW} \left(\frac{Span}{2} \right) = 0 \cdot \text{kip} \cdot \text{ft}$ Due to DW



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DESIGN EXAMPLE: FIB-36**

$$M_{LL.IM.mid} := M_{LL.IM} \left(\frac{\text{Span}}{2} \right) = 1628 \cdot \text{kip} \cdot \text{ft} \quad \text{Due to LL}$$

$$M_{LL.IM.fatigue.mid} := M_{LL.IM.f} \left(\frac{\text{Span}}{2} \right) = 695 \cdot \text{kip} \cdot \text{ft} \quad \text{Due to LL}$$

$$M_{Str1.mid} := 1.25 \cdot M_{DC.mid} + 1.5 \cdot M_{DW.mid} + 1.75 \cdot M_{LL.IM.mid} = 5381 \cdot \text{kip} \cdot \text{ft}$$

Strength I limit state

$$M_{Srv1.mid} := 1.0 \cdot M_{DC.mid} + 1.0 \cdot M_{DW.mid} + 1.0 \cdot M_{LL.IM.mid} = 3654 \cdot \text{kip} \cdot \text{ft}$$

Service I limit state

$$M_{Srv3.mid} := 1.0 \cdot M_{DC.mid} + 1.0 \cdot M_{DW.mid} + 0.8 \cdot M_{LL.IM.mid} = 3328 \cdot \text{kip} \cdot \text{ft}$$

Service III limit state

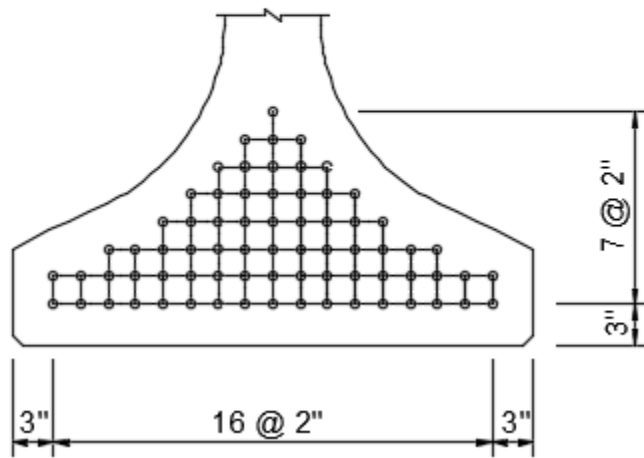
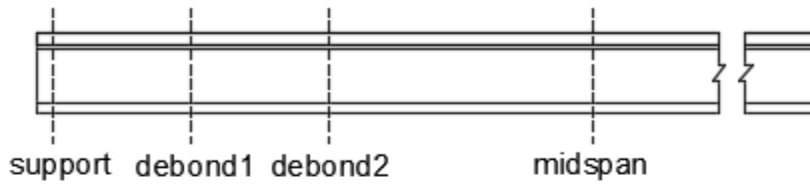
$$M_{fatigue.mid} := 1.75 \cdot M_{LL.IM.fatigue.mid} = 1216 \cdot \text{kip} \cdot \text{ft}$$

Fatigue I limit state

5. STRAND PATTERN

for FIB-36 Beam

The pattern of strands need to be adjusted according to limit states



support := 0ft

Distance from left support

debond1 := 10ft

Debond place1, "0" if no need debonding

debond2 := 20ft

Debond place2, "0" if no need debonding

$$\text{midspan} := \frac{\text{Span}}{2} = 43.833 \text{ ft}$$

Max number of strands in each row, from bottom up

$$n_{\max} := \begin{pmatrix} 17 \\ 17 \\ 13 \\ 9 \\ 7 \\ 5 \\ 3 \\ 1 \end{pmatrix}$$

$$n := \begin{pmatrix} 17 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Number of strands in each row, from bottom up

Adjust the numbers to satisfy Service III and Strength I limit states (Sections 9 and 11 in this example)

Note: Service I can be satisfied by debonding (Section 8 in this example)

$$n_{\text{total}} := \sum_{i=0}^{\text{rows}(n)-1} n_i = 39$$

Total number of strands

$$A_p := n \cdot A_{\text{pf}} = \begin{pmatrix} 3.043 \\ 3.043 \\ 0.895 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{in}^2$$

Total area of strands in each row

$$A_{p,\text{total}} := \sum_{i=0}^{\text{rows}(n)-1} A_{p_i} = 6.981 \cdot \text{in}^2$$

Total area of strands

$$d_p := \begin{pmatrix} h_g - 3\text{in} \\ h_g - 5\text{in} \\ h_g - 7\text{in} \\ h_g - 9\text{in} \\ h_g - 11\text{in} \\ h_g - 13\text{in} \\ h_g - 15\text{in} \\ h_g - 17\text{in} \end{pmatrix} = \begin{pmatrix} 33 \\ 31 \\ 29 \\ 27 \\ 25 \\ 23 \\ 21 \\ 19 \end{pmatrix} \cdot \text{in}$$

Distance from top beam section to the center of each strand row



**CFRP -PC TRAINING COURSE
DESIGN EXAMPLE: FIB-36**

$$d_{p.strands.c.g} := \frac{\sum_{i=0}^{rows(n)-1} (A_{p_i} \cdot d_{p_i})}{A_{p.total}} = 31.615 \cdot \text{in}$$

Distance from top beam section to the gravity center of strand group

$$d_{p.strands.c.g.bot} := h_g - d_{p.strands.c.g} = 4.385 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group

6. TRANSFORMED SECTION PROPERTIES

6.1 Transformed Section Properties - Non-Composite Section (Girder+Strands)

$$n_p = 4.397 \quad \text{Elastic modulus ratio, } E_p/E_c$$

$$A_{g.tr} := A_g + (n_p - 1)A_{p.total} = 830.295 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g.tr.top} := \frac{A_g \cdot y_{g.top} + \sum_{i=0}^{\text{rows}(n)-1} [A_{p_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g.tr}} = 19.856 \cdot \text{in}$$

Distance from top fiber (beam) to N.A

$$y_{g.tr.bot} := h_g - y_{g.tr.top} = 16.144 \cdot \text{in} \quad \text{Distance from bottom fiber (beam) to N.A}$$

$$I_{g.tr} := I_g + A_g \cdot (y_{g.tr.top} - y_{g.top})^2 + \sum_{i=0}^{\text{rows}(n)-1} \left[(n_p - 1) \cdot A_{p_i} \cdot (d_{p_i} - y_{g.tr.top})^2 \right] = 1.31 \times 10^5 \cdot \text{in}^4$$

$$S_{g.tr.top} := \frac{I_{g.tr}}{y_{g.tr.top}} = 6.596 \times 10^3 \cdot \text{in}^3 \quad \text{Moment inertia}$$

$$S_{g.tr.bot} := \frac{I_{g.tr}}{y_{g.tr.bot}} = 8.112 \times 10^3 \cdot \text{in}^3$$

$$e_{g.tr.cg.strands} := y_{g.tr.bot} - d_{p.strands.c.g.bot} = 11.76 \cdot \text{in}$$

eccentricity of strand group

**6.2 Transformed Section Properties - Composite Section
(Girder+Deck+Strands+Bars)**

Assume #5 GFRP bar at 12" spacing, longitudinal direction, in both top and bottom of the deck slab

$$n_b = 1.271 \quad \text{Elastic modulus ratio, } E_{gfrp}/E_c$$

$$A_{deck.rebar} := \frac{b_e}{12in} \cdot A_{GFRP} \cdot 2 = 5.522 \cdot in^2$$

$$y_{rebar} := h_g + h_{buildup} + \frac{t_{slab}}{2} = 3.437 \text{ ft} \quad \text{Distance from c.g. of rebar group (in deck slab) to bottom of girder section}$$

$$A_{g.comp.tr} := A_{g.comp} + (n_p - 1)A_{p.total} + (n_b - 1) \cdot A_{deck.rebar} = 1.669 \times 10^3 \cdot in^2$$

Area

$$y_{g.comp.tr.top} := \frac{A_{g.comp} \cdot y_{g.comp.top} + \sum_{i=0}^{rows(n)-1} [A_{p_i} \cdot (n_p - 1) \cdot d_{p_i}] \dots + (n_b - 1) \cdot A_{deck.rebar} \cdot \frac{t_{slab}}{2}}{A_{g.comp.tr}} = 16.726 \cdot in$$

Distance from top fiber (deck) to N.A

$$y_{g.comp.tr.bot} := h_{g.comp} - y_{g.comp.tr.top} = 28.774 \cdot in \quad \text{Distance from bottom fiber (beam) to N.A}$$

$$y_{g.comp.tr.topbeam} := y_{g.comp.tr.top} - t_{slab} - h_{buildup} = 7.226 \cdot in$$

Distance from top fiber of the beam to N.A

$$I_{g.comp.tr} := I_{g.comp} + A_{g.comp} \cdot (y_{g.comp.tr.top} - y_{g.comp.top})^2 \dots = 4.389 \times 10^5 \cdot in^4$$

$$+ \sum_{i=0}^{rows(n)-1} \left[(n_p - 1) \cdot A_{p_i} \cdot (d_{p_i} - y_{g.comp.tr.top})^2 \right] \dots$$

$$+ (n_b - 1) \cdot A_{deck.rebar} \cdot \left(\frac{t_{slab}}{2} - y_{g.comp.tr.top} \right)^2 \quad \text{Moment inertia}$$

$$S_{g.comp.tr.top} := \frac{I_{g.comp.tr}}{y_{g.comp.tr.top}} = 2.624 \times 10^4 \cdot \text{in}^3$$

$$S_{g.comp.tr.bot} := \frac{I_{g.comp.tr}}{y_{g.comp.tr.bot}} = 1.525 \times 10^4 \cdot \text{in}^3$$

$$S_{g.comp.tr.topbeam} := \frac{I_{g.comp.tr}}{y_{g.comp.tr.topbeam}} = 6.073 \times 10^4 \cdot \text{in}^3$$

$$e_{g.comp.tr.cg.strands} := y_{g.comp.tr.bot} - d_{p.strands.c.g.bot} = 24.389 \cdot \text{in}$$

Eccentricity of strand group

7. PRESTRESS LOSSES

$$\Delta f_p = \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pTH}$$

Total prestress loss
(AASHTO CFRP 1.9.2.1)

Δf_{pES} : prestress loss due to elastic shortening

Δf_{pLT} : long-term time-dependent prestress losses, due to shrinkage, creep, and relaxation

Δf_{pTH} : prestress losses due to temperature change (Not considered in this example)

7.1 Prestress Loss due to Elastic Shortening

$$\Delta f_{pES} = \frac{E_f}{E_{ct}} f_{cgp}$$

Prestress loss due to elastic shortening
(AASHTO CFRP 1.4.2)

$$f_{pi} = 239 \cdot \text{ksi}$$

Jacking stress

$$F_{pi} := A_{p,\text{total}} \cdot f_{pi} = 1.665 \times 10^3 \cdot \text{kip}$$

Total jacking force

$$e_{g,\text{tr},\text{cg},\text{strands}} = 11.76 \cdot \text{in}$$

Eccentricity of strand group to N.A. of the transformed non-composite section

$$M_{\text{sf},\text{beam},\text{midspan}} := M_{\text{sf},\text{beam}} \left(\frac{\text{Span}}{2} \right) = 807.152 \cdot \text{kip} \cdot \text{ft}$$

Moment at midspan due to self weight of beam

$$f_{cgp} := \frac{F_{pi}}{A_{g,\text{tr}}} + \frac{F_{pi} \cdot e_{g,\text{tr},\text{cg},\text{strands}}^2}{I_{g,\text{tr}}} - \frac{M_{\text{sf},\text{beam},\text{midspan}} \cdot e_{g,\text{tr},\text{cg},\text{strands}}}{I_{g,\text{tr}}} = 2.894 \cdot \text{ksi}$$

Concrete stress at gravity center of the prestressing force at transfer

$$\Delta f_{pES} := \frac{E_p}{E_{ci,\text{beam}}} f_{cgp} = 14.277 \cdot \text{ksi}$$

Prestress loss due to elastic shortening
(AASHTO CFRP 1.4.2)

$$f_{pt} := f_{pi} - \Delta f_{pES} = 224.27 \cdot \text{ksi}$$

Prestress stress at transfer

7.2 Long-Term Time-Dependent Prestress Losses (Refined Estimation)

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df}$$

Total long-term time-dependent losses
id - time between transfer and deck placement
df - time between deck placement and the final time

(LRFD 5.9.3.4.1-1)

-----Time-Dependent losses between transfer and deck placement-----

Shrinkage of girder concrete (i->d)

$$\Delta f_{pSR} = \epsilon_{bid} \cdot E_p \cdot K_{id} \quad \text{(LRFD 5.9.3.4)}$$

$$k_s := \max\left(1.45 - 0.13 \frac{A_g}{\text{Perimeter}_{\text{beam}} \cdot \text{in}}, 1.0\right) = 1.000$$

Factor for the effect of the volume-to-surface ratio of the component
(LRFD 5.4.2.3.2)

$$k_{hs} := 2.00 - 0.014RH = 0.950$$

Humidity factor for shrinkage
(LRFD 5.4.2.3.3)

$$k_{hc} := 1.56 - 0.008RH = 0.960$$

Humidity factor for creep
(LRFD 5.4.2.3.2)

$$k_f := \frac{5}{1 + \frac{f_{ci.beam}}{\text{ksi}}} = 0.714$$

Factor for the effect of concrete strength
(LRFD 5.4.2.3.2)

$$k_{td}(t_0, t_1) := \frac{t_1 - t_0}{12 \left(\frac{100 - 4 \cdot \frac{f_{ci.beam}}{\text{ksi}}}{\frac{f_{ci.beam}}{\text{ksi}} + 20} \right) + (t_1 - t_0)}$$

