

# AASHTO CFRP- Prestressed Concrete Design Training Course



Florida Department of  
TRANSPORTATION

UNIVERSITY of **HOUSTON**

CULLEN COLLEGE of ENGINEERING  
Department of Civil & Environmental Engineering

# Design of Pretensioned Concrete Bridge Beams with Carbon Fiber-Reinforced Polymer (CFRP) Systems



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# COURSE OUTLINE

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1. Introduction & References
2. Prestressing CFRP
3. Flexural Design
4. Shear Design
5. Prestressed Piles
6. Design Examples



# 4. SHEAR DESIGN



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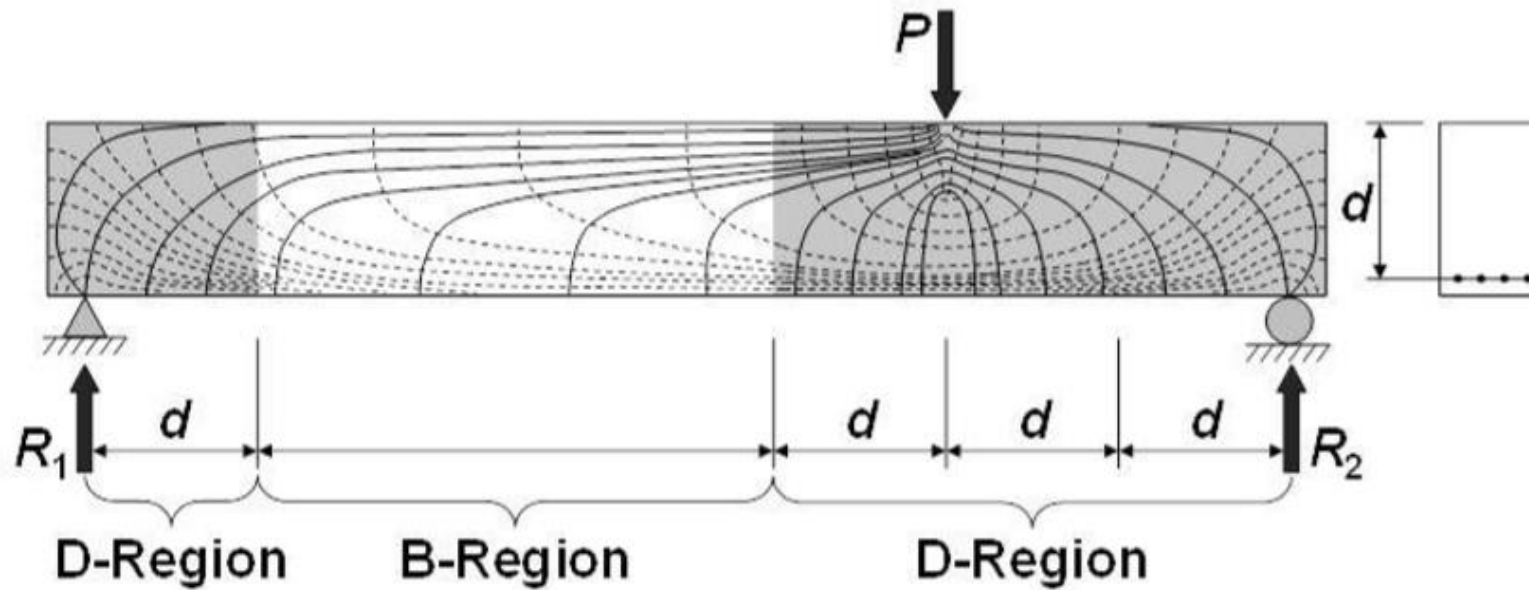
# SHEAR DESIGN

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- Transverse Shear
- Interface Shear Transfer
- Minimum Longitudinal Reinforcement
- Combined Shear and Torsion

# SHEAR DESIGN

## B- and D-region



B-region -> Sectional model, conventional beam theory applicable

D-region -> Strut-and-Tie method

# SHEAR DESIGN

## Transverse Shear Design

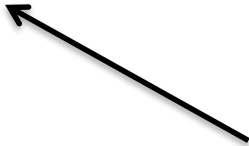
- Regions Requiring Transverse Reinforcement
- Shear Resistance
- Minimum Transverse Reinforcement
- Maximum Spacing of Transverse Reinforcement
- Deep Component

# SHEAR DESIGN

## Regions Requiring Transverse Reinforcement

Transverse reinforcement shall be provided where

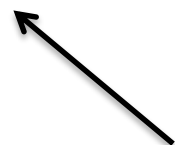
$$V_u > 0.5\phi(V_c + V_p)$$



factored shear force

Or where consideration of torsion is required by

$$T_u > 0.25\phi T_{cr}$$

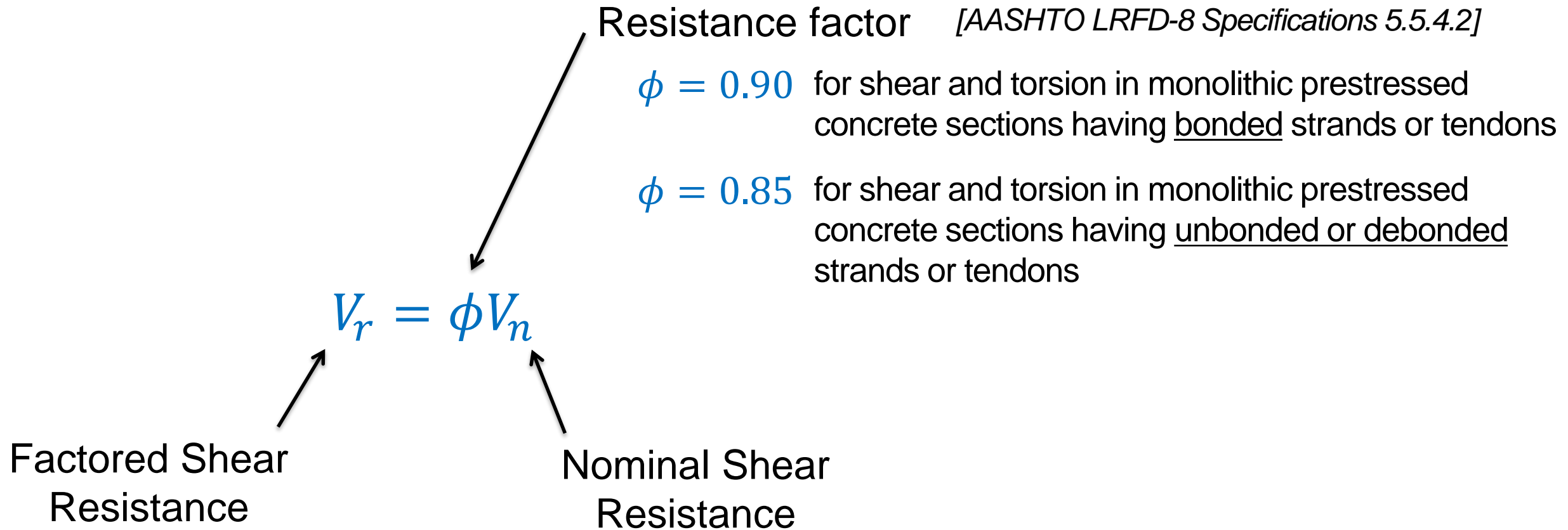


factored torsional moment



# SHEAR DESIGN

## Shear Resistance



# SHEAR DESIGN

## Nominal Shear Resistance

Contribution by concrete  $V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$

Contribution by transverse reinforcement

$$V_f = \frac{A_{fv}f_{fv}d_v \cot\theta}{s}$$

For steel  $V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha) \sin\alpha}{s}$

Contribution by prestressing force in the direction of the shear force ( $V_p = 0$  for straight strands, no draped)

$$V_p = p_e \cdot n_h \cdot \sin\Psi$$

Effective shear depth

Effective web width

$$V_n = \min \left\{ \begin{array}{l} V_n = V_c + V_f + V_p \\ V_n = 0.25f'_c b_v d_v + V_p \end{array} \right.$$

# SHEAR DESIGN

## Nominal Shear Resistance

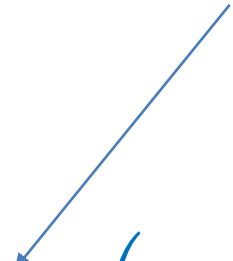
Contribution by transverse FRP reinforcement

$$V_f = \frac{A_{fv} f_{fv} d_v \cot \theta}{s}$$

$$f_{fv} = 0.004 E_f \leq f_{fb}$$

**FDOT Standard Specification 932-3:**

the strength of 90 bent bar should be greater than 60% guaranteed tensile strength of straight bar


$$f_{fb} = \left( 0.05 \frac{r_b}{d_b} + 0.3 \right) f_{fd} \leq f_{fd}$$

$f_{fv}$  = design tensile strength of transverse reinforcement

$f_{fb}$  = design tensile strength of bended portion of FRP reinforcing bars

$f_{fd}$  = design tensile strength of FRP reinforcing bars considering reductions of service environment