Time development factor
(LRFD 5.4.2.3.2)

$$k_{td}(t_i, t_d) = 0.772$$

Time development factor
Time from transfer to deck placement

$$k_{td}(t_i, t_f) = 0.997$$

Time development factor
Time from transfer to final time

$$k_{td}(t_d, t_f) = 0.996$$

Time development factor
Time from deck placement to final time

$$\epsilon_{bid} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}(t_i, t_d) \cdot 0.48 \cdot 10^{-3} = 2.516 \times 10^{-4}$$

Shrinkage strain of girder
(LRFD 5.4.2.3.3)

$$\Psi_b(t_1, t_0) := 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td}(t_0, t_1) \cdot t_0^{-0.118}$$

Creep coefficient
(LRFD 5.4.2.3.2)

$$\Psi_b(t_d, t_i) = 1.006$$

Time development factor
Time from transfer to deck placement

$$\Psi_b(t_f, t_i) = 1.298$$

Time development factor
Time from transfer to final time

$$\Psi_b(t_f, t_d) = 0.738$$

Time development factor
Time from deck placement to final time

$$K_{id} := \frac{1}{1 + \frac{E_p}{E_{ci.beam}} \cdot \frac{A_{p.total}}{A_{g.tr}} \cdot \left(1 + \frac{A_{g.tr} \cdot e_{g.tr.cg.strands}^2}{I_{g.tr}} \right) \cdot (1 + 0.7 \cdot \Psi_b(t_f, t_i))} = 0.871$$

Transformed section coefficient
(LRFD 5.9.3.4.2)

$$\Delta f_{pSR} := \epsilon_{bid} \cdot E_p \cdot K_{id} = 4.924 \cdot \text{ksi}$$

Prestress loss due to
Shrinkage of girder concrete
(LRFD 5.9.3.4)

Creep of girder concrete (i->d)

$$\Delta f_{pCR} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \Psi_b(t_d, t_i) \cdot K_{id} = 12.508 \cdot \text{ksi}$$

Prestress loss due to
Creep of girder concrete
(LRFD 5.9.3.4)

Relaxation of prestressing CFRP (i->d)

$$f_{pu} = 341 \cdot \text{ksi} \quad \text{Design tensile strength}$$

$$f_{pt} = 224 \cdot \text{ksi} \quad \text{Prestressing stress at transfer}$$

$$\Delta f_{pR}(t_0, t_1) := \begin{cases} \left[\left(0.019 \frac{f_{pt}}{f_{pu}} - 0.0066 \right) \cdot \log[24 \cdot (t_1 - t_0)] \cdot f_{pu} \right] & \text{if StrandType}_{CFRP} = 1 \\ \left[\left(0.013 \frac{f_{pt}}{f_{pu}} - 0.006 \right) \cdot \log[24 \cdot (t_1 - t_0)] \cdot f_{pu} \right] & \text{if StrandType}_{CFRP} = 2 \end{cases}$$

Prestress loss due to relaxation
(AASHTO CFRP 1.9.2.5.2)

$$\Delta f_{pR1} := \Delta f_{pR}(t_i, t_d) = 6.953 \cdot \text{ksi} \quad \begin{array}{l} \text{Prestress loss due to relaxation} \\ \text{Time from transfer to deck placement} \end{array}$$

Total time-dependent prestress loss between transfer and deck placement

$$\Delta f_{pLT.id} := \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} = 24.385 \cdot \text{ksi}$$

-----Time-Dependent losses between deck placement to final time-----

Shrinkage of girder concrete (d->f)

$$\Delta f_{pSD} = \epsilon_{bdf} \cdot E_p \cdot K_{df} \quad (\text{LRFD 5.9.3.4.3})$$

$$\epsilon_{bdf} := k_s \cdot k_{hs} \cdot k_f \cdot k_{td}(t_d, t_f) \cdot 0.48 \cdot 10^{-3} = 3.246 \times 10^{-4} \quad \begin{array}{l} \text{Shrinkage strain of girder} \\ (\text{LRFD 5.4.2.3.3}) \end{array}$$

$$e_{g.comp.tr.cg.strands} = 24.389 \cdot \text{in} \quad \begin{array}{l} \text{Eccentricity of prestressing force with} \\ \text{respect to composite section} \\ (\text{LRFD 5.9.3.4.3}) \end{array}$$

$$I_{g.comp.tr} = 4.389 \times 10^5 \cdot \text{in}^4 \quad \begin{array}{l} \text{Moment inertia for transformed composite} \\ \text{section} \end{array}$$

$$A_{g,comp.tr} = 1.669 \times 10^3 \cdot \text{in}^2$$

Area for transformed composite section

$$K_{df} := \frac{1}{1 + \frac{E_p}{E_{ci,beam}} \cdot \frac{A_{p,total}}{A_{g,comp.tr}} \cdot \left(1 + \frac{A_{g,comp.tr} \cdot e_{g,comp.tr} \cdot e_{g,comp.tr} \cdot \text{cg.strands}^2}{I_{g,comp.tr}} \right) \cdot (1 + 0.7 \cdot \Psi_b(t_f, t_i))} = 0.886$$

Transformed section coefficient
(LRFD 5.9.3.4.3)

$$\Delta f_{pSD} := \epsilon_{bdf} \cdot E_p \cdot K_{df} = 6.465 \cdot \text{ksi}$$

Prestress loss due to
Shrinkage of girder concrete
(LRFD 5.9.3.4.3)

Creep of girder concrete (d->f)

$$P_{pLT,id} := \Delta f_{pLT,id} \cdot A_{p,total} = 170.229 \cdot \text{kip}$$

Prestress force loss between transfer to
deck placement

$$M_{DL,nc} := M_{sf,slab} \left(\frac{\text{Span}}{2} \right) + M_{forms} \left(\frac{\text{Span}}{2} \right) = 1.053 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Permanent dead load moment at midspan
(exclude self weight of beam) acting on
non-composite section

$$M_{DL,comp} := M_{barrier} \left(\frac{\text{Span}}{2} \right) + M_{fws} \left(\frac{\text{Span}}{2} \right) + M_{utility} \left(\frac{\text{Span}}{2} \right) = 165.237 \cdot \text{kip} \cdot \text{ft}$$

Permanent dead load moment at midspan
acting on composite section

$$\Delta f_{cd} := (-1) \left[\frac{P_{pLT,id}}{A_{g,tr}} + \frac{P_{pLT,id} \cdot (e_{g,tr} \cdot \text{cg.strands})^2}{I_{g,tr}} \dots \right] = -1.629 \cdot \text{ksi}$$

$$+ \frac{M_{DL,nc} \cdot e_{g,tr} \cdot \text{cg.strands}}{I_{g,tr}} + \frac{M_{DL,comp} \cdot e_{g,comp.tr} \cdot \text{cg.strands}}{I_{g,comp.tr}}$$

Change in concrete stress at centroid of
prestressing strands due to long-term
losses between transfer and deck
placement, combined with deck weight
and superimposed loads

$$\Delta f_{pCD} := \frac{E_p}{E_{ci.beam}} f_{cgp} \cdot (\Psi_b(t_f, t_i) - \Psi_b(t_d, t_i)) \cdot K_{df} + \frac{E_p}{E_{c.beam}} \Delta f_{cd} \cdot \Psi_b(t_f, t_d) \cdot K_{df} = -0.99 \cdot \text{ksi}$$

Prestress loss due to
Creep of girder concrete
(LRFD 5.9.3.4.3)

Relaxation of prestressing CFRP (d->f)

$$f_{pu} = 341 \cdot \text{ksi}$$

Design tensile strength

$$f_{pt} = 224 \cdot \text{ksi}$$

Prestress stress at transfer

$$\Delta f_{pR2} := \Delta f_{pR}(t_d, t_f) = 10.814 \cdot \text{ksi}$$

Prestress loss due to relaxation
Time from transfer to deck placement

Shrinkage of deck concrete (d->f)

$$k_{s.deck} := \max\left(1.45 - 0.13 \frac{A_{slab}}{2 \cdot b_e \cdot \text{in}}, 1.0\right) = 1.000$$

Factor for the effect of the
volume-to-surface ratio of the component
(LRFD 5.4.2.3.2)

$$k_{hs} = 0.950$$

Humidity factor for shrinkage
(LRFD 5.4.2.3.3)

$$k_{hc} = 0.960$$

Humidity factor for creep
(LRFD 5.4.2.3.2)

$$k_{f.deck} := \frac{5}{1 + \frac{f_{c.slub}}{\text{ksi}}} = 0.769$$

Factor for the effect of concrete strength
(LRFD 5.4.2.3.2)

$$k_{td.deck}(t_0, t_1) := \frac{t_1 - t_0}{12 \left(\frac{100 - 4 \cdot \frac{f_{c.slub}}{\text{ksi}}}{\frac{f_{c.slub}}{\text{ksi}} + 20} \right) + (t_1 - t_0)}$$

Time development factor
(LRFD 5.4.2.3.2)

$$k_{td.deck}(t_d, t_f) = 0.996$$

Time development factor
Time from deck placement to final time

$$\epsilon_{ddf} := k_{s.deck} \cdot k_{hs} \cdot k_{f.deck} \cdot k_{td.deck}(t_d, t_f) \cdot 0.48 \cdot 10^{-3} = 3.495 \times 10^{-4}$$

Shrinkage strain of deck
(LRFD 5.4.2.3.3)

$$\Psi_{b.deck}(t_1, t_0) := 1.9 \cdot k_{s.deck} \cdot k_{hc} \cdot k_{f.deck} \cdot k_{td.deck}(t_0, t_1) \cdot t_0^{-0.118}$$

Creep coefficient
(LRFD 5.4.2.3.2)

$$\Psi_{b.deck}(t_f, t_d) = 0.795$$

Time development factor
Time from deck placement to final time

$$e_{deck.comp} := y_{g.comp.tr.top} - \frac{t_{slab}}{2} = 12.476 \cdot \text{in}$$

Eccentricity of deck with respect to N.A. of transformed composite section

$$\Delta f_{cdf} := \frac{\epsilon_{ddf} \cdot A_{slab} \cdot E_{c.slub}}{1 + 0.7 \cdot \Psi_{b.deck}(t_f, t_d)} \cdot \left(\frac{1}{A_{g.comp.tr}} - \frac{e_{g.tr.cg.strands} \cdot e_{deck.comp}}{I_{g.comp.tr}} \right) = 0.242 \cdot \text{ksi}$$

Change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

$$\Delta f_{pSS} := \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cdf} \cdot K_{df} \cdot (1 + 0.7 \cdot \Psi_b(t_f, t_d)) = 1.43 \cdot \text{ksi}$$

Prestress loss due to shrinkage of deck concrete
(LRFD 5.9.3.4.3)

Total time-depended prestress loss between deck placement and final time

$$\Delta f_{pLT.df} := \Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS} = 14.86 \cdot \text{ksi}$$

Total time-depended prestress loss between deck placement and final time

$$\Delta f_{pLT} := \Delta f_{pLT.id} + \Delta f_{pLT.df} = 39.245 \cdot \text{ksi}$$

7.3 Summary -Prestress Loss

$$\Delta f_p := \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pTH} = 53.522 \cdot \text{ksi}$$

Total prestress loss
(AASHTO CFRP 1.9.2.1)

$$\text{Loss\%} := \frac{\Delta f_p}{f_{pi}} = 22.4\%$$

Total prestress loss percentage

$$\Delta f_{pES} = 14.277 \cdot \text{ksi}$$

Prestress loss due to elastic shortening

$$\Delta f_{pTH} = 0$$

Prestress loss due to temperature change (not considered)

$$\Delta f_{pLT} = 39.245 \cdot \text{ksi}$$

Total long-term time-dependent prestress loss

$$\Delta f_{pLT.id} = 24.385 \cdot \text{ksi}$$

Long-term time-dependent prestress loss between transfer and deck placement

$$\Delta f_{pLT.df} = 14.86 \cdot \text{ksi}$$

Long-term time-dependent prestress loss between deck placement and final time

$$f_{pi} = 239 \cdot \text{ksi}$$

Jacking stress

$$f_{pt} = 224 \cdot \text{ksi}$$

Prestressing stress at transfer

$$f_{pe} := f_{pi} - \Delta f_p = 185 \cdot \text{ksi}$$

Effect prestressing stress after all losses

$$\text{Check}_{pe.limit} := \begin{cases} \text{"OK"} & \text{if } f_{pe} \leq f_{pe.limit} \\ \text{"NG"} & \text{if } f_{pe} > f_{pe.limit} \end{cases}$$

Service stress limit (after all losses)
(AASHTO CFRP Table 1.9.1.1)

$\text{Check}_{pe.limit} = \text{"OK"}$

**8. STRESS LIMITS FOR CONCRETE
-TEMPORARY STRESSES BEFORE LOSSES (LRFD 5.9.2.3.1)**

for temporary stresses before losses (at release) (Compression +, Tension -)

$$f_{\text{compressive.allow.before.loss}} := 0.65 \cdot f_{\text{ci.beam}} = 3.900 \cdot \text{ksi}$$

Compressive stress limit for concrete
(LRFD 5.9.2.3.1a)

Note: assume minimum tension reinforcement satisfy SDG 4.3.1-1 and LRFD 5.9.2.3.1

$$f_{\text{tensile.allow.15%.before.loss}} := -0.24 \cdot \left(\frac{f_{\text{ci.beam}}}{\text{ksi}} \right)^{0.5} \text{ ksi} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete in outer
15% regions
(FDOT SDG 4.3.1)

$$f_{\text{tensile.allow.70%.bond.before.loss}} := -0.24 \cdot \left(\frac{f_{\text{ci.beam}}}{\text{ksi}} \right)^{0.5} \text{ ksi} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete in middle
70% regions, for bonded reinforcements
(FDOT SDG 4.3.1, LRFD 5.9.2.3.1b)

$$f_{\text{tensile.allow.70%.unbond.before.loss}} := -\min \left[0.0948 \cdot \left(\frac{f_{\text{ci.beam}}}{\text{ksi}} \right)^{0.5} \text{ ksi}, 0.2 \text{ ksi} \right] = -0.2 \cdot \text{ksi}$$

Tensile stress limit for concrete in middle
70% regions, for bonded reinforcements
(FDOT SDG 4.3.1, LRFD 5.9.2.3.1b)

At release, loads include self-weight of beam and the prestressing force

$$P_{\text{pi}} := A_{\text{p.total}} \cdot f_{\text{pi}} = 1.665 \times 10^3 \cdot \text{kip}$$

Total prestress force at release

8.1 At Mid-Span Section

midspan = 43.833 ft

$$f_{c.topbeam.midspan} := \frac{P_{pi}}{A_{g,tr}} + \frac{M_{sf.beam(midspan)} - P_{pi} \cdot e_{g.tr.cg.strands}}{S_{g,tr.top}} = 0.505 \cdot ksi$$

Concrete stress at top fiber of the beam at mid-span

$$f_{c.botbeam.midspan} := \frac{P_{pi}}{A_{g,tr}} - \frac{M_{sf.beam(midspan)} - P_{pi} \cdot e_{g.tr.cg.strands}}{S_{g,tr.bot}} = 3.226 \cdot ksi$$

Concrete stress at bottom fiber of the beam at mid-span

$$f_{compressive.allow.before.loss} = 3.900 \cdot ksi$$

$$f_{tensile.allow.70\%.bond.before.loss} = -0.588 \cdot ksi$$

in the pretensioned beams, at the midspan, "bonded reinforcements"

$$Check_{top.midspan} := \begin{cases} \text{if } f_{c.topbeam.midspan} \geq 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } f_{c.topbeam.midspan} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \\ \text{if } f_{c.topbeam.midspan} < 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } -f_{c.topbeam.midspan} \leq -f_{tensile.allow.70\%.bond.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \end{cases}$$

Check_{top.midspan} = "OK"

Check concrete stress limits at the midspan, at release

$$Check_{bot.midspan} := \begin{cases} \text{if } f_{c.botbeam.midspan} \geq 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } f_{c.botbeam.midspan} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \\ \text{if } f_{c.botbeam.midspan} < 0 \text{ ksi} \\ \quad \left| \begin{array}{l} \text{"OK"} \text{ if } -f_{c.botbeam.midspan} \leq -f_{tensile.allow.70\%.bond.before.loss} \\ \text{"NG"} \text{ otherwise} \end{array} \right. \end{cases}$$

Check_{bot.midspan} = "OK"

Check concrete stress limits at the midspan, at release

8.2 At Support (transfer length) Section

After transfer length, the prestressing forces are transferred to concrete, so the section that needs to check is located at transfer length distance from the support

$$\text{support}_{\text{check}} := \text{support} + L_{\text{transfer}} = 2.5 \text{ ft}$$

$$f_{c.\text{topbeam.at.release2}} := \frac{P_{\text{pi}}}{A_{\text{g.tr}}} + \frac{M_{\text{sf.beam}}(\text{support}_{\text{check}}) - P_{\text{pi}} \cdot e_{\text{g.tr.cg.strands}}}{S_{\text{g.tr.top}}} = -0.801 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at support

$$f_{c.\text{botbeam.at.release2}} := \frac{P_{\text{pi}}}{A_{\text{g.tr}}} - \frac{M_{\text{sf.beam}}(\text{support}_{\text{check}}) - P_{\text{pi}} \cdot e_{\text{g.tr.cg.strands}}}{S_{\text{g.tr.bot}}} = 4.287 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the beam at support

$$f_{\text{compressive.allow.before.loss}} = 3.9 \cdot \text{ksi}$$

$$f_{\text{tensile.allow.15%.before.loss}} = -0.588 \cdot \text{ksi}$$

$$\text{Check}_{\text{sp.top}} := \begin{cases} \text{"OK"} & \text{if } f_{c.\text{topbeam.at.release2}} \geq 0 \text{ksi} \wedge \frac{f_{c.\text{topbeam.at.release2}}}{f_{\text{compressive.allow.before.loss}}} \leq 1 \\ \text{"OK"} & \text{if } f_{c.\text{topbeam.at.release2}} < 0 \text{ksi} \wedge \frac{-f_{c.\text{topbeam.at.release2}}}{-f_{\text{compressive.allow.before.loss}}} \leq 1 \\ \text{"NG"} & \text{otherwise} \end{cases}$$

in the pretensioned beams, at the support, "bonded reinforcements" check concrete stress limits at top fiber of beam

Check_{sp.top} = "OK"

$$\text{Check}_{\text{sp.bot}} := \begin{cases} \text{"OK"} & \text{if } f_{c.\text{botbeam.at.release2}} \geq 0 \text{ksi} \wedge \frac{f_{c.\text{botbeam.at.release2}}}{f_{\text{compressive.allow.before.loss}}} \leq 1 \\ \text{"OK"} & \text{if } f_{c.\text{botbeam.at.release2}} < 0 \text{ksi} \wedge \frac{-f_{c.\text{botbeam.at.release2}}}{-f_{\text{compressive.allow.before.loss}}} \leq 1 \\ \text{"NG"} & \text{otherwise} \end{cases}$$

in the pretensioned beams, at the support, "bonded reinforcements" check concrete stress limits at bot fiber of beam

Check_{sp.bot} = "NG"



**CFRP -PC TRAINING COURSE
DESIGN EXAMPLE: FIB-36**

$Check_{db} := \begin{cases} \text{"No Need to Debond Strands"} & \text{if } Check_{sp.top} = \text{"OK"} \wedge Check_{sp.bot} = \text{"OK"} \\ \text{"Need to Debond Strand"} & \text{otherwise} \end{cases}$

$Check_{db} = \text{"Need to Debond Strand"}$

Need to debond strands?

Debond some strands at support to satisfy concrete stress limites (*if necessary*)

$$\begin{array}{ccc}
 \begin{pmatrix} 17 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & n_{\max.\text{debond}} := n \cdot 40\% = & \begin{pmatrix} 6.8 \\ 6.8 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 n = & & \\
 & & \text{Adjust} \\
 & & \begin{pmatrix} 6 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 & & n_{\text{dbs}} :=
 \end{array}$$

Note: the number of partially debonded strands should NOT exceed 30% of the total number of strands (SDG 4.3.1); in any horizontal row, the number of debonded strands should NOT exceed 40% of the total strands in that row (LRFD 5.9.4.3.3.)

$$\begin{array}{ccc}
 n_{\text{after.dbs}} := n - n_{\text{dbs}} = & \begin{pmatrix} 11 \\ 13 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{array}{l} \text{Number of bonded strands in each row} \\ \text{after debonding at support dbs} \\ \text{bottom up, 0...8} \end{array}
 \end{array}$$

$$n_{\text{after.dbs.total}} := \sum_{i=0}^{\text{rows}(n)-1} n_{\text{after.dbs}_i} = 29$$

Remaining total number of debonded strands after debonding support

$$\text{Check}_{30\%} := \begin{cases} \text{"OK"} & \text{if } \frac{n_{\text{total}} - n_{\text{after.dbs.total}}}{n_{\text{total}}} \leq 30\% \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check if total debonded strands exceed 30% of total strands (SDG 4.3.1)

Check_{30%} = "OK"

$$\text{Check}_{40\%} := \begin{cases} \text{"NG"} & \text{if for } i \in 0 \dots \text{rows}(n) - 1 \\ & \left| \begin{array}{l} n_{\text{dbs}_i} > 40\% \cdot n_i \\ \text{break} \end{array} \right. \\ \text{"OK"} & \text{otherwise} \end{cases}$$

Check if debonded strands in each row exceed 40% of total strands in that row (LRFD 5.9.4.3.3)

Check_{40%} = "OK"

$$P_{pi.dbs} := n_{after.dbs.total} \cdot A_{pf} \cdot f_{pi} = 1.238 \times 10^3 \cdot \text{kip}$$

Total prestress force at release, after debonding support

$$A_{p.dbs} := n_{after.dbs} \cdot A_{pf} = \begin{pmatrix} 1.969 \\ 2.327 \\ 0.895 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \cdot \text{in}^2$$

Total area of strands in each row, after debonding support

$$A_{p.total.dbs} := \sum_{i=0}^{rows(n)-1} A_{p.dbs_i} = 5.191 \cdot \text{in}^2$$

Total area of strands, after debonding support

$$d_{p.strands.c.g.dbs} := \frac{\sum_{i=0}^{rows(n)-1} (A_{p.dbs_i} \cdot d_{p_i})}{A_{p.total.dbs}} = 31.414 \cdot \text{in}$$

Distance from top beam section to the gravity center of strand group, after debonding support

$$d_{p.strands.c.g.bot.dbs} := h_g - d_{p.strands.c.g.dbs} = 4.586 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group, after debonding support

Need to recalculate transformed section properties - Non-Composite (Girder+Strands), at support section, after debonding support

$$A_{g.tr.dbs} := A_g + (n_p - 1)A_{p.total.dbs} = 824.215 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g.tr.top.dbs} := \frac{A_g \cdot y_{g.top} + \sum_{i=0}^{rows(n)-1} [A_{p.dbs_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g.tr.dbs}} = 19.765 \cdot \text{in}$$

Distance from top fiber (beam) to N.A after debonding support

$$y_{g.tr.bot.dbs} := h_g - y_{g.tr.top.dbs} = 16.235 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A after debonding support

$$I_{g.tr.dbs} := I_g + A_g \cdot (y_{g.tr.top.dbs} - y_{g.top})^2 \dots = 1.3 \times 10^5 \cdot \text{in}^4$$

$$+ \sum_{i=0}^{\text{rows}(n)-1} \left[(n_p - 1) \cdot A_{p.dbs_i} \cdot (d_{p_i} - y_{g.tr.top.dbs})^2 \right]$$

Moment inertia, after debonding support

$$S_{g.tr.top.dbs} := \frac{I_{g.tr.dbs}}{y_{g.tr.top.dbs}} = 6.579 \times 10^3 \cdot \text{in}^3$$

$$S_{g.tr.bot.dbs} := \frac{I_{g.tr.dbs}}{y_{g.tr.bot.dbs}} = 8.009 \times 10^3 \cdot \text{in}^3$$

$$e_{g.tr.cg.strands.dbs} := y_{g.tr.bot.dbs} - d_{p.strands.c.g.bot.dbs} = 11.649 \cdot \text{in}$$

eccentricity of strand group
after debonding support

Check concrete stress limits at support section, after debonding support

$$f_{c.topbeam.support} := \frac{P_{pi.dbs}}{A_{g.tr.dbs}} + \frac{M_{sf.beam}(\text{support}_{check}) - P_{pi.dbs} \cdot e_{g.tr.cg.strands.dbs}}{S_{g.tr.top.dbs}}$$

$$f_{c.topbeam.support} = -0.527 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at support

$$f_{c.botbeam.support} := \frac{P_{pi.dbs}}{A_{g.tr.dbs}} - \frac{M_{sf.beam}(\text{support}_{check}) - P_{pi.dbs} \cdot e_{g.tr.cg.strands.dbs}}{S_{g.tr.bot.dbs}}$$

$$f_{c.botbeam.support} = 3.17 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the beam at support

$$f_{compressive.allow.before.loss} = 3.9 \cdot \text{ksi}$$

$$f_{tensile.allow.15\%.before.loss} = -0.588 \cdot \text{ksi}$$

in the pretensioned beams, at the support, "bonded reinforcements"

```

Checktop.dbs := if fc.topbeam.support ≥ 0ksi
                | "OK" if fc.topbeam.support ≤ fcompressive.allow.before.loss
                | "NG" otherwise
                if fc.topbeam.support < 0ksi
                | "OK" if -fc.topbeam.support ≤ -ftensile.allow.15%.before.loss
                | "NG" otherwise
    
```

Check_{top.dbs} = "OK"

Check concrete stress limits at the support, top fiber, at release

```

Checkbot.dbs := if fc.botbeam.support ≥ 0ksi
                | "OK" if fc.botbeam.support ≤ fcompressive.allow.before.loss
                | "NG" otherwise
                if fc.botbeam.support < 0ksi
                | "OK" if -fc.botbeam.support ≤ -ftensile.allow.15%.before.loss
                | "NG" otherwise
    
```

Check_{bot.dbs} = "OK"

Check concrete stress limits at the support, bottom fiber, at release

8.3 At Debond1 Section

$$\text{debond1} = 10 \text{ ft}$$

debond distance, defined at strand pattern

$$\frac{\text{debond1}}{\text{Span}} = 11.4\%$$

Check if the debond distance is within 15% span range to beam ends

Adjust

$n = \begin{pmatrix} 17 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$n_{\text{dbs}} = \begin{pmatrix} 6 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	Adjust	$n_{\text{db1}} := \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$n_{\text{after.db1}} := n - n_{\text{db1}} =$	$\begin{pmatrix} 15 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	<p>Number of bonded strands in each row after debond 1 (db1) bottom up, 0...8</p>
--	---	--------	--	--	--	---

Note: the number of partially debonded strands should NOT exceed 30% of the total number of strands (SDG 4.3.1); in any horizontal row, the number of debonded strands should NOT exceed 40% of the total strands in that row (LRFD 5.9.4.3.3.)