# SHEAR DESIGN

## Effective web width and shear depth $b_v, d_v$

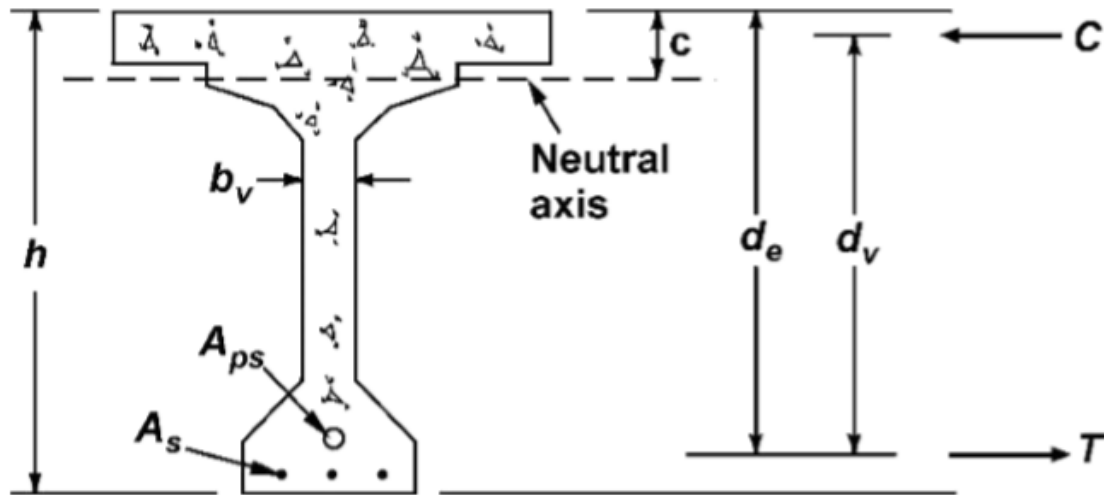


Figure C5.7.2.8-1—Illustration of the Terms  $b_v$  and  $d_v$

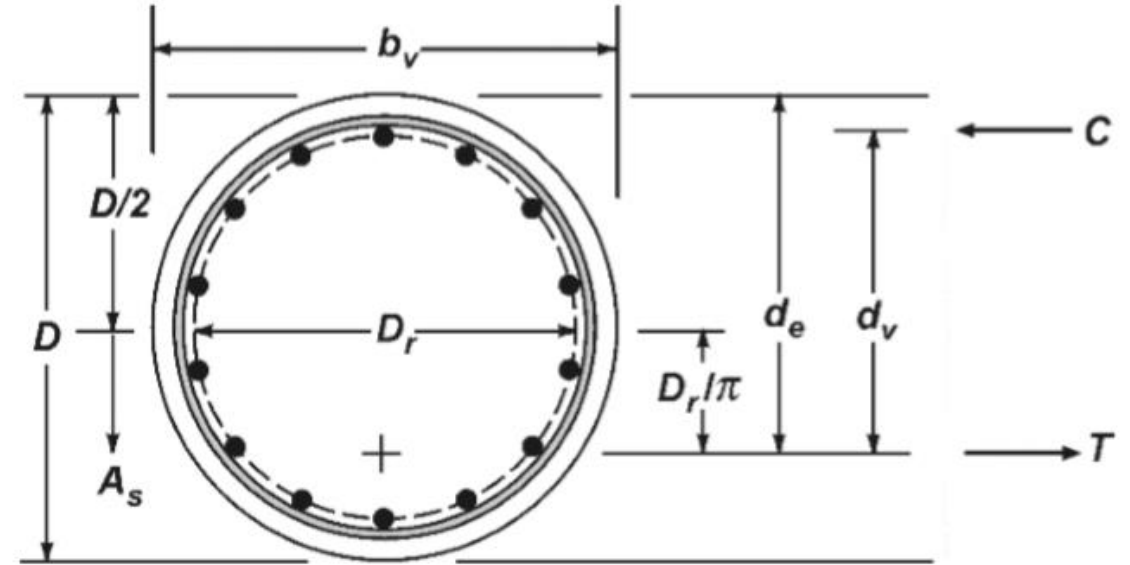


Figure C5.7.2.8-2—Illustration of Terms  $b_v, d_v$ , and  $d_e$  for Circular Sections

$b_v$ : effective web width taken as the minimum web thickness within the depth  $d_v$

$d_v$ : effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of  $0.9d_e$  or  $0.72h$



# SHEAR DESIGN

## Procedures for Determining $\beta$ and $\theta$

### Simplified Procedure

$$\beta = 5.0k$$

$$\theta = 45^\circ$$

$k$  is the ratio of neutral axis to reinforcement depth

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f$$

$$\rho_f = \frac{A_f}{bd}$$

$$n_f = \frac{E_f}{E_c}$$

# SHEAR DESIGN

## Procedures for Determining $\beta$ and $\theta$

### General Procedure

- Sections containing **at least** the minimum amount of transverse reinforcement

$$A_v \geq A_{v,min}$$

- Sections containing **less than** the minimum amount of transverse reinforcement

$$A_v < A_{v,min}$$

# SHEAR DESIGN

when  $A_v \geq A_{v,min}$

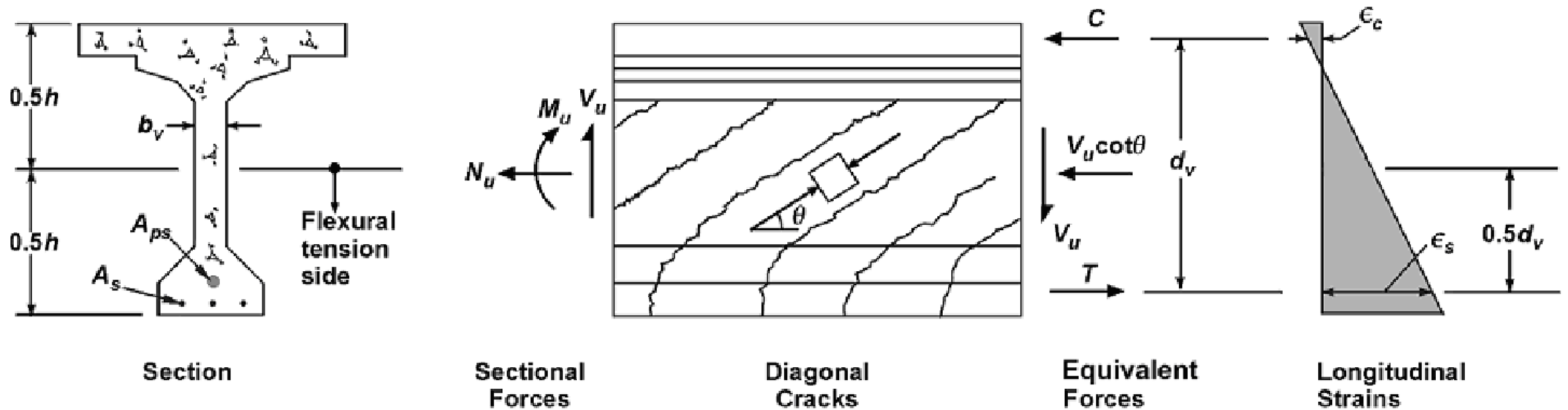


Figure 5.7.3.4.2-1—Illustration of Shear Parameters for Section Containing at Least the Minimum Amount of Transverse Reinforcement,  $V_p = 0$

# SHEAR DESIGN

when  $A_v \geq A_{v,min}$

$$\beta = \frac{4.8}{1 + 750\varepsilon_f} \quad \theta = 29 + 3500\varepsilon_f$$

Net longitudinal tensile strain

$$\varepsilon_f = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{pf}f_{p0}}{E_f A_{pf}}$$

Where consideration of torsion,  $V_u$  shall be replaced by  $V_{eff}$

For solid sections:

$$V_{eff} = \sqrt{V_u^2 + \left(\frac{0.9p_h T_u}{2A_0}\right)^2}$$

For hollow sections:

$$V_{eff} = V_u + \frac{T_u d_s}{2A_0}$$



# SHEAR DESIGN

Net longitudinal tensile strain

$$\epsilon_f = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{pf}f_{p0}}{E_f A_{pf}}$$