$n_{\text{after.db1.total}} := \sum_{i=0}^{\text{rows}(n)-1} n_{\text{after.db1}_i} = 37$	<p>Number of bonded strands after debond 1</p>
---	--

$P_{\text{pi.db1}} := n_{\text{after.db1.total}} \cdot A_{\text{pf}} \cdot f_{\text{pi}} = 1.58 \times 10^3 \cdot \text{kip}$	<p>Total prestress force at release, after debond1</p>
---	--

$A_{\text{p.db1}} := n_{\text{after.db1}} \cdot A_{\text{pf}} =$	$\begin{pmatrix} 2.685 \\ 3.043 \\ 0.895 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \cdot \text{in}^2$	<p>Total area of strands in each row, after debond1</p>
--	--	---

$A_{\text{p.total.db1}} := \sum_{i=0}^{\text{rows}(n)-1} A_{\text{p.db1}_i} = 6.623 \cdot \text{in}^2$	<p>Total area of strands, after debond1</p>
--	---

$d_{\text{p.strands.c.g.db1}} := \frac{\sum_{i=0}^{\text{rows}(n)-1} (A_{\text{p.db1}_i} \cdot d_{\text{p}_i})}{A_{\text{p.total.db1}}} = 31.541 \cdot \text{in}$	<p>Distance from top beam section to the gravity center of strand group, after debond1</p>
---	--

$$d_{p.strands.c.g.bot.db1} := h_g - d_{p.strands.c.g.db1} = 4.459 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group, after debond 1

Need to recalculate transformed section properties - Non-Composite (Girder+Strands), in debond 1 region

$$A_{g.tr.db1} := A_g + (n_p - 1)A_{p.total.db1} = 829.079 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g.tr.top.db1} := \frac{A_g \cdot y_{g.top} + \sum_{i=0}^{rows(n)-1} [A_{p.db1_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g.tr.db1}} = 1.653 \text{ ft}$$

Distance from top fiber (beam) to N.A after debond 1

$$y_{g.tr.bot.db1} := h_g - y_{g.tr.top.db1} = 1.347 \text{ ft}$$

Distance from bottom fiber (beam) to N.A after debond 1

$$I_{g.tr.db1} := I_g + A_g \cdot (y_{g.tr.top.db1} - y_{g.top})^2 + \sum_{i=0}^{rows(n)-1} [(n_p - 1) \cdot A_{p.db1_i} \cdot (d_{p_i} - y_{g.tr.top.db1})^2] \dots = 1.308 \times 10^5 \cdot \text{in}^4$$

Moment inertia after debond 1

$$S_{g.tr.top.db1} := \frac{I_{g.tr.db1}}{y_{g.tr.top.db1}} = 6.592 \times 10^3 \cdot \text{in}^3$$

$$S_{g.tr.bot.db1} := \frac{I_{g.tr.db1}}{y_{g.tr.bot.db1}} = 8.09 \times 10^3 \cdot \text{in}^3$$

$$e_{g.tr.cg.strands.db1} := y_{g.tr.bot.db1} - d_{p.strands.c.g.bot.db1} = 11.704 \cdot \text{in}$$

eccentricity of strand group after debond 1

Check concrete stress limits at debond1 section, after debond1

$$f_{c.topbeam.debond1} := \frac{P_{pi.db1}}{A_{g.tr.db1}} + \frac{M_{sf.beam}(debond1) - P_{pi.db1} \cdot e_{g.tr.cg.strands.db1}}{S_{g.tr.top.db1}} = -0.306 \cdot ksi$$

Concrete stress at top fiber of the beam at debond1 section

$$f_{c.botbeam.debond1} := \frac{P_{pi.db1}}{A_{g.tr.db1}} - \frac{M_{sf.beam}(debond1) - P_{pi.db1} \cdot e_{g.tr.cg.strands.db1}}{S_{g.tr.bot.db1}} = 3.707 \cdot ksi$$

Concrete stress at bottom fiber of the beam at debond1 section

$$f_{compressive.allow.before.loss} = 3.9 \cdot ksi$$

$$f_{tensile.allow.debond1} := \begin{cases} f_{tensile.allow.15\%.before.loss} & \text{if } \frac{debond1}{Span} \leq 15\% \\ f_{tensile.allow.70\%.bond.before.loss} & \text{if } \frac{debond1}{Span} > 15\% \end{cases} = -0.588 \cdot ksi$$

in the pretensioned beams, at the debond1 section, "bonded reinforcements"

$$Check_{top.db1} := \begin{cases} \text{if } f_{c.topbeam.debond1} \geq 0ksi \\ \quad \begin{cases} \text{"OK"} & \text{if } f_{c.topbeam.debond1} \leq f_{compressive.allow.before.loss} \\ \text{"NG"} & \text{otherwise} \end{cases} \\ \text{if } f_{c.topbeam.debond1} < 0ksi \\ \quad \begin{cases} \text{"OK"} & \text{if } -f_{c.topbeam.debond1} \leq -f_{tensile.allow.debond1} \\ \text{"NG"} & \text{otherwise} \end{cases} \end{cases}$$

Check_{top.db1} = "OK"

Check concrete stress limits at the debond1 section, top fiber, at release

```
Checkbot.db1 := if fc.botbeam.debond1 ≥ 0ksi  
                | "OK" if fc.botbeam.debond1 ≤ fcompressive.allow.before.loss  
                | "NG" otherwise  
                if fc.botbeam.debond1 < 0ksi  
                | "OK" if -fc.botbeam.debond1 ≤ -ftensile.allow.debond1  
                | "NG" otherwise
```

Check_{bot.db1} = "OK"

Check concrete stress limits at the
debond1 section, bottom fiber, at release

8.4 At Debond2 Section

$$\text{debond2} = 20 \text{ ft}$$

debond distance, defined at strand pattern

$$\frac{\text{debond2}}{\text{Span}} = 22.8\%$$

$$n = \begin{pmatrix} 17 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad n_{\text{dbs}} = \begin{pmatrix} 6 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad n_{\text{db1}} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Adjust

$$n_{\text{db2}} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

make all 0,
means no debonding after
debond2

Note: the number of partially debonded strands should NOT exceed 30% of the total number of strands (SDG 4.3.1); in any horizontal row, the number of debonded strands should NOT exceed 40% of the total strands in that row (LRFD 5.9.4.3.3.)

$$n_{\text{after.db2}} := n - n_{\text{db2}} = \begin{pmatrix} 17 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Number of bonded strands in each row
after debond 2 (db2)
bottom up, 0...8

$$n_{\text{after.db2.total}} := \sum_{i=0}^{\text{rows}(n)-1} n_{\text{after.db2}_i} = 39$$

Number of bonded strands after debond 2

$$P_{\text{pi.db2}} := n_{\text{after.db2.total}} \cdot A_{\text{pf}} \cdot f_{\text{pi}} = 1.665 \times 10^3 \cdot \text{kip}$$

Total prestress force at release, after
debond2

$$A_{\text{p.db2}} := n_{\text{after.db2}} \cdot A_{\text{pf}} = \begin{pmatrix} 3.043 \\ 3.043 \\ 0.895 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \cdot \text{in}^2$$

Total area of strands in each row, after
debond2

$$A_{p,\text{total.db2}} := \sum_{i=0}^{\text{rows}(n)-1} A_{p,\text{db2}_i} = 6.981 \cdot \text{in}^2 \quad \text{Total area of strands, after debond2}$$

$$d_{p,\text{strands.c.g.db2}} := \frac{\sum_{i=0}^{\text{rows}(n)-1} (A_{p,\text{db2}_i} \cdot d_{p_i})}{A_{p,\text{total.db2}}} = 31.615 \cdot \text{in}$$

Distance from top beam section to the gravity center of strand group, after debond2

$$d_{p,\text{strands.c.g.bot.db2}} := h_g - d_{p,\text{strands.c.g.db2}} = 4.385 \cdot \text{in}$$

Distance from bottom beam section to the gravity center of strand group, after debond2

Need to recalculate transformed section properties - Non-Composite (Girder+Strands), in debond 2 region

$$A_{g,\text{tr.db2}} := A_g + (n_p - 1)A_{p,\text{total.db2}} = 830.295 \cdot \text{in}^2 \quad \text{Area}$$

$$y_{g,\text{tr.top.db2}} := \frac{A_g \cdot y_{g,\text{top}} + \sum_{i=0}^{\text{rows}(n)-1} [A_{p,\text{db2}_i} \cdot (n_p - 1) \cdot d_{p_i}]}{A_{g,\text{tr.db2}}} = 19.856 \cdot \text{in}$$

Distance from top fiber (beam) to N.A after debond2

$$y_{g,\text{tr.bot.db2}} := h_g - y_{g,\text{tr.top.db2}} = 16.144 \cdot \text{in}$$

Distance from bottom fiber (beam) to N.A after debond2

$$I_{g,\text{tr.db2}} := I_g + A_g \cdot (y_{g,\text{tr.top.db2}} - y_{g,\text{top}})^2 + \sum_{i=0}^{\text{rows}(n)-1} [(n_p - 1) \cdot A_{p,\text{db2}_i} \cdot (d_{p_i} - y_{g,\text{tr.top.db2}})^2] = 1.31 \times 10^5 \cdot \text{in}^4$$

Moment inertia after debond2

$$S_{g.tr.top.db2} := \frac{I_{g.tr.db2}}{y_{g.tr.top.db2}} = 6.596 \times 10^3 \cdot \text{in}^3$$

$$S_{g.tr.bot.db2} := \frac{I_{g.tr.db2}}{y_{g.tr.bot.db2}} = 8.112 \times 10^3 \cdot \text{in}^3$$

$$e_{g.tr.cg.strands.db2} := y_{g.tr.bot.db2} - d_{p.strands.c.g.bot.db2} = 11.76 \cdot \text{in}$$

eccentricity of strand group
after debond2

Check concrete stress limits at debond2 section, after debond2

$$f_{c.topbeam.deb2} := \frac{P_{pi.db2}}{A_{g.tr.db2}} + \frac{M_{sf.beam}(deb2) - P_{pi.db2} \cdot e_{g.tr.cg.strands.db2}}{S_{g.tr.top.db2}} = 0.071 \cdot \text{ksi}$$

Concrete stress at top fiber of the beam at
debond2 section

$$f_{c.botbeam.deb2} := \frac{P_{pi.db2}}{A_{g.tr.db2}} - \frac{M_{sf.beam}(deb2) - P_{pi.db2} \cdot e_{g.tr.cg.strands.db2}}{S_{g.tr.bot.db2}} = 3.455 \cdot \text{ksi}$$

Concrete stress at bottom fiber of the
beam at debond2 section

$$f_{compressive.allow.before.loss} = 3.9 \cdot \text{ksi}$$

$$f_{tensile.allow.deb2} := \begin{cases} f_{tensile.allow.15\%.before.loss} & \text{if } \frac{deb2}{Span} \leq 15\% \\ f_{tensile.allow.70\%.bond.before.loss} & \text{if } \frac{deb2}{Span} > 15\% \end{cases} = -0.588 \cdot \text{ksi}$$

in the pretensioned beams, at the
debond2 section, "bonded
reinforcements"

```

Checktop.db2 := if fc.topbeam.debond2 ≥ 0ksi
                | "OK" if fc.topbeam.debond2 ≤ fcompressive.allow.before.loss
                | "NG" otherwise
                if fc.topbeam.debond2 < 0ksi
                | "OK" if -fc.topbeam.debond2 ≤ -ftensile.allow.debond2
                | "NG" otherwise
    
```

Check_{top.db2} = "OK"

Check concrete stress limits at the debond2 section, top fiber, at release

```

Checkbot.db2 := if fc.botbeam.debond2 ≥ 0ksi
                | "OK" if fc.botbeam.debond2 ≤ fcompressive.allow.before.loss
                | "NG" otherwise
                if fc.botbeam.debond2 < 0ksi
                | "OK" if -fc.botbeam.debond2 ≤ -ftensile.allow.debond2
                | "NG" otherwise
    
```

Check_{bot.db2} = "OK"

Check concrete stress limits at the debond2 section, bottom fiber, at release



CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FIB-36

8.5 Summary

at midspan

midspan = 43.833 ft

Check_{top.midspan} = "OK"

Check_{bot.midspan} = "OK"

at support

$L_{transfer} = 2.5$ ft

Check_{sp.top} = "OK"

Check_{sp.bot} = "NG"

Check_{db} = "Need to Debond Strand"

Need to debond strands?

Check_{top.dbs} = "OK"

Check_{bot.dbs} = "OK"

at debond1

debond1 = 10 ft

Check_{top.db1} = "OK"

Check_{bot.db1} = "OK"

at debond2

debond2 = 20 ft

Check_{top.db2} = "OK"

Check_{bot.db2} = "OK"

Strand pattern

without debond	debond number at support	debond number at debond1	debond number at debond2
$n = \begin{pmatrix} 17 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$n_{dbs} = \begin{pmatrix} 6 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$n_{db1} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$n_{db2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

**9. STRESS LIMITS FOR CONCRETE
STRESSES AT SERVICE LIMIT STATE (AFTER ALL LOSSES)
(LRFD 5.9.2.3.2)**

(Compression +, Tension -)

for stresses after all losses

$$P_{pe} := A_{p.total} \cdot f_{pe} = 1.292 \times 10^3 \cdot \text{kip}$$

Total prestress force, at service, after all losses

$$f_{compressive.allow.after.loss1.deck} := 0.45 \cdot f_{c.slab} = 2.475 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$$f_{compressive.allow.after.loss1.beam} := 0.45 \cdot f_{c.beam} = 3.825 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$$f_{compressive.allow.after.loss2.deck} := 0.60 \cdot f_{c.slab} = 3.3 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$f_{compressive.allow.after.loss2.beam} := 0.60 \cdot f_{c.beam} = 5.1 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$f_{tensile.allow.after.loss.beam} := \begin{cases} \left[-\min \left[0.24 \cdot \left(\frac{f_{ci.beam}}{\text{ksi}} \right)^{0.5}, \text{ksi}, 0.6\text{ksi} \right] \right] & \text{if Environment} = \text{"Extremely"} \\ \left[-\min \left[0.0948 \cdot \left(\frac{f_{ci.beam}}{\text{ksi}} \right)^{0.5}, \text{ksi}, 0.3\text{ksi} \right] \right] & \text{otherwise} \end{cases}$$



**CFRP -PC TRAINING COURSE
DESIGN EXAMPLE: FIB-36**

$$f_{\text{tensile.allow.after.loss.beam}} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete, after all losses (LRFD 5.9.2.3.2b) (Service III)

9.1 Service I Limit State: under Effective Prestress and Permanent Loads

Critical section: mid span

for deck, check compression limit

Note : slab under load of utility, wearing surface, and barriers (due to construction stages)

$$f_{\text{compressive.allow.after.loss1.deck}} = 2.475 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$$M_{\text{permanent2}} := M_{\text{utility}}(\text{midspan}) + M_{\text{fws}}(\text{midspan}) + M_{\text{barrier}}(\text{midspan}) = 165.237 \cdot \text{kip} \cdot \text{ft}$$

$$f_{\text{c.deck.top1}} := \frac{M_{\text{permanent2}}}{S_{\text{g.comp.tr.top}}} = 0.076 \cdot \text{ksi}$$

Concrete stress at the top fiber of deck

$$\text{Check}_{\text{topdeck1}} := \begin{cases} \text{"OK"} & \text{if } f_{\text{c.deck.top1}} \leq f_{\text{compressive.allow.after.loss1.deck}} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{\text{topdeck1}} = \text{"OK"}$$

Concrete stress at the top fiber of deck

for beam, check compression limit

$$f_{\text{compressive.allow.after.loss1.beam}} = 3.825 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress and permanent loads (LRFD 5.9.2.3.2a)

$$M_{\text{permanent1}} := M_{\text{sf.beam}}(\text{midspan}) + M_{\text{sf.slabs}}(\text{midspan}) + M_{\text{forms}}(\text{midspan}) = 1.86 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

$$f_{c.beam.top1} := \left(\frac{P_{pe}}{A_{g.tr}} - \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.top}} \right) + \frac{M_{permanent1}}{S_{g.tr.top}} + \frac{M_{permanent2}}{S_{g.comp.tr.topbeam}}$$

M.permanent1 (self weight of beam, slab, and forms) was applied to non-composite section; M.permanent2 (utility, wearing surface, and barriers) was applied to composite section

$$f_{c.beam.top1} = 2.669 \cdot \text{ksi}$$

Concrete stress at the top fiber of beam

$$\text{Check}_{topbeam1} := \begin{cases} \text{"OK"} & \text{if } f_{c.beam.top1} \leq f_{compressive.allow.after.loss1.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{topbeam1} = \text{"OK"}$$

Concrete stress at the top fiber of beam

9.2 Service I Limit State:under Effective Prestress, Permanent Loads, and Transient Loads

Note: loading during shipping and handling is not included in this example

Critical section: mid span

for deck, check compression limit

$$f_{compressive.allow.after.loss2.deck} = 3.3 \cdot \text{ksi}$$

Compressive stress limit for deck concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$M_{LL.IM.mid} = 1.628 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment due to Live Loads (truck and lane)

$$f_{c.deck.top2} := f_{c.deck.top1} + \frac{M_{LL.IM.mid}}{S_{g.comp.tr.top}} = 0.82 \cdot \text{ksi}$$

Concrete stress at the top fiber of deck

$$\text{Check}_{topdeck2} := \begin{cases} \text{"OK"} & \text{if } f_{c.deck.top2} \leq f_{compressive.allow.after.loss2.deck} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check_{topdeck2} = "OK"

Concrete stress at the top fiber of deck

for beam, check compression limit

$$f_{\text{compressive.allow.after.loss2.beam}} = 5.1 \cdot \text{ksi}$$

Compressive stress limit for beam concrete due to (Service I) the sum of effective prestress, permanent loads, and transient loads (LRFD 5.9.2.3.2a)

$$f_{\text{c.beam.top2}} := f_{\text{c.beam.top1}} + \frac{M_{\text{LL.IM.mid}}}{S_{\text{g.comp.tr.topbeam}}} = 2.991 \cdot \text{ksi}$$

Concrete stress at the top fiber of beam

$$\text{Check}_{\text{topbeam2}} := \begin{cases} \text{"OK"} & \text{if } f_{\text{c.beam.top2}} \leq f_{\text{compressive.allow.after.loss2.beam}} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check_{topbeam2} = "OK"

Concrete stress at the top fiber of beam

9.3 Service III Limit State

Note: loading during shipping and handling is not included in this example

Critical section: mid span

for beam, check tensile limit at the bottom fiber

$$f_{\text{tensile.allow.after.loss.beam}} = -0.588 \cdot \text{ksi}$$

Tensile stress limit for concrete, after all losses (LRFD 5.9.2.3.2b) (Service III)

$$f_{\text{c.beam.bot}} := \frac{P_{\text{pe}}}{A_{\text{g.tr}}} + \frac{P_{\text{pe}} \cdot e_{\text{g.tr.cg.strands}}}{S_{\text{g.tr.bot}}} \dots = -0.478 \cdot \text{ksi}$$

$$+ \frac{-M_{\text{permanent1}}}{S_{\text{g.tr.bot}}} + \frac{-M_{\text{permanent2}}}{S_{\text{g.comp.tr.bot}}} + (0.8) \frac{-M_{\text{LL.IM.mid}}}{S_{\text{g.comp.tr.bot}}}$$

Concrete stress at the bottom fiber of beam

$$\text{Check}_{\text{botbeam}} := \begin{cases} \text{"OK"} & \text{if } -f_{c.\text{beam.bot}} \leq -f_{\text{tensile.allow.after.loss.beam}} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check_{botbeam} = "OK"

Concrete stress at the bottom fiber of beam

9.4 Summary

Check_{topdeck1} = "OK"	$\frac{f_{c.\text{deck.top1}}}{f_{\text{compressive.allow.after.loss1.deck}}} = 0.031$	Service I
--	--	-----------

Check_{topbeam1} = "OK"	$\frac{f_{c.\text{beam.top1}}}{f_{\text{compressive.allow.after.loss1.beam}}} = 0.698$	Service I
--	--	-----------

Check_{topdeck1} = "OK"	$\frac{f_{c.\text{deck.top2}}}{f_{\text{compressive.allow.after.loss2.deck}}} = 0.249$	Service I
--	--	-----------

Check_{topbeam1} = "OK"	$\frac{f_{c.\text{beam.top2}}}{f_{\text{compressive.allow.after.loss2.beam}}} = 0.586$	Service I
--	--	-----------

Check_{botbeam} = "OK"	$\frac{-f_{c.\text{beam.bot}}}{-f_{\text{tensile.allow.after.loss.beam}}} = 0.813$	Service III
---------------------------------------	--	-------------

10. FATIGUE LIMIT STATE

According to AASHTO CFRP 1.5.2,

In regions of compressive stress due to unfactored loads and prestress in reinforced concrete components, fatigue shall be checked only if this stress compressive strength is less than the maximum tensile live load stress resulting from the Fatigue I load combination.

Fatigue of the reinforcement need not be checked for prestressed components designed to have extreme fiber tensile stress due to the Service III Limit State within the stress limit specified in LRFD Table 5.9.2.3.2b-1

at midspan

$$f_{c.topbeam} := \frac{P_{pe}}{A_{g.tr}} - \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.top}} + \frac{M_{permanent1}}{S_{g.tr.top}} + \frac{M_{permanent2}}{S_{g.comp.tr.topbeam}} = 2.669 \cdot ksi$$

compressive stress at top fiber of the beam due to unfactored loads and prestress

$$f_{t.bot.fatigue} := \frac{M_{fatigue}(midspan)}{S_{g.comp.tr.bot}} = 0.956 \cdot ksi$$

$$Check_{fatigue} := \begin{cases} \text{"Need to Check Fatigue"} & \text{if } \frac{f_{c.topbeam}}{f_{t.bot.fatigue}} < 1 \\ \text{"No Need to Check Fatigue"} & \text{otherwise} \end{cases}$$

$$Check_{fatigue} = \text{"No Need to Check Fatigue"}$$

Fatigue of the reinforcement need not be checked since the prestressed beam is designed to have extreme fiber tensile stress due to the Service III Limit State within the stress limit specified in LRFD Table 5.9.2.3.2b-1 (shown in section 9.3 of this example)

11. STRENGTH LIMIT STATE -MOMENT CAPACITY

Note: for strength calculations, deck reinforcement is conservatively ignored

$$P_{pe} = 1.292 \times 10^3 \cdot \text{kip}$$

Total prestress force, at service, after all losses

$$M_u := M_{Str1.mid} = 5.381 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Strength I limit state moment at midspan

$$\phi_{\text{moment}} = 0.75$$

Reduction factor for CFRP prestressed girder - Moment

$$\alpha_{1,\text{beam}} := \min \left(\max \left(0.85 - 0.02 \cdot \frac{f_{c,\text{beam}} - 10\text{ksi}}{\text{ksi}}, 0.75 \right), 0.85 \right) = 0.85$$

$$\beta_{1,\text{beam}} := \min \left(\max \left(0.85 - 0.05 \cdot \frac{f_{c,\text{beam}} - 4\text{ksi}}{\text{ksi}}, 0.65 \right), 0.85 \right) = 0.65$$

$$\beta_1 := \beta_{1,\text{beam}} = 0.65$$

$$\alpha_{1,\text{slab}} := \min \left(\max \left(0.85 - 0.02 \cdot \frac{f_{c,\text{slab}} - 10\text{ksi}}{\text{ksi}}, 0.75 \right), 0.85 \right) = 0.85$$

$$\beta_{1,\text{slab}} := \min \left(\max \left(0.85 - 0.05 \cdot \frac{f_{c,\text{slab}} - 4\text{ksi}}{\text{ksi}}, 0.65 \right), 0.85 \right) = 0.775$$

Concrete stress block factor
(LRFD 5.6.2.2)

$$\epsilon_{pu} = 0.015$$

Rupture strain of CFRP strand

$$\epsilon_{pe} := \frac{f_{pe}}{E_p} = 8.231 \times 10^{-3}$$

Effective prestress strain of CFRP

$$\epsilon_{cu} := 0.003$$

Concrete crushing strain

$$A_p = \begin{pmatrix} 3.043 \\ 3.043 \\ 0.895 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{in}^2$$

Area of strands in each row
bottom up, 0..8

$$d_{pf} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ d_{pf_i} \leftarrow d_{p_i} + t_{slab} + h_{buildup} \\ d_{pf} \end{cases} = \begin{pmatrix} 42.5 \\ 40.5 \\ 38.5 \\ 36.5 \\ 34.5 \\ 32.5 \\ 30.5 \\ 28.5 \end{pmatrix} \cdot \text{in}$$

Distance from top slab section
(composite section) to the center of each
strand row
bottom up, 0..8

11.1 Compression-Controlled Section

$$cc_C = 9.032 \cdot \text{in}$$

Concrete compression depth
This number is defined later to satisfy C=T

$$aa_C := \beta_1 \cdot cc_C = 5.871 \cdot \text{in}$$

$$\epsilon_{cc.C} := \epsilon_{cu} = 0.003$$

$$\epsilon_{f.C} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 \\ \epsilon_{f.C_i} \leftarrow \epsilon_{pe} + \epsilon_{cc.C} \cdot \left(\frac{d_{pf_i} - cc_C}{cc_C} \right) \\ \epsilon_{f.C} \end{cases} = \begin{pmatrix} 0.019 \\ 0.019 \\ 0.018 \\ 0.017 \\ 0.017 \\ 0.016 \\ 0.015 \\ 0.015 \end{pmatrix}$$

CFRP strain in each row
bottom up, 0..8

$$f_{f,C} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 & = \\ f_{f,C_i} \leftarrow \epsilon_{f,C_i} \cdot E_p & \\ f_{f,C} & \end{cases} \cdot \text{ksi} \quad \begin{pmatrix} 435 \\ 420 \\ 405 \\ 390 \\ 375 \\ 360 \\ 345 \\ 330 \end{pmatrix}$$

CFRP stress in each row
bottom up, 0...8

$$\text{Check}_{fp,C} := \begin{cases} \text{"CFRP Rupture Before Concrete Crushing, Assumption Not Satisfied"} & \text{if } f_{f,C_0} > f_{pu} \\ \text{"Compression Controlled Section, Assumption Satisfied"} & \text{otherwise} \end{cases}$$

Check_{fp,C} = "CFRP Rupture Before Concrete Crushing, Assumption Not Satisfied"

$$T_{f,C} := \begin{cases} \text{for } i \in 0 \dots \text{rows}(n) - 1 & = \\ T_{f,C_i} \leftarrow f_{f,C_i} \cdot A_{p_i} & \\ T_{f,C} & \end{cases} \cdot \text{kip} \quad \begin{pmatrix} 1323 \\ 1278 \\ 363 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

CFRP tension force in each row
bottom up, 0...8

$$T_{CFRP,C} := \sum_{i=0}^{\text{rows}(n)-1} T_{f,C_i} = 2.964 \times 10^3 \cdot \text{kip} \quad \text{Total tension force in all CFRP strands}$$

Define different depth of compression zone

$$tt1 := t_{slab} = 8.5 \cdot \text{in}$$

$$tt2 := tt1 + h_{buildup} = 9.5 \cdot \text{in}$$

$$tt3 := tt2 + h_{beam.top.flange} = 13 \cdot \text{in}$$

$$h_{tf} := h_{beam.top.flange} \quad \text{Beam, top flange thickness}$$

$$b_{tf} := b_{beam.top.flange} \quad \text{Beam, top flange width}$$

$$C_{\text{concrete.C}} := \begin{cases} \alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot aa_C \cdot b_e & \text{if } aa_C \leq tt1 \\ \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \dots & \text{if } tt1 < aa_C \leq tt2 \\ + \left[\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot (aa_C - tt1) \cdot b_{\text{tf}} \right] & \\ \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \dots & \text{if } tt2 < aa_C \leq tt3 \\ + \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot h_{\text{buildup}} \cdot b_{\text{tf}} \right) \dots & \\ + \left[\alpha_{1.\text{beam}} \cdot f_{c.\text{beam}} \cdot (aa_C - tt2) \cdot b_{\text{tf}} \right] & \\ \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \dots & \text{if } aa_C > tt3 \\ + \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot h_{\text{buildup}} \cdot b_{\text{tf}} \right) \dots & \\ + \left(\alpha_{1.\text{beam}} \cdot f_{c.\text{beam}} \cdot h_{\text{tf}} \cdot b_{\text{tf}} \right) \dots & \\ + \left[\alpha_{1.\text{beam}} \cdot f_{c.\text{beam}} \cdot (aa_C - tt3) \cdot b_{\text{web}} \right] & \end{cases}$$