For usual levels of prestressing, a value of  $0.6 f_{pu}$  will be appropriate for CFRP strands

area of prestressing CFRP on the flexural tension side of the member (exclude debonded strands)

effective shear depth

# SHEAR DESIGN

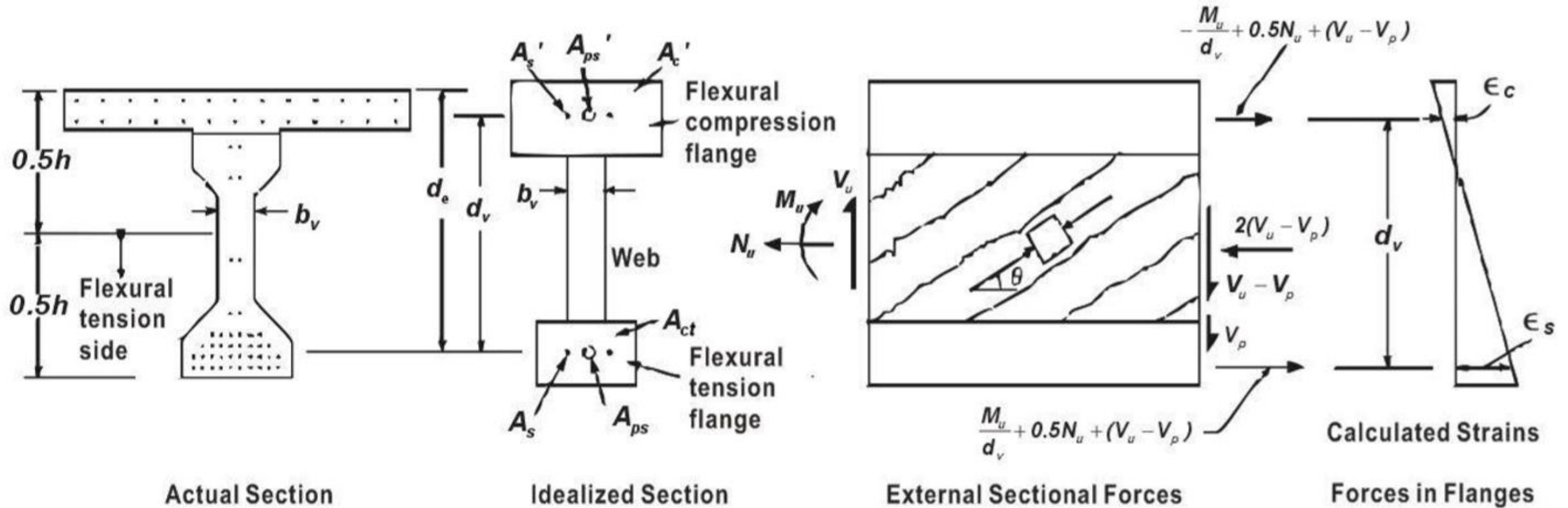


Figure C5.7.3.4.2-3—More Accurate Calculation Procedure for Determining  $\epsilon_s$

# SHEAR DESIGN

when  $A_v < A_{v,min}$

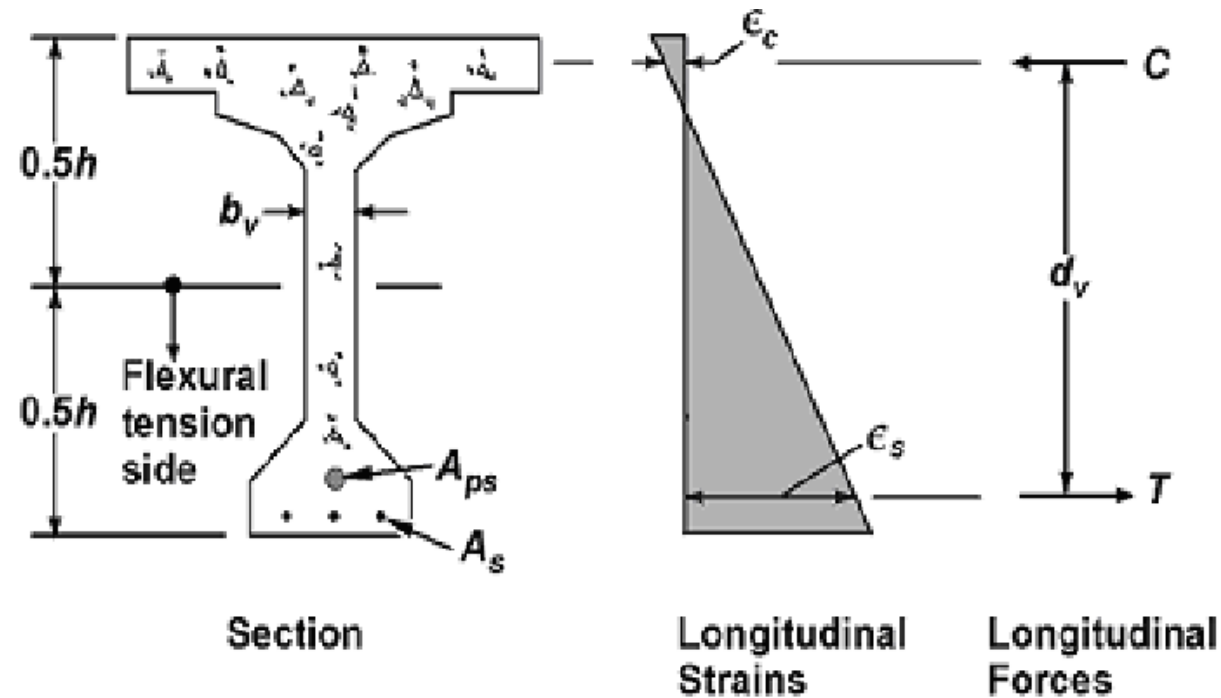


Figure 5.7.3.4.2-2—Longitudinal Strain,  $\epsilon_s$ , for Sections Containing Less than the Minimum Amount of Transverse Reinforcement

# SHEAR DESIGN

when  $A_v < A_{v,min}$

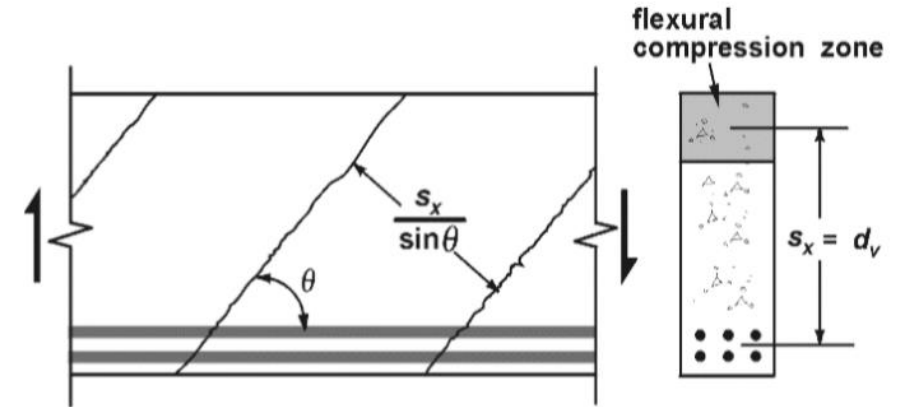
$$\beta = \frac{4.8}{1 + 750\varepsilon_f} \cdot \frac{51}{39 + s_{xe}}$$

$$\theta = 29 + 3500\varepsilon_f$$

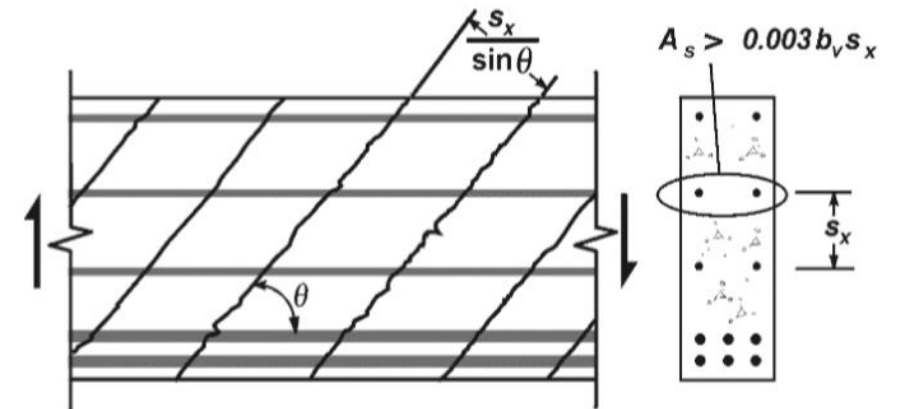
$$s_{xe} = s_x \frac{1.38}{a_g + 0.63}$$

$$12 \text{ in.} \leq s_{xe} \leq 80 \text{ in.}$$

$a_g$ : maximum aggregate size



(a) Member without transverse reinforcement and with concentrated longitudinal reinforcement