Compression force from concrete

Note: here FIB 36 section dimensions are simplified

$$C_{\text{concrete.C}} = 2.964 \times 10^3 \cdot \text{kip}$$

$$M_{n,C} := \begin{cases} \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } aa_C \leq tt1 \\ + (-1) \cdot \left[(\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot aa_C \cdot b_e) \cdot \left(\frac{aa_C}{2} \right) \right] \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } tt1 < aa_C \leq tt2 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot t_{\text{slab}} \cdot b_e) \cdot \left(\frac{tt1}{2} \right) \dots \\ + (-1) \cdot \left[\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (aa_C - tt1) \cdot b_{tf} \right] \cdot \left(tt1 + \frac{aa_C - tt1}{2} \right) \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } tt2 < aa_C \leq tt3 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot t_{\text{slab}} \cdot b_e) \cdot \left(\frac{tt1}{2} \right) \dots \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot h_{\text{buildup}} \cdot b_{tf}) \cdot \left(tt1 + \frac{h_{\text{buildup}}}{2} \right) \dots \\ + (-1) \cdot \left[\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_C - tt2) \cdot b_{tf} \right] \cdot \left(tt2 + \frac{aa_C - tt2}{2} \right) \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,C_i} \cdot d_{pf_i}) \dots & \text{if } aa_C > tt3 \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot t_{\text{slab}} \cdot b_e) \cdot \left(\frac{tt1}{2} \right) \dots \\ + (-1) \cdot (\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot h_{\text{buildup}} \cdot b_{tf}) \cdot \left(tt1 + \frac{h_{\text{buildup}}}{2} \right) \dots \\ + (-1) \cdot (\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot h_{tf} \cdot b_{tf}) \cdot \left(tt2 + \frac{h_{tf}}{2} \right) \dots \\ + (-1) \cdot \left[\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_C - tt3) \cdot b_{\text{web}} \right] \cdot \left(tt3 + \frac{aa_C - tt2}{2} \right) \end{cases}$$

$$M_{n,C} = 9.439 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment capacity (with respect to top fiber of the composite section)

$$\text{Check}_{CT,C} := C_{\text{concrete},C} - T_{\text{CFRP},C} = 0 \cdot \text{kip}$$

Tension = Compression

$$\text{Check}_{fp,C} = \text{"CFRP Rupture Before Concrete Crushing, Assumption Not Satisfied"}$$

$$cc_C \equiv 9.032 \text{ in}$$

Adjust cc until C-T=0

$$\text{SectionControl} := \begin{cases} \text{"Tension Controlled Section"} & \text{if } f_{f.C_0} > f_{pu} \\ \text{"Compression Controlled Section"} & \text{otherwise} \end{cases}$$

SectionControl = "Tension Controlled Section"

11.2 Tension-Controlled Section

$$cc_T = 7.12 \cdot \text{in}$$

Concrete compression depth
This number is defined later to satisfy C=T

$$aa_T := \beta_1 \cdot cc_T = 4.628 \cdot \text{in}$$

$$\epsilon_{pf.T} := \epsilon_{pu} = 0.0152$$

CFRP strain in bottom row is rupture strain

$$\epsilon_{cc.T} := (\epsilon_{pu} - \epsilon_{pe}) \cdot \left(\frac{cc_T}{d_{pf_0} - cc_T} \right) = 1.394 \times 10^{-3}$$

Concrete strain at the top fiber of the section (deck)

$$\text{Check}_{cc.T} := \begin{cases} \text{"OK"} & \text{if } \epsilon_{cc.T} \leq \epsilon_{cu} \\ \text{"NG, Concrete Strain Exceeds } \epsilon_{cu}. \text{ Compression-Controlled Section"} & \text{otherwise} \end{cases}$$

Check_{cc.T} = "OK"

$$\epsilon_{f.T} := \begin{cases} \text{for } i \in 0..rows(n) - 1 \\ \epsilon_{f.T_i} \leftarrow \epsilon_{pe} + \epsilon_{cc.T} \cdot \left(\frac{d_{pf_i} - cc_T}{cc_T} \right) \\ \epsilon_{f.T} \end{cases} = \begin{pmatrix} 0.0152 \\ 0.0148 \\ 0.0144 \\ 0.0140 \\ 0.0136 \\ 0.0132 \\ 0.0128 \\ 0.0124 \end{pmatrix}$$

CFRP strain in each row bottom up, 0...8

$$f_{f.T} := \begin{cases} \text{for } i \in 0.. \text{rows}(n) - 1 \\ f_{f.T_i} \leftarrow \epsilon_{f.T_i} \cdot E_p \\ f_{f.T} \end{cases} = \begin{pmatrix} 341 \\ 332 \\ 323 \\ 314 \\ 306 \\ 297 \\ 288 \\ 279 \end{pmatrix} \cdot \text{ksi}$$

CFRP stress in each row
bottom up, 0..8

$$\text{Check}_{f.p.T} := \begin{cases} \text{"CFRP Rupture, Assumption Not Satisfied"} & \text{if } f_{f.T_0} > f_{pu} \\ \text{"OK"} & \text{otherwise} \end{cases}$$

Check_{f.p.T} = "OK"

CFRP not rupture

$$T_{f.T} := \begin{cases} \text{for } i \in 0.. \text{rows}(n) - 1 \\ T_{f.T_i} \leftarrow f_{f.T_i} \cdot A_{p_i} \\ T_{f.T} \end{cases} = \begin{pmatrix} 1037 \\ 1010 \\ 289 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kip}$$

CFRP tension force in each row
bottom up, 0..8

$$T_{\text{CFRP.T}} := \sum_{i=0}^{\text{rows}(n)-1} T_{f.T_i} = 2.336 \times 10^3 \cdot \text{kip}$$

Total tension force in all CFRP strands

$$C_{\text{concrete.T}} := \begin{cases} \alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot a_{aT} \cdot b_e & \text{if } a_{aT} \leq t_{t1} \\ \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \dots & \text{if } t_{t1} < a_{aT} \leq t_{t2} \\ + \left[\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot (a_{aT} - t_{\text{slab}}) \cdot b_{\text{beam.bot..flange}} \right] & \\ \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \dots & \text{if } t_{t2} < a_{aT} \leq t_{t3} \\ + \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot h_{\text{buildup}} \cdot b_{\text{beam.bot..flange}} \right) \dots & \\ + \left[\alpha_{1.\text{beam}} \cdot f_{c.\text{beam}} \cdot (a_{aT} - t_{t2}) \cdot b_{\text{beam.bot..flange}} \right] & \\ \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \dots & \text{if } a_{aT} > t_{t3} \\ + \left(\alpha_{1.\text{slab}} \cdot f_{c.\text{slab}} \cdot h_{\text{buildup}} \cdot b_{\text{beam.bot..flange}} \right) \dots & \\ + \left(\alpha_{1.\text{beam}} \cdot f_{c.\text{beam}} \cdot h_{\text{beam.top.flange}} \cdot b_{\text{beam.bot..flange}} \right) \dots & \\ + \left[\alpha_{1.\text{beam}} \cdot f_{c.\text{beam}} \cdot (a_{aT} - t_{t3}) \cdot b_{\text{web}} \right] & \end{cases}$$

Compression force from concrete

Note: here FIB 36 section dimensions are simplified

$$C_{\text{concrete.T}} = 2.337 \times 10^3 \cdot \text{kip}$$

$$M_{n,T} := \begin{cases} \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } aa_T \leq tt1 \\ + (-1) \cdot \left[\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot aa_T \cdot b_e \right] \cdot \left(\frac{aa_T}{2} \right) & \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } tt1 < aa_T \leq tt2 \\ + (-1) \cdot \left(\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \cdot \left(\frac{tt1}{2} \right) \dots & \\ + (-1) \cdot \left[\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot (aa_T - tt1) \cdot b_{tf} \right] \cdot \left(tt1 + \frac{aa_T - tt1}{2} \right) & \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } tt2 < aa_T \leq tt3 \\ + (-1) \cdot \left(\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \cdot \left(\frac{tt1}{2} \right) \dots & \\ + (-1) \cdot \left(\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot h_{\text{buildup}} \cdot b_{tf} \right) \cdot \left(tt1 + \frac{h_{\text{buildup}}}{2} \right) \dots & \\ + (-1) \cdot \left[\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_T - tt2) \cdot b_{tf} \right] \cdot \left(tt2 + \frac{aa_T - tt2}{2} \right) & \\ \sum_{i=0}^{\text{rows}(n)-1} (T_{f,T_i} \cdot d_{pf_i}) \dots & \text{if } aa_T > tt3 \\ + (-1) \cdot \left(\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot t_{\text{slab}} \cdot b_e \right) \cdot \left(\frac{tt1}{2} \right) \dots & \\ + (-1) \cdot \left(\alpha_{1,\text{slab}} \cdot f_{c,\text{slab}} \cdot h_{\text{buildup}} \cdot b_{tf} \right) \cdot \left(tt1 + \frac{h_{\text{buildup}}}{2} \right) \dots & \\ + (-1) \cdot \left(\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot h_{tf} \cdot b_{tf} \right) \cdot \left(tt2 + \frac{h_{tf}}{2} \right) \dots & \\ + (-1) \cdot \left[\alpha_{1,\text{beam}} \cdot f_{c,\text{beam}} \cdot (aa_T - tt3) \cdot b_{\text{web}} \right] \cdot \left(tt3 + \frac{aa_T - tt3}{2} \right) & \end{cases}$$

$$M_{n,T} = 7.56 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment capacity (with respect to top fiber of the composite section)

$$\text{Check}_{CT,T} := C_{\text{concrete},T} - T_{\text{CFRP},T} = 0 \cdot \text{kip}$$

Equilibrium check

$$\text{Check}_{fp,T} = \text{"OK"}$$

CFRP strain limits check

$$\text{Check}_{cc,T} = \text{"OK"}$$

Concrete strain limits check

$$cc_T \equiv 7.12 \text{in}$$

Adjust cc until C-T~0

11.3 Summary

SectionControl = "Tension Controlled Section"

$$cc := \begin{cases} cc_T & \text{if SectionControl = "Tension Controlled Section"} \\ cc_C & \text{if SectionControl = "Compression Controlled Section"} \end{cases} = 7.12 \cdot \text{in}$$

Compression Depth

$$M_n := \begin{cases} M_{n,T} & \text{if SectionControl = "Tension Controlled Section"} \\ M_{n,C} & \text{if SectionControl = "Compression Controlled Section"} \end{cases} = 7.56 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Moment capacity

$$M_r := \phi_{\text{moment}} \cdot M_n = 5670 \cdot \text{kip} \cdot \text{ft}$$

$$M_u = 5381 \cdot \text{kip} \cdot \text{ft}$$

$$\text{Check}_M := \begin{cases} \text{"OK"} & \text{if } M_r \geq M_u \\ \text{"NG"} & \text{if } M_r < M_u \end{cases}$$

$$\text{Check}_M = \text{"OK"}$$

Strength I, moment capacity

12. MINIMUM REINFORCEMENT -PRESTRESSING CFRP

For tension-controlled beams, the minimum reinforcement requirements ensure the factored moment capacity provided is at least equal to the lesser of the cracking moment M_{cr} and 1.33 times the factored moment required by the applicable strength load combinations (M_u).

(AASHTO CFRP 1.7.3.3.1)

$$M_{cr} = \gamma_3 \cdot \left[(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}) \cdot S_c - M_{dnc} \cdot \left(\frac{S_c}{S_{nc}} - 1 \right) \right]$$

Cracking moment
(AASHTO CFRP 1.7.3.3.1-1)

$$f_{r.beam} = 0.7 \cdot \text{ksi}$$

Modulus of rupture of beam concrete
(SDG 1.4.1B)

$$\gamma_1 := 1.6$$

flexural cracking variability factor

$$\gamma_2 := 1.1$$

Prestress variability factor
1.1 for bonded prestressing CFRP
1.0 for unbonded prestressing CFRP
(AASHTO CFRP 1.7.3.3.1)

$$\gamma_3 := 1.0$$

$$f_{cpe} := \frac{P_{pe}}{A_{g.tr}} + \frac{P_{pe} \cdot e_{g.tr.cg.strands}}{S_{g.tr.bot}} = 3.428 \cdot \text{ksi}$$

Compressive stress in concrete due to effective prestress forces only (after all losses) at extreme fiber of section where tensile stress is caused by externally applied loads

$$M_{dnc} := M_{sf.beam}(\text{midspan}) + M_{sf.slabs}(\text{midspan}) \dots = 1.86 \times 10^3 \cdot \text{kip} \cdot \text{ft} \\ + M_{forms}(\text{midspan})$$

Total unfactored dead load moment acting on the noncomposite section

$$M_{cr} := \gamma_3 \cdot \left[(\gamma_1 \cdot f_{r.beam} + \gamma_2 \cdot f_{cpe}) \cdot S_{g.comp.tr.bot} - M_{dnc} \cdot \left(\frac{S_{g.comp.tr.bot}}{S_{g.tr.bot}} - 1 \right) \right] = 4.579 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Cracking moment
(AASHTO CFRP 1.7.3.3.1-1)



CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FIB-36

$$M_{\text{req}} := \min(M_{\text{cr}}, 1.33M_{\text{u}}) = 4.579 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Required minimum amount of CFRP shall develop the moment capacity M_{req}

$$\text{SectionCT} := \begin{cases} \text{"C"} & \text{if SectionControl} = \text{"Compression Controlled Section"} \\ \text{"T"} & \text{if SectionControl} = \text{"Tension Controlled Section"} \end{cases}$$

$$\text{Check}_{\text{minCFRP}} := \begin{cases} \text{"Compression Controlled Section, Not Applicable"} & \text{if SectionCT} = \text{"C"} \\ \text{"OK"} & \text{if SectionCT} = \text{"T"} \wedge M_{\text{n}} \geq M_{\text{req}} \\ \text{"NG"} & \text{if SectionCT} = \text{"T"} \wedge M_{\text{n}} < M_{\text{req}} \end{cases}$$

$$\text{Check}_{\text{minCFRP}} = \text{"OK"}$$

minimum amount of prestressing CFRP

13. TRANSVERSE SHEAR DESIGN

$$V_u \leq V_r = \phi_{\text{shear}} \cdot V_n$$

(AASHTO CFRP 1.8.2.1)

$$\phi_{\text{shear}} = 0.9$$

Reduction factor for CFRP prestressed girder - Shear
(AASHTO CFRP 1.8.2.1, LRFD 5.5.4.2)

$$V_n = \min(V_{n1}, V_{n2})$$

$$V_{n1} = V_c + V_f + V_p$$

V_c, V_f, V_p are shear resistance contribution by concrete, FRP stirrup, and CFRP prestressing strands

$$V_{n2} = 0.25f_c \cdot b_v \cdot d_v + V_p$$

$$V_u > 0.5 \cdot \phi_{\text{shear}} \cdot (V_c + V_p)$$

Transverse reinforcement should be provided in this case (LRFD 5.7.3.2-1)

13.1 Critical Section for Shear

critical section for shear shall be taken as d_v from the internal face of support

$$d_{e,\text{shear}} := h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot}} = 41.115 \cdot \text{in}$$

Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcements

$$d_v := \max\left(d_{e,\text{shear}} - \frac{\beta_1 \cdot c_c}{2}, 0.9 \cdot d_{e,\text{shear}}, 0.72 \cdot h_{g,\text{comp}}\right) = 38.801 \cdot \text{in}$$

Effective shear depth
(LRFD 5.7.2.8)

$$\text{shearcheck} := d_v + \frac{\text{PadWidth}}{2} = 3.65 \text{ ft}$$

Critical shear section, from left support

$$b_v := b_{\text{web}} = 7 \cdot \text{in}$$

Width of web
(LRFD 5.7.2.7)

13.2 Contribution by Prestressing CFRP

$$V_p := 0$$

Straight strands, no draped, $V_p=0$

13.3 Contribution by Concrete, V_c

$$V_c = 0.0316 \cdot \beta \cdot \sqrt{f_{c.beam}} \cdot b_v \cdot d_v$$

(LRFD 5.7.3.3-3)

Determine parameters β and θ

Note: assume the section containing at least the minimum amount of transverse reinforcement

$$\epsilon_{f, shear} = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{pf} \cdot f_{p0}}{E_f \cdot A_{pf}}$$

Net longitudinal tensile strain
(AASHTO CFRP 1.8.3.2)

$$N_{u, shear} := 0$$

Factored axial force at shear critical section

$$V_{u, shear} := V_{Str1}(shearcheck) = 271.846 \cdot \text{kip}$$

Factored shear at shear critical section

$$M_{Str1}(shearcheck) = 885.949 \cdot \text{kip} \cdot \text{ft}$$

$$M_{u, shear} := \max(M_{Str1}(shearcheck), |V_{u, shear} - V_p| \cdot d_v) = 885.949 \cdot \text{kip} \cdot \text{ft}$$

Factored moment at shear critical section
(AASHTO CFRP 1.8.3.2)

$$f_{p0, shear} := 0.6 \cdot f_{pu} = 204.469 \cdot \text{ksi}$$

(AASHTO CFRP 1.8.3.2)

$$A_{pf.shear} := \begin{cases} A_{p.total.db1} & \text{if } shearcheck < debond1 \\ A_{p.total.db1} & \text{if } debond1 \leq shearcheck < debond2 \\ A_{p.total.db2} & \text{if } shearcheck \geq debond2 \end{cases}$$

$$A_{pf.shear} = 5.191 \cdot \text{in}^2$$

Area of prestressing CFRP on the flexural tension side of the member, exclude debonded strands

$$\epsilon_{f.shear1} := \min \left(\epsilon_{pu}, \frac{\left(\frac{|M_{u.shear}|}{d_v} + 0.5N_{u.shear} + |V_{u.shear} - V_p| - A_{pf.shear} \cdot f_{p0.shear} \right)}{E_p \cdot A_{pf.shear}} \right)$$

$$\epsilon_{f.shear1} = -4.418 \times 10^{-3}$$

$$A_{ct.shear} := A_g - b_{tf} \cdot (h_{tf} + 1.5\text{in}) - b_{web} \cdot (h_g - 0.5 \cdot h_{web} - h_{tf}) - A_{p.total} = 389.849 \cdot \text{in}^2$$

Area of concrete on the flexural tension side of the member (LRFD B5.2)
Approximate

$$\epsilon_{f.shear2} := \min \left(\epsilon_{pu}, \frac{\left(\frac{|M_{u.shear}|}{d_v} + 0.5N_{u.shear} + |V_{u.shear} - V_p| - A_{pf.shear} \cdot f_{p0.shear} \right)}{A_{ct.shear} \cdot E_{c.beam} + E_p \cdot A_{pf.shear}} \right)$$

$$\epsilon_{f.shear2} = -2.444 \times 10^{-4}$$

$$\epsilon_{f.shear} := \begin{cases} \epsilon_{f.shear1} & \text{if } \epsilon_{f.shear1} \geq 0 \\ \max(-0.0004, \epsilon_{f.shear2}) & \text{if } \epsilon_{f.shear1} < 0 \end{cases} = -2.444 \times 10^{-4}$$

$$\beta := \frac{4.8 \cdot \text{deg}}{1 + 750 \cdot \epsilon_{f.shear}} = 5.9 \cdot \text{deg} \quad (\text{LRFD 5.7.3.4.2-1})$$

$$\theta := (29 + 3500 \cdot \epsilon_{f.shear}) \cdot \text{deg} = 28.1 \cdot \text{deg} \quad (\text{LRFD 5.7.3.4.2-3})$$

$$V_c := 0.0316 \cdot \frac{\beta}{\text{deg}} \cdot \sqrt{\frac{f_{c.beam}}{\text{ksi}}} \cdot \frac{b_v}{\text{in}} \cdot \frac{d_v}{\text{in}} \cdot \text{kip} = 147.064 \cdot \text{kip} \quad (\text{LRFD 5.7.3.3-3})$$

13.4 Required Shear Reinforcement

$$\text{CheckVf} := \begin{cases} \text{"Need Shear Reinforcement"} & \text{if } V_{u,\text{shear}} \geq 0.5 \cdot \phi_{\text{shear}} \cdot (V_c + V_p) \\ \text{"Shear Reinforcement Not Needed"} & \text{if } V_{u,\text{shear}} < 0.5 \cdot \phi_{\text{shear}} \cdot (V_c + V_p) \end{cases}$$

CheckVf = "Need Shear Reinforcement"

$$d_{v,\text{shear}} := d_{\text{GFRP}} = 0.625 \cdot \text{in} \quad \text{Diameter of GFRP stirrup}$$

$$A_{v,\text{shear}} := A_{\text{GFRP}} = 0.307 \cdot \text{in}^2 \quad \text{Area of each GFRP bar}$$

$$E_{v,\text{shear}} := E_{\text{GFRP}} = 6.5 \times 10^3 \cdot \text{ksi} \quad \text{Elastic modulus of GFRP stirrup}$$

$$f_{v,\text{shear}} := f_{\text{fu,GFRP}} = 66.396 \cdot \text{ksi} \quad \text{Tensile strength of GFRP stirrup}$$

$$\phi_{\text{bend,GFRP}} = 0.6 \quad \text{take 0.6 in this design example}$$

$$V_n := \min \left(\frac{V_{u,\text{shear}}}{\phi_{\text{shear}}}, 0.25 \cdot f_{c,\text{beam}} \cdot b_v \cdot d_v + V_p \right) = 302.052 \cdot \text{kip}$$

$$V_{\text{FRP,req}} := \max(0, V_n - V_c - V_p) = 154.987 \cdot \text{kip} \quad \text{Required shear provided by FRP stirrup}$$

$$V_{\text{FRP,req}} = \frac{f_{fv} \cdot A_{v,\text{shear, total}} \cdot d_v}{s_{\text{shear}}} \quad \text{(AASHTO GFRP 2.10.3.2.2)}$$

$$f_{fv} := \min(0.004 \cdot E_{v,\text{shear}} \cdot \phi_{\text{bend,GFRP}} \cdot f_{v,\text{shear}}, 22 \text{ksi}) = 22 \cdot \text{ksi}$$

Max bended FRP tensile stress
(AASHTO GFRP 2.10.3.2.2), modified with
FDOT Material Spec 932-3, which limits
the shear strength to 22 ksi

$$\text{leg}_{\text{stirrup}} := 2 \quad \text{Number of legs of stirrup}$$

$$A_{v,\text{shear, total}} := \text{leg}_{\text{stirrup}} \cdot A_{v,\text{shear}} = 0.614 \cdot \text{in}^2$$



CFRP -PC TRAINING COURSE DESIGN EXAMPLE: FIB-36

$$s_{\text{shear.req}} := \begin{cases} 1000\text{in} & \text{if } V_{\text{FRP.req}} = 0 \\ \frac{f_{\text{fv}} \cdot A_{\text{v.shear.total}} \cdot d_{\text{v}}}{V_{\text{FRP.req}}} & \text{otherwise} \end{cases} = 3.38 \cdot \text{in}$$

Required FRP stirrup spacing from calculation
(if not needed, output 1000 inch spacing)

$$s_{\text{shear.max}} := \begin{cases} \min(0.75 \cdot h_{\text{g}}, 24\text{in}) & \text{if } V_{\text{FRP.req}} \leq 4.0 \cdot \sqrt{\frac{f_{\text{c.beam}}}{\text{ksi}}} \cdot b_{\text{v}} \cdot d_{\text{v}} \cdot \text{ksi} \\ \min(0.375 \cdot h_{\text{g}}, 12\text{in}) & \text{if } V_{\text{FRP.req}} > 4.0 \cdot \sqrt{\frac{f_{\text{c.beam}}}{\text{ksi}}} \cdot b_{\text{v}} \cdot d_{\text{v}} \cdot \text{ksi} \end{cases} = 24 \cdot \text{in}$$

Maximum FRP stirrup spacing
(ACI 440.4R 5.3)

$$A_{\text{v.min}} = 0.05 \cdot \frac{s \cdot b_{\text{w}}}{f_{\text{fv}}}$$

Minimum GFRP transverse reinforcement
(AASHTO GFRP 2.10.2.2.1)

$$s_{\text{shear.minAf}} := \frac{A_{\text{v.shear.total}} \cdot \frac{f_{\text{fv}}}{\text{ksi}}}{0.05 \cdot b_{\text{web}}} = 38.569 \cdot \text{in}$$

Max spacing corresponding to minimum
GFRP transverse reinforcement

$$s_{\text{shear}} := \min(s_{\text{shear.req}}, s_{\text{shear.max}}, s_{\text{shear.minAf}}) = 3.38 \cdot \text{in}$$

$$s_{\text{shear.provide}} := \text{floor}\left(\frac{s_{\text{shear}}}{\text{in}}\right) \cdot \text{in} = 3 \cdot \text{in}$$

Provided transverse reinforcement spacing

$$V_{\text{FRP}} := \frac{f_{\text{fv}} \cdot A_{\text{v.shear.total}} \cdot d_{\text{v}}}{s_{\text{shear.provide}}} = 175 \cdot \text{kip}$$

Provided shear resistance from FRP
transverse reinforcement

13.5 Summary

$$V_{u,\text{shear}} = 272 \cdot \text{kip}$$

$$V_c = 147 \cdot \text{kip}$$

$$V_p = 0$$

$$V_{\text{FRP}} = 175 \cdot \text{kip}$$

$$d_{v,\text{shear}} = 0.625 \cdot \text{in}$$

Diameter of GFRP stirrup

$$\text{leg}_{\text{stirrup}} = 2$$

Number of legs of GFRP stirrup

$$s_{\text{shear,provide}} = 3 \cdot \text{in}$$

Spacing of GFRP stirrup

$$V_{n,\text{provide}} := V_c + V_{\text{FRP}} + V_p = 322 \cdot \text{kip}$$

$$\text{CheckShearCapacity} := \begin{cases} \text{"OK"} & \text{if } V_{n,\text{provide}} \geq \min \left(\frac{V_{u,\text{shear}}}{\phi_{\text{shear}}}, 0.25 \cdot f_{c,\text{beam}} \cdot b_v \cdot d_v + V_p \right) \\ \text{"NG"} & \text{otherwise} \end{cases}$$

CheckShearCapacity = "OK"

Transverse shear capacity

14. INTERFACE SHEAR DESIGN

$$V_{u1} := V_{Str1}(\text{shearcheck}) = 271.846 \cdot \text{kip} \quad \text{Factored interface shear force}$$

$$V_{ri} = \phi_{\text{shear}} \cdot V_{ni} \geq V_{ui} \quad (\text{LRFD 5.7.4.3})$$

$$\phi_{\text{shear}} = 0.9$$

Assumed numbers:

$$c_{vi} := 0.28 \text{ksi} \quad \text{Cohesion factor}$$

$$\mu_{vi} := 1 \quad \text{Friction factor}$$

$$K_{1,vi} := 0.3 \quad \text{Fraction of concrete strength available to resist interface shear}$$

$$K_{2,vi} := 1.8 \text{ksi} \quad \text{Limiting interface shear resistance}$$

Factored interface shear force, per ft

$$d_{vi} := \begin{cases} h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot.db1}} - \frac{t_{\text{slab}}}{2} & \text{if } \text{shearcheck} < \text{debond1} \\ h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot.db1}} - \frac{t_{\text{slab}}}{2} & \text{if } \text{debond1} \leq \text{shearcheck} < \text{debond2} \\ h_{g,\text{comp}} - d_{p,\text{strands.c.g.bot.db2}} - \frac{t_{\text{slab}}}{2} & \text{if } \text{shearcheck} \geq \text{debond2} \end{cases}$$

$$d_{vi} = 36.664 \cdot \text{in} \quad \text{Distance between the centroid of the tension strands and the mid-thickness of the slab}$$

$$V_{hi} := \frac{V_{u1}}{d_{vi}} = 88.975 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{Factored interface shear force, per ft}$$