(b) Member without transverse reinforcement but with well distributed longitudinal reinforcement

Figure 5.7.3.4.2-3—Definition of Crack Spacing Parameter,  $s_x$



# SHEAR DESIGN

## Minimum Shear Reinforcement

### ACI 440.4R-04:

$$A_{v,min} = 0.75 \sqrt{f'_c} \frac{b_w s}{f_{fv}}$$

$$f_{fv} = \phi_{bend} f_{fu} \leq 0.004 \cdot E_f$$

$$0.25 \leq \phi_{bend} = \left( 0.11 + 0.05 \frac{r}{d_b} \right) f_{fu} \leq 1.0$$

### AASHTO GFRP-2:

$$A_{v,min} = 0.05 \frac{b_v s}{f_{fv}}$$

$$f_{fv} = 0.004 E_f \leq f_{fb}$$

$$f_{fb} = \left( 0.05 \frac{r_b}{d_b} + 0.3 \right) f_{fd} \leq f_{fd}$$

# SHEAR DESIGN

## Maximum Spacing of FRP Transverse Reinforcement

### ACI 440.4R-04:

$$\text{If } V_{frp} \leq 4.0\sqrt{f'_c}b_vd_v$$

$$s_{max} = 0.75h \leq 24 \text{ in.}$$

$$\text{If } V_{frp} > 4.0\sqrt{f'_c}b_vd_v$$

$$s_{max} = 0.375h \leq 12 \text{ in.}$$

### AASHTO GFRP-2:

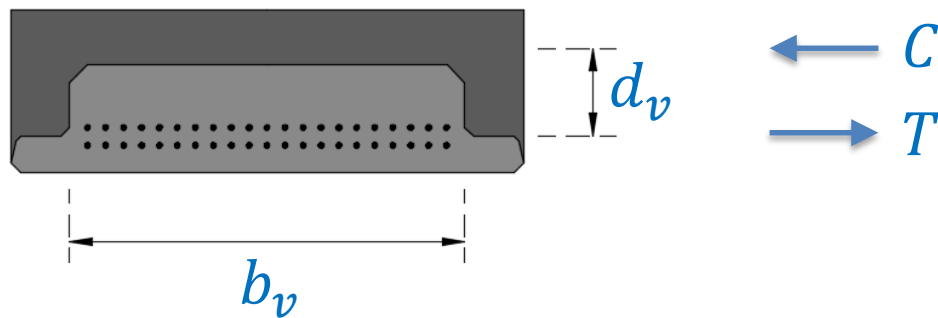
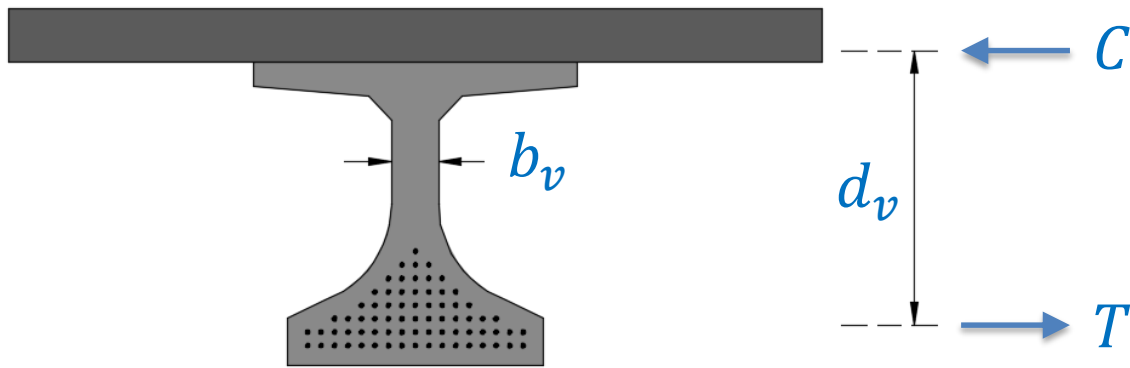
$$s_{max} = 0.5d \leq 24 \text{ in.}$$



Minimum radius and tail length of a stirrup bend

# SHEAR DESIGN

e.g. FIB/FSB



- Find critical section and corresponding  $V_u$

- Compute  $\beta$  and  $\theta$

- Concrete contribution

$$V_c = 0.0316\beta\lambda\sqrt{f'_c}b_vd_v$$

- CFRP prestressing strand contribution (if straight strands, no draped)

$$V_p = 0$$

# SHEAR DESIGN

## e.g. FIB/FSB (cont'd)

- Need to compute the amount of FRP transverse reinforcement for shear?

No, if  $V_u \leq 0.5\phi(V_c + V_p)$   $\longrightarrow$  Provide  $\frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min}$ , check spacing  $s_{max}$

Yes, if  $V_u > 0.5\phi(V_c + V_p)$   $\longrightarrow$  Compute  $\left(\frac{A_v}{s}\right)_{req}$

$$\begin{array}{l}
 \phi V_n \geq V_u \\
 V_n = V_c + V_f + V_p \leq 0.25f'_c b_v d_v + V_p \\
 V_f = \frac{f_{fv} A_v d_v}{s} \\
 \left(\frac{A_v}{s}\right)_{min}
 \end{array}
 \left. \vphantom{\begin{array}{l} \phi V_n \geq V_u \\ V_n = V_c + V_f + V_p \leq 0.25f'_c b_v d_v + V_p \\ V_f = \frac{f_{fv} A_v d_v}{s} \\ \left(\frac{A_v}{s}\right)_{min} \end{array}} \right\}
 \begin{array}{l}
 V_{f,req} \\
 \left(\frac{A_v}{s}\right)_{req}
 \end{array}
 \longrightarrow
 \begin{array}{l}
 \text{Provide } \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min} \\
 \text{Check spacing } s_{min}, s_{max}
 \end{array}$$

# SHEAR DESIGN

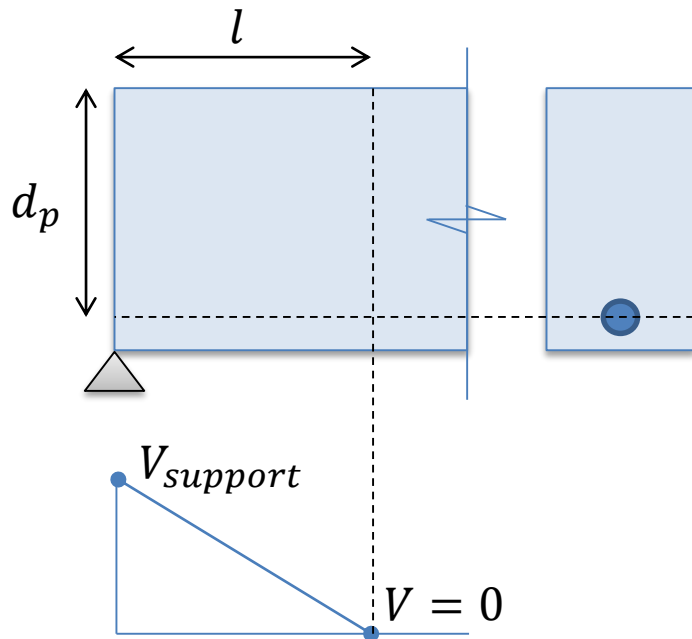
- a) Components in which the distance from the point of **zero shear** to the face of the support is less than  $2d_p$
- b) Components in which a load causing more than  $\frac{1}{2}$  of the shear at a support is closer than  $2d_p$  from the face of the support

} **Deep Component**  
use

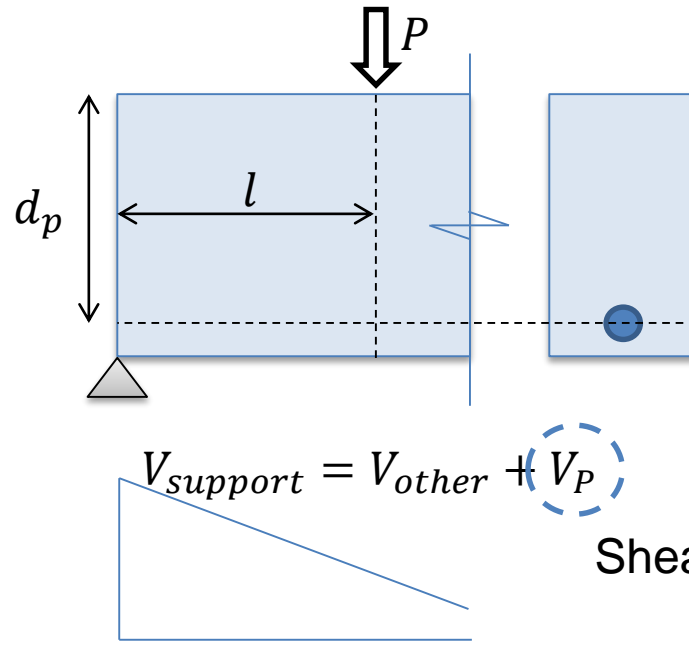
[AASHTO CFRP-1 Specifications 1.6.3]  
[AASHTO LRFD-8 Specifications 5.8.2]

e.g.

a) if  $l < 2d_p$



b) if  $V_P > 0.5V_{support}$  and  $l < 2d_p$



# SHEAR DESIGN

## Interface Shear Transfer

- Interface shear resistance
- Cohesion and friction factors
- Interface shear force for girder/slab bridges
- Interface shear in box girder bridges
- Minimum area of interface shear reinforcement

# SHEAR DESIGN

Interface shear transfer shall be considered across a given plane at:

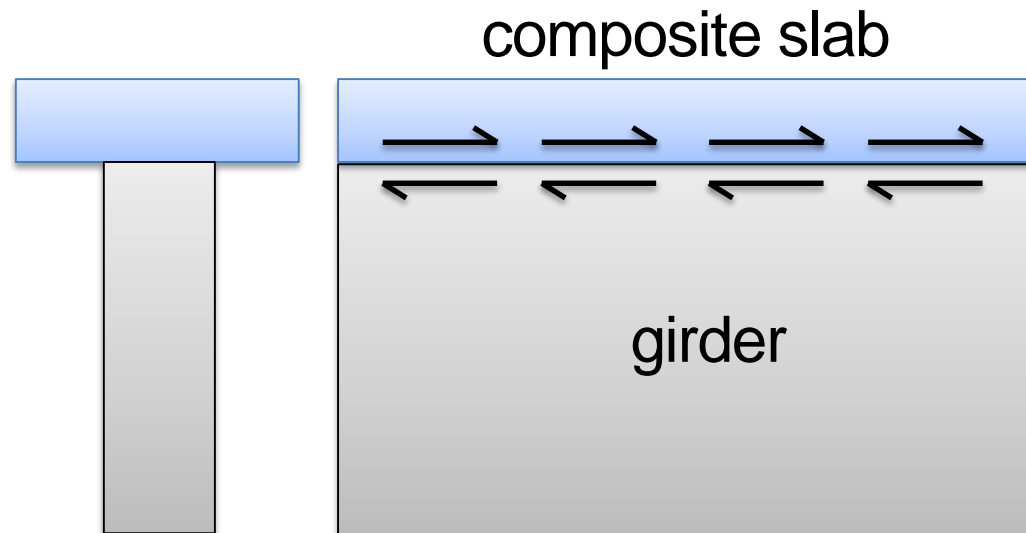
- ✓ An existing or potential **crack**
- ✓ An interface between **dissimilar materials**
- ✓ An interface between **two concretes cast** at different time
- ✓ The interface between **different elements** of the cross-section



# SHEAR DESIGN

Shear displacement along an interface plane may be resisted by

- ✓ Cohesion
- ✓ Aggregate interlock
- ✓ Shear friction



$$v_h = \frac{VQ}{Ib} \longrightarrow v_h \cong \frac{V}{bd} \xrightarrow{\text{unit length}} v_h \cong \frac{V}{d}$$

# SHEAR DESIGN

## Interface Shear Resistance

$$V_{ri} = \phi V_{ni} \geq V_{ui}$$

Resistance factor

Factored Interface Shear Resistance

Factored interface shear force due to total load based on the applicable strength and extreme event load combinations

Nominal Interface Shear Resistance

The diagram illustrates the design equation for interface shear resistance. The equation is  $V_{ri} = \phi V_{ni} \geq V_{ui}$ . Arrows point from the following text to the corresponding parts of the equation: 'Resistance factor' points to  $\phi$ ; 'Factored Interface Shear Resistance' points to  $V_{ri}$ ; 'Nominal Interface Shear Resistance' points to  $V_{ni}$ ; and 'Factored interface shear force due to total load based on the applicable strength and extreme event load combinations' points to  $V_{ui}$ .

# SHEAR DESIGN

## Nominal Interface Shear Resistance

Nominal Interface Shear Resistance

$$V_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c) \quad [Replaced f_y \text{ with } f_{fv}]$$

$V_{ni}$  used in the design shall satisfy:

$$V_{ni} \leq K_1 f'_c A_{cv}$$

$$V_{ni} \leq K_2 A_{cv}$$

# SHEAR DESIGN

## Nominal Interface Shear Resistance

cohesion factor

friction factor

Use the minimum transfer shear strength  $f_{fv,min} = 22$  ksi for GFRP & CFRP bars (per *FDOT Materials Spec. 932-3*), due to the potential for premature failure modes, until more testing completed

$$V_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c)$$

[Replaced  $f_y$  with  $f_{fv}$ ]

permanent net compressive force normal to the shear plane,  $P_c = 0$  if tension

area of reinforcement crossing the shear plane

area of concrete engaged in shear transfer  $A_{cv} = b_{vi}L_{vi}$

# SHEAR DESIGN

## Cohesion and Friction Factors $c, \mu, K_1, K_2$

e.g.

- For a cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.

$$\begin{aligned}c &= 0.28 \text{ ksi} \\ \mu &= 1.0 \\ K_1 &= 0.3 \\ K_2 &= 1.8 \text{ ksi for normal weight concrete} \\ &= 1.3 \text{ ksi for lightweight concrete}\end{aligned}$$

- For normal weight concrete placed monolithically:

$$\begin{aligned}c &= 0.40 \text{ ksi} \\ \mu &= 1.4 \\ K_1 &= 0.25 \\ K_2 &= 1.5 \text{ ksi}\end{aligned}$$

- For normal weight concrete placed against a clean concrete surface, free of laitance, with surface intentionally roughened to an amplitude of 0.25 in.:

$$\begin{aligned}c &= 0.24 \text{ ksi} \\ \mu &= 1.0 \\ K_1 &= 0.25 \\ K_2 &= 1.5 \text{ ksi}\end{aligned}$$

- For concrete placed against a clean concrete surface, free of laitance, but not intentionally roughened:

$$\begin{aligned}c &= 0.075 \text{ ksi} \\ \mu &= 0.6 \\ K_1 &= 0.2 \\ K_2 &= 0.8 \text{ ksi}\end{aligned}$$

# SHEAR DESIGN

## Interface Shear Force for Girder/Slab Bridges

Factored interface shear stress

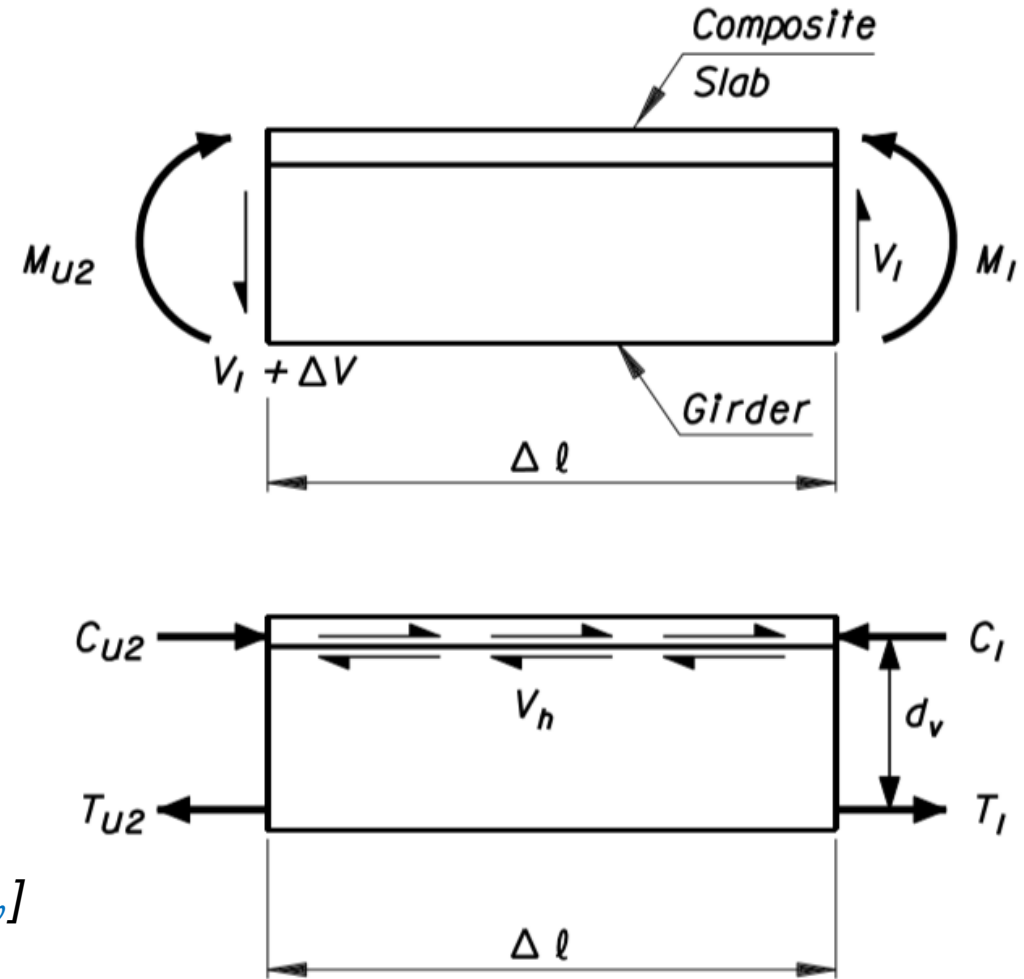
$$v_{ui} = \frac{V_{u1}}{b_{vi}d_v}$$

Factored interface shear force (kips/ft)