Required nominal interface shear resistance, per ft

$$V_{ni} := \frac{V_{hi}}{\phi_{\text{shear}}} = 98.861 \cdot \frac{\text{kip}}{\text{ft}} \quad (\text{LRFD 5.7.4.3})$$

Required interface shear reinforcement, per ft

$$V_{ni} = c \cdot A_{cv} + \mu \cdot (A_{vf} \cdot f_y + P_c)$$

Nonimal interface shear resistance
(LRFD 5.7.4.3)

$$b_{vi} := b_{tf} = 48 \cdot \text{in}$$

Interface width considered to be engaged in
shear transfer

$$A_{cv} := b_{vi} = 576 \cdot \frac{\text{in}^2}{\text{ft}}$$

Area of concrete considered to be engaged
in shear transfer, per 1-ft length

$$P_{c.vi} := 0$$

Permanent net compression force normal
to the shear plane

$$A_{vf.req} := \frac{\frac{V_{ni} - c_{vi} \cdot A_{cv}}{\mu_{vi}} - P_{c.vi}}{f_{fv}} = -2.837 \cdot \frac{\text{in}^2}{\text{ft}}$$

Required interface shear reinforcement

Minimum interface shear reinforcement, per ft

$$A_{vf.min1} := 0.05 \cdot \frac{A_{cv}}{\frac{f_{fv}}{\text{ksi}}} = 1.309 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$A_{vf.min2} := \frac{\frac{1.33V_{ni} - c_{vi} \cdot A_{cv}}{\mu_{vi}} - P_{c.vi}}{f_{fv}} = -1.354 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$A_{vf.min} := \min(A_{vf.min1}, A_{vf.min2}) = -1.354 \cdot \frac{\text{in}^2}{\text{ft}}$$

Minimum interface shear reinforcemen
(LRFD 5.7.4.2)

Design interface shear reinforcement, per ft

$$A_{vf,design,need} := \max(A_{vf,req}, A_{vf,min}, 0) = 0 \cdot \frac{\text{in}^2}{\text{ft}}$$

Design interface reinforcement need to provide

$$A_{vf,provide} := \frac{A_{v, shear, total}}{s_{shear, provide}} = 2.454 \cdot \frac{\text{in}^2}{\text{ft}}$$

Interface shear reinforcement already provided by GFRP stirrup

$$Check_{iv0} := \begin{cases} \text{"OK"} & \text{if } A_{vf,provide} \geq A_{vf,design,need} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check_{iv0} = "OK"

Interface shear reinforcement,
Provided > Required

Maximum interface shear resistance, per ft

$$V_{ni,provide} := c_{vi} \cdot A_{cv} + \mu_{vi} \cdot (A_{vf,provide} \cdot f_{fv} + P_{c,vi}) = 215 \cdot \frac{\text{kip}}{\text{ft}}$$

Provided nominal interface shear resistance (LRFD 5.7.4.3)

$$V_{ni,max} := \min(K_{1,vi} \cdot f_{c,beam} \cdot A_{cv}, K_{2,vi} \cdot A_{cv}) = 1037 \cdot \frac{\text{kip}}{\text{ft}}$$

$$K_{1,vi} \cdot f_{c,beam} \cdot A_{cv} = 1.469 \times 10^3 \cdot \frac{\text{kip}}{\text{ft}}$$

Design interface shear resistance limitation (LRFD 5.7.4.3)

$$K_{2,vi} \cdot A_{cv} = 1.037 \times 10^3 \cdot \frac{\text{kip}}{\text{ft}}$$

$$Check_{iv,max} := \begin{cases} \text{"OK"} & \text{if } V_{ni,provide} \leq \min(K_{1,vi} \cdot f_{c,beam} \cdot A_{cv}, K_{2,vi} \cdot A_{cv}) \\ \text{"NG"} & \text{otherwise} \end{cases}$$

Check_{iv,max} = "OK"

Maximum interface shear resistance

Summary

$$V_{ni.provide} = 215.276 \cdot \frac{\text{kip}}{\text{ft}}$$

Nominal interface shear resistance

$$V_{hi} = 88.975 \cdot \frac{\text{kip}}{\text{ft}}$$

Factored interface shear force, per ft

$$\text{Check}_{vi.capacity} := \begin{cases} \text{"OK"} & \text{if } \phi_{shear} \cdot V_{ni.provide} \geq V_{hi} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{vi.capacity} = \text{"OK"}$$

Check capacity

$$\text{Check}_{iv.max} = \text{"OK"}$$

Check max nominal resistance

15. MINIMUM LONGITUDINAL REINFORCEMENT

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy

$$\sum_{x=1}^n (A_{px} \cdot f_{px}) \geq \frac{|M_u|}{d_v \cdot \phi_f} + 0.5 \cdot \frac{N_u}{\phi_c} + \left(\left| \frac{V_u}{\phi_v} - V_p \right| - 0.5 \cdot V_f \right) \cdot \frac{1}{\tan(\theta)}$$

(AASHTO CFRP 1.8.3.3)

$$N_u := 0$$

Axial load

$$\phi_{\text{moment}} = 0.75$$

Resistance factor for bending

$$\phi_{\text{shear}} = 0.9$$

Resistance factor for shear

$$\theta = 28.145 \cdot \text{deg}$$

Angle of inclination of diagonal compressive stresses

$$d_v = 38.801 \cdot \text{in}$$

Effective shear depth

$$V_p = 0$$

Shear capacity contribution by prestressing CFRP

$$f_{pe} = 185 \cdot \text{ksi}$$

Effective prestressing stress

At the support location

$$\text{support} = 0$$

$$V_{f,\text{support}} := \min \left(V_{\text{FRP}}, \frac{V_{\text{Str1}}(\text{support})}{\phi_{\text{shear}}} \right) = 174.594 \cdot \text{kip} \quad (\text{AASHTO CFRP 1.8.3.3})$$

$$\text{right} := \frac{|M_{\text{Str1}}(\text{support})|}{d_v \cdot \phi_{\text{moment}}} + \left(\left| \frac{V_{\text{Str1}}(\text{support})}{\phi_{\text{shear}}} - V_p \right| - 0.5 \cdot V_{f,\text{support}} \right) \cdot \frac{1}{\tan(\theta)} = 442 \cdot \text{kip}$$

the crack plane crosses the c.g of the strand group at a distance

$$x_{\text{crackplane}} := \text{BearingDistance} + d_{\text{p.strands.c.g.bot.dbs}} \cdot \frac{1}{\tan(\theta)} = 17.573 \cdot \text{in}$$

$$L_{\text{transfer}} = 30 \cdot \text{in}$$

Transfer length of prestressing CFRP
(AASHTO CFRP 1.9.3.2)

$$f_{\text{px}} := f_{\text{pe}} \cdot \frac{x_{\text{crackplane}}}{L_{\text{transfer}}}$$

Approximate prestressing stress at the
crack plane

$$\text{left} := \sum_{i=0}^{\text{rows}(n)-1} (A_{\text{p.dbs}_i} \cdot f_{\text{px}}) = 563 \cdot \text{kip}$$

$$\text{Check}_{\text{long.rebar}} := \begin{cases} \text{"OK, No Additional Longitudinal Reinforcement is Required"} & \text{if } \text{left} \geq \text{right} \\ \text{"NG, Additional Longitudinal Reinforcement is Required"} & \text{otherwise} \end{cases}$$

$$\text{Check}_{\text{long.rebar}} = \text{"OK, No Additional Longitudinal Reinforcement is Required"}$$

16. DEFLECTION AND CAMBER

Immediate deflection, due to prestressing force and DL

downward -> positive

Deflection due to prestressing force at transfer

$$\Delta = \frac{M \cdot L^2}{8 \cdot E \cdot I}$$

$$\Delta_{p1} := \frac{-(A_{p,\text{total.db1}} \cdot f_{pt}) \cdot e_{g,\text{tr.cg.strands}} \cdot (\text{Span})^2}{8 \cdot E_{ci,\text{beam}} \cdot I_g} = -3.258 \cdot \text{in}$$

$$\Delta_{p2} := \frac{-[(A_{p,\text{total.db1}} - A_{p,\text{total.db2}}) \cdot f_{pt}] \cdot e_{g,\text{tr.cg.strands}} \cdot (\text{Span} - 2 \cdot \text{debond1})^2}{8 \cdot E_{ci,\text{beam}} \cdot I_g} = -0.535 \cdot \text{in}$$

$$\Delta_{p3} := \frac{-[(A_{p,\text{total.db2}} - A_{p,\text{total.db1}}) \cdot f_{pt}] \cdot e_{g,\text{tr.cg.strands}} \cdot (\text{Span} - 2 \cdot \text{debond2})^2}{8 \cdot E_{ci,\text{beam}} \cdot I_g} = -0.066 \cdot \text{in}$$

$$\Delta_p := \Delta_{p1} + \Delta_{p2} + \Delta_{p3} = -3.86 \cdot \text{in}$$

Deflection due to self weight of beam

$$\Delta = \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I}$$

$$\Delta_{\text{sf.beam}} := \frac{5 \cdot w_{\text{beam}} \cdot \text{Span}^4}{384 \cdot E_{ci,\text{beam}} \cdot I_g} = 1.921 \cdot \text{in}$$

Deflection due to self weight of slab, buildup, and permanent forms

$$\Delta_{\text{permanent1}} := \frac{5 \cdot (w_{\text{slab}} + w_{\text{buildup}} + w_{\text{forms}}) \cdot \text{Span}^4}{384 \cdot E_{\text{c.beam}} \cdot I_{\text{g}}} = 2.233 \cdot \text{in}$$

Deflection due to barrier, future wearing surface, and utility

$$\Delta_{\text{permanent2}} := \frac{5 \cdot (w_{\text{barrier}} + w_{\text{fws}} + w_{\text{utility}}) \cdot \text{Span}^4}{384 \cdot E_{\text{c.beam}} \cdot I_{\text{g.comp}}} = 0.103 \cdot \text{in}$$

17. SUMMARY OF ALL CHECKS

All "Check"

7. PRESTRESS LOSSES

7.3 Summary

Check_{pe.limit} = "OK"

Service stress limit (after all losses)

8. STRESS LIMITS FOR CONCRETE -TEMPORARY STRESSES BEFORE LOSSES

8.1 At Mid-Span Section

Check_{top.midspan} = "OK"

Check concrete stress limits at the midspan, at release

Check_{bot.midspan} = "OK"

Check concrete stress limits at the midspan, at release

8.2 At Support Section

Check_{sp.top} = "OK"

Check concrete stress limits at the support, top fiber, at release

Check_{sp.bot} = "NG"

Check concrete stress limits at the support, bottom fiber, at release

Check_{db} = "Need to Debond Strand"

Need to debond strands?

Check_{30%} = "OK"

Total debonded strands shall not exceed 30% of total strands (SDG 4.3.1)

Check_{40%} = "OK"

Debonded strands in each row shall not exceed 40% of total strands in that row (LRFD 5.9.4.3.3)

Check_{top.dbs} = "OK"

Check concrete stress limits at the support after debond, top fiber, at release

Check_{bot.dbs} = "OK"

Check concrete stress limits at the support after debond, bottom fiber, at release

8.3 At Debond1 Section

Check_{top.db1} = "OK"

Check concrete stress limits at the debond1 section, top fiber, at release

Check_{bot.db1} = "OK"

Check concrete stress limits at the debond1 section, bottom fiber, at release

8.4 At Debond2 Section

Check_{top.db2} = "OK"

Check concrete stress limits at the debond2 section, top fiber, at release

Check_{bot.db2} = "OK"

Check concrete stress limits at the debond2 section, bottom fiber, at release

8.5 Summary

9. STRESS LIMITS FOR CONCRETE STRESSES AT SERVICE LIMIT STATE

9.1 Service I Limit State:under Effective Prestress and Permanent Loads

Check_{topdeck1} = "OK"

Concrete stress at the top fiber of deck

Check_{topbeam1} = "OK"

Concrete stress at the top fiber of beam

9.2 Service I Limit State:under Effective Prestress, Permanent loads, and Transient loads

Check_{topdeck2} = "OK"

Concrete stress at the top fiber of deck

Check_{topbeam2} = "OK"

Concrete stress at the top fiber of beam

9.3 Service III Limit State

Check_{botbeam} = "OK"

Concrete stress at the bottom fiber of beam

9.4 Summary

10. FATIGUE LIMIT STATE

Check_{fatigue} = "No Need to Check Fatigue"



11. STRENGTH LIMIT STATE -MOMENT CAPACITY

11.1 Compression-Controlled Section

Check_{fp.C} = "CFRP Rupture Before Concrete Crushing, Assumption Not Satisfied"

Check_{CT.C} = 0·kip Equilibrium

11.2 Tension-Controlled Section

Check_{fp.T} = "OK" CFRP not rupture

Check_{cc.T} = "OK" Concrete strain limits

Check_{CT.T} = 0·kip Equilibrium

11.3 Summary

SectionControl = "Tension Controlled Section"

Check_M = "OK" Strength I, moment capacity

12. MINIMUM REINFORCEMENT -PRESTRESSING CFRP

Check_{minCFRP} = "OK" Minimum amount of prestressing CFRP

13. TRANSVERSE SHEAR DESIGN

13.5 Summary

CheckShearCapacity = "OK" Transverse shear capacity

14. INTERFACE SHEAR DESIGN

Check_{iv0} = "OK" Interface shear reinforcement,
Provided > Required

Check_{iv.max} = "OK" Maximum interface shear resistance

Check_{vi.capacity} = "OK" Check interface shear capacity

15. MINIMUM LONGITUDINAL REINFORCEMENT

Check_{long.rebar} = "OK, No Additional Longitudinal Reinforcement is Required"