$$V_{ui} = v_{ui}A_{cv} = v_{ui} \cdot 12b_{vi}$$

If the net force  $P_c$  is tensile, additional reinforcement shall be provided as

$$A_{vpc} = \phi \frac{P_c}{f_{fv}} \quad [\text{Replaced } f_y \text{ with } f_{fv}]$$



# SHEAR DESIGN

## Interface Shear in Box Girder Bridges

Adequate shear transfer reinforcement shall be provided at the web/flange in box girders to transfer flange longitudinal forces at the strength limit state



# SHEAR DESIGN

## Minimum Area of Interface Shear Reinforcement

$$A_{vf.min} = \frac{0.05A_{cv}}{f_{fv}} \quad \text{[Replaced } f_y \text{ with } f_{fv}\text{]}$$

$A_{cv}$  = Area of concrete considered to be engaged in interface shear transfer ( $in^2$ )

$A_{vf}$  = Area of interface shear reinforcement crossing the shear plane within the area  $A_{cv}$  ( $in^2$ )

$f'_{fv}$  = Design transverse strength for shear for FRP bar. Use the minimum transfer shear strength  $f_{fv,min} = 22$  ksi for GFRP & CFRP bars (per *FDOT Materials Spec. 932-3*), due to the potential for premature failure modes, until more testing completed.

# SHEAR DESIGN

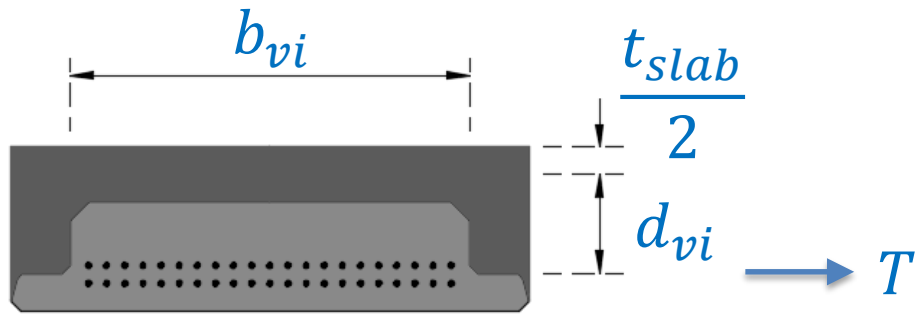
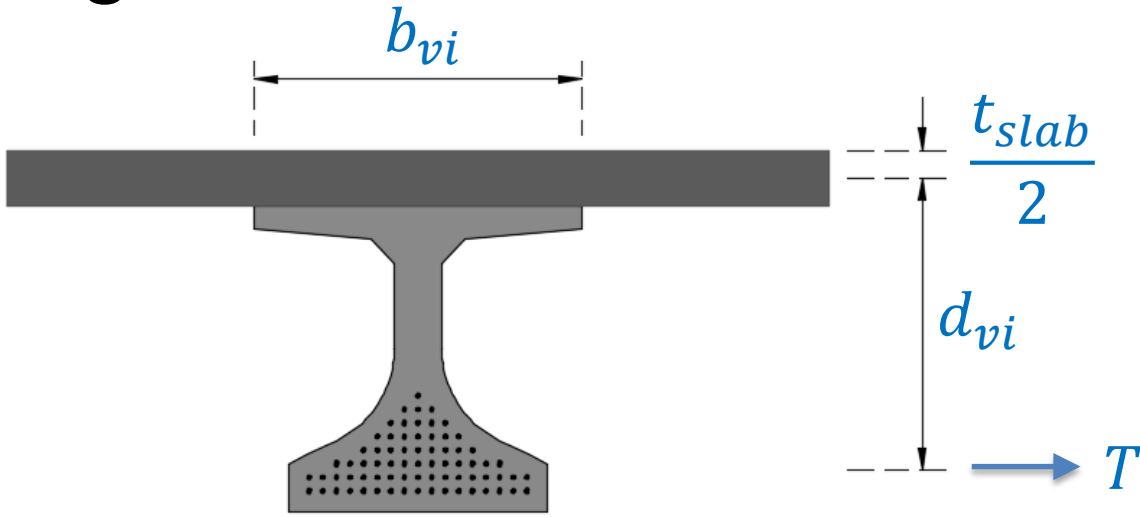
## Minimum Area of Interface Shear Reinforcement

For Cast-In-Place concrete slab on clean concrete girder surfaces free of laitance, additional requirements apply

- The minimum interface shear reinforcement, need not exceed the lesser of  $A_{vf.min}$  and the amount needed to resist  $1.33 \cdot V_{ui} / \phi$
- The minimum interface shear reinforcement requirements shall be **waived** for girder/slab interfaces with surface roughened to an amplitude of 0.25 inch where the factored interface shear stress is less than 0.21 ksi, and all vertical transverse shear reinforcement is extended across the interface and adequately anchored in the slab

# SHEAR DESIGN

e.g. FIB/FSB



- Find critical section and corresponding interface shear force  $V_{ui}$
- Factored interface shear force per unit length  $v_{hi}$  (kip/ft)

$$v_{hi} = \frac{V_{ui}}{d_{vi}}$$

- Required nominal interface shear force per unit length  $v_{ni,req}$  (kip/ft)

$$v_{ni,req} = \frac{v_{hi}}{\phi}$$

# SHEAR DESIGN

e.g. FIB/FSB (cont'd)

- Required nominal interface shear reinforcement per unit length (in<sup>2</sup>/ft)

$$v_{ni,req} = \frac{v_{hi}}{\phi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c)$$

$$v_{ni} \leq K_1 f'_c A_{cv}$$

$$v_{ni} \leq K_2 A_{cv}$$

$$A_{cf,min}$$

$A_{cf,req}$



Provide

$$A_{cf} \geq A_{cf,req}$$

# SHEAR DESIGN

## Longitudinal Reinforcement

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy

$$\sum_{x=1}^n A_{px} f_{px} \geq \frac{|M_u|}{d_v \phi_f} + 0.5 \frac{N_u}{\phi_c} + \left[ \left| \frac{V_u}{\phi_v} - V_p \right| - 0.5 V_f \right] \cot \theta$$

[Replaced  $V_s$  with  $V_f$ ]

$\phi_f, \phi_c, \phi_v$  are resistance factor of flexure, compression/tension, and shear, respectively

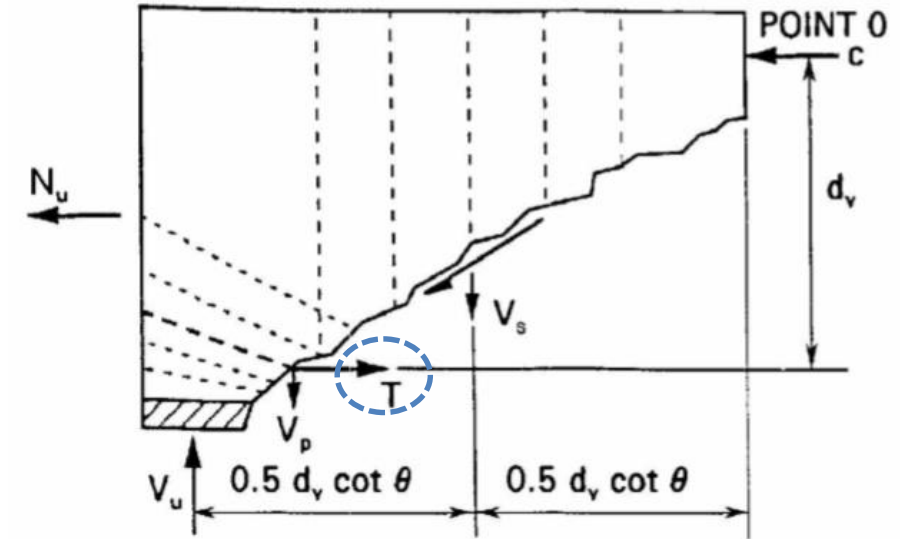


Figure C5.7.3.5-1—Forces Assumed in Resistance Model Caused by Moment and Shear

# SHEAR DESIGN

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## Combined Shear and Torsion

- Transverse Reinforcement
- Torsion Resistance
- Longitudinal Reinforcement

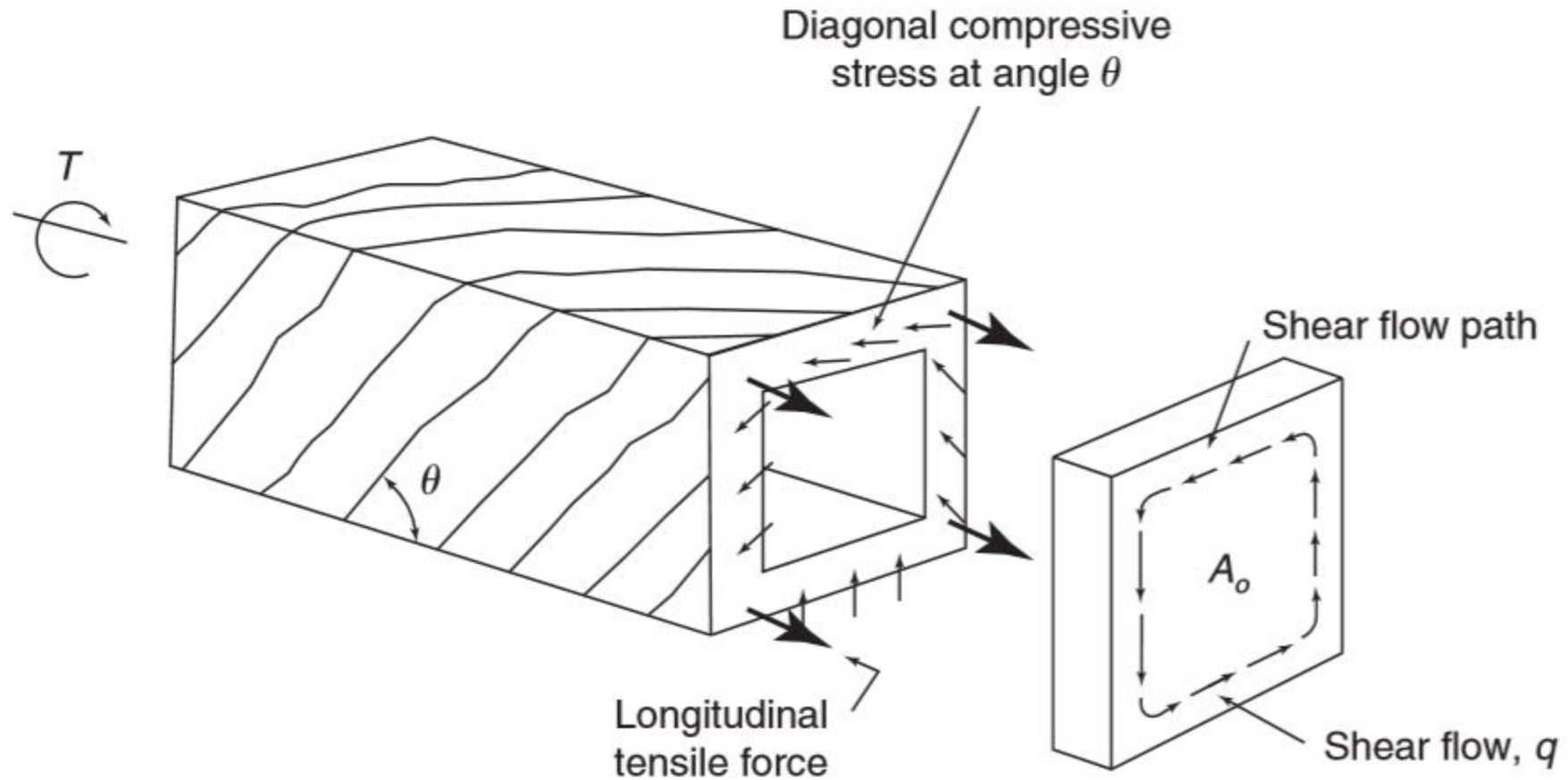
# SHEAR DESIGN

## Transverse Reinforcement





# SHEAR DESIGN



Thin-walled tube analogy.

# SHEAR DESIGN

## Nominal Torsional Resistance

$$T_n = \frac{2A_0 A_t f_{pu} \cot \theta}{S}$$

Area enclosed by the shear flow path, including an area of holes therein

Angle of inclination of diagonal compressive stresses

[Replaced  $f_y$  with  $f_{pu}$ ]

For solid member: area of one leg of enclosed transverse torsion reinforcement in solid members,  
For hollow member: total area of transverse torsion reinforcement in the exterior web and flange

# SHEAR DESIGN

## Longitudinal Reinforcement

In solid sections:

$$A_{pf}f_{pf} + A_s f_y \geq \frac{|M_u|}{\phi d_v} + \frac{0.5N_u}{\phi} + \cot \theta \sqrt{\left(\left|\frac{V_u}{\phi} - V_p\right| - 0.5V_f\right)^2 + \left(\frac{0.45\rho_h T_u}{2A_0\phi}\right)^2}$$

In box sections, longitudinal reinforcement for torsion (in addition to flexural reinforcement)

$$A_l \geq \frac{T_u \rho_h}{2A_0 f_{pu}}$$

[Replaced  $f_y$  with  $f_{pu}$ ]

[Replaced  $V_s$  with  $V_f$ ]

# Questions?



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# 4. SHEAR DESIGN

## 4.1 Review Questions: Fundamentals



# REVIEW QUESTIONS

**4.1.1) For GFRP transverse reinforcements, does the maximum amount of transverse reinforcement requirement similar to steel transverse reinforcements still apply:\_\_\_\_\_**

- a. True
- b. False

# REVIEW QUESTIONS

Minimum GFRP Transverse Reinforcement

$$A_{v,min} = 0.05 \frac{b_v s}{f_{fv}}$$

Maximum GFRP Transverse Reinforcement

$$V_f \leq 0.25 f'_c b_v d_v$$



# REVIEW QUESTIONS

4.1.1) For GFRP transverse reinforcements, does the maximum amount of transverse reinforcement requirement similar to steel transverse reinforcements still apply:\_\_\_\_\_

- a. True
- b. False

# REVIEW QUESTIONS

## 4.1.2) The shear strength of PC members with GFRP transverse reinforcements\_\_\_\_\_

- a. Is comparable to the shear strength of PC members with steel transverse reinforcements
- b. Is lower than the shear strength of PC members with steel transverse reinforcements
- c. Is higher than to the shear strength of PC members with steel transverse reinforcements
- a. Cannot be compared to the shear strength of PC members with steel transverse reinforcements

# REVIEW QUESTIONS

## Nominal Shear Resistance

Contribution by concrete  $V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$

Contribution by GFRP transverse reinforcement

$$V_f = \frac{A_{fv}f_{fv}d_v \cot\theta}{s}$$

$$V_n = \min \left\{ \begin{array}{l} V_n = V_c + V_f + V_p \\ V_n = 0.25f'_c b_v d_v + V_p \end{array} \right.$$

Contribution by prestressing force in the direction of the shear force ( $V_p = 0$  for straight strands, no draped)

$$V_p = p_e \cdot n_h \cdot \sin \Psi$$

Effective shear depth

Effective web width

# REVIEW QUESTIONS

## 4.1.2) The shear strength of PC members with GFRP transverse reinforcements\_\_\_\_\_

- a. Is comparable to the shear strength of PC members with steel transverse reinforcements
- b. Is lower than the shear strength of PC members with steel transverse reinforcements
- c. Is higher than to the shear strength of PC members with steel transverse reinforcements
- a. Cannot be compared to the shear strength of PC members with steel transverse reinforcements

# REVIEW QUESTIONS

4.1.3) The required tail length of FRP stirrups is at least equal to or more than \_\_\_\_\_ times the bar diameter

- a. 4
- b. 8
- c. 12
- a. 16

# REVIEW QUESTIONS

## Maximum Spacing of FRP Transverse Reinforcement

### ACI 440.4R-04:

If  $V_{frp} \leq 4.0\sqrt{f'_c}b_vd_v$

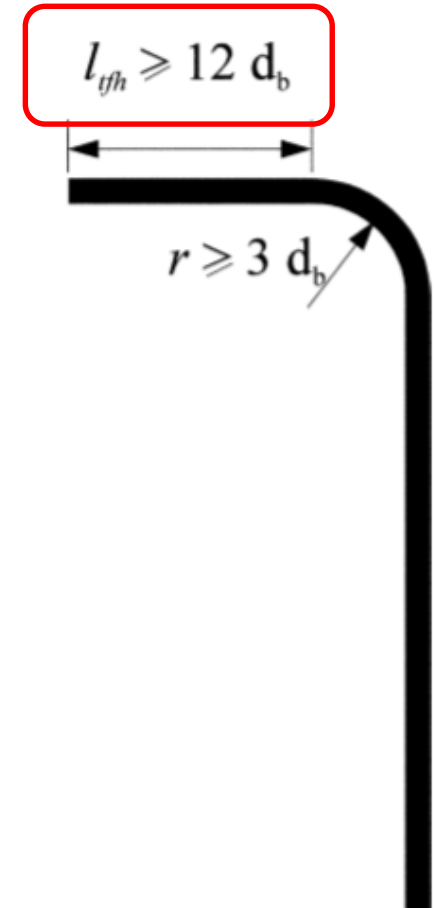
$$s_{max} = 0.75h \leq 24 \text{ in.}$$

If  $V_{frp} > 4.0\sqrt{f'_c}b_vd_v$

$$s_{max} = 0.375h \leq 12 \text{ in.}$$

### AASHTO GFRP-2:

$$s_{max} = 0.5d \leq 24 \text{ in.}$$



Minimum radius and tail length of a stirrup bend

# REVIEW QUESTIONS

4.1.3) The required tail length of GFRP stirrups is at least equal to or more than \_\_\_\_\_ times the bar diameter

- a. 4
- b. 8
- c. 12
- a. 16

# REVIEW QUESTIONS

4.1.4) The maximum spacing of transverse GFRP reinforcement is generally \_\_\_\_\_ per AASHTO GFRP-2.

- a. 12 in.
- b. 24 in.
- c.  $0.5d^*$
- d. Min ( $0.5d^*$ , 24 in.)

\*Flexural reinforcement depth



# REVIEW QUESTIONS

## Maximum Spacing of FRP Transverse Reinforcement

### ACI 440.4R-04:

$$\text{If } V_{frp} \leq 4.0\sqrt{f'_c}b_vd_v \quad S_{max} = 0.75h \leq 24 \text{ in.}$$

$$\text{If } V_{frp} > 4.0\sqrt{f'_c}b_vd_v \quad S_{max} = 0.375h \leq 12 \text{ in.}$$



Minimum radius and tail length of a stirrup bend

### AASHTO GFRP-2:

$$S_{max} = 0.5d \leq 24 \text{ in.}$$

# REVIEW QUESTIONS

4.1.4) The maximum spacing of transverse GFRP reinforcement is generally \_\_\_\_\_ per AASHTO GFRP-2.

- a. 12 in.
- b. 24 in.
- c.  $0.5d^*$
- d. Min ( $0.5d^*$ , 24 in.)

\*Flexural reinforcement depth

# REVIEW QUESTIONS

---

4.1.5) GFRP stirrups can be bent on site with EOR approval?

- a. True
- b. False

# REVIEW QUESTIONS

- Field bending or straightening of GFRP bars not possible
- All stirrups are **pre-bent**



# REVIEW QUESTIONS

---

4.1.5) GFRP stirrups can be bent on site with EOR approval?

- a. True
- b. **False**

# 4. SHEAR DESIGN

## 4.2 Design Example: FIB 36



# DESIGN EXAMPLE: FIB

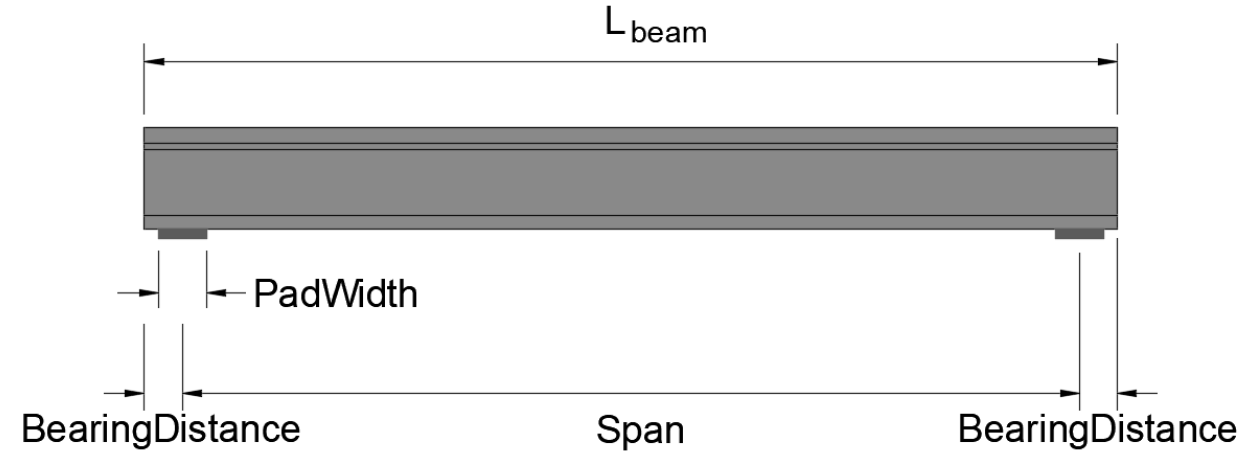
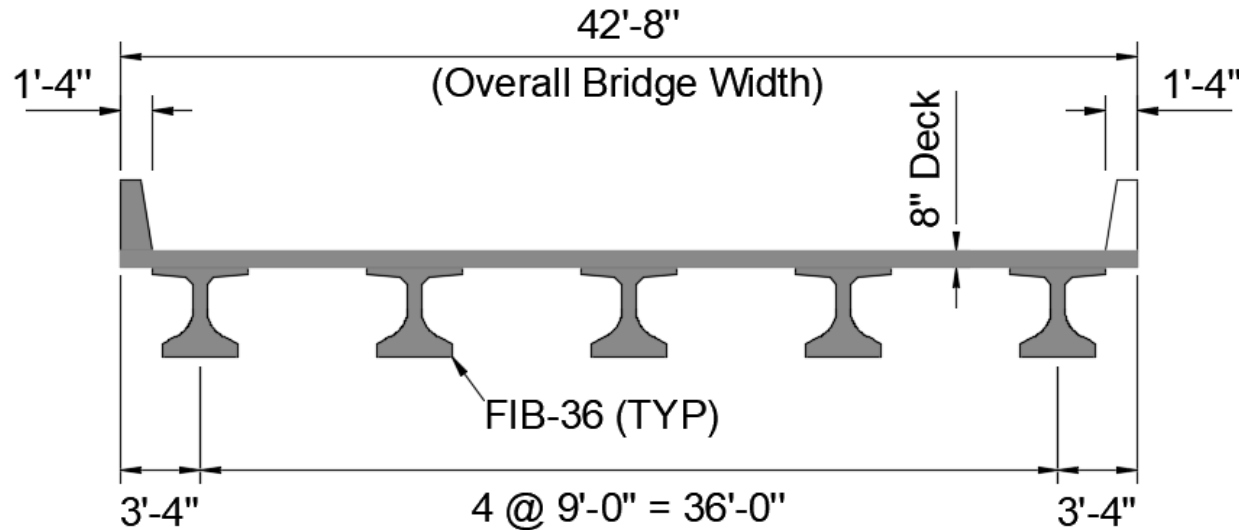
---

- Transverse Shear
- Interface Shear Transfer
- Minimum Longitudinal Reinforcement



# DESIGN EXAMPLE: FIB

**Geometry** This example is simplified from "SR 687/4th Street, NB bridge" project



Beam Span = 87.667 ft

BeamSpacing = 9 ft

BridgeWidth = 42.667 ft

Slab thickness  $t_{slab} = 8.5$  in

Beam depth  $h_{beam} = 36$  in



# DESIGN EXAMPLE: FIB

## Concrete

$$\text{Beam } f'_{c.beam} = 8.5 \text{ ksi}$$

$$\text{Beam } f'_{ci.beam} = 6 \text{ ksi}$$

$$\text{Beam } E_{c.beam} = 5112 \text{ ksi}$$

$$\text{Beam } E_{ci.beam} = 4557 \text{ ksi}$$

$$\text{Slab } f'_{c.slab} = 5.5 \text{ ksi}$$

$$\text{Slab } E_{c.slab} = 4428 \text{ ksi}$$

$$\text{Unit weight } \gamma_c = 150 \text{ pcf}$$

*Note: 145 pcf is permitted*

## CFRP strand

$$\text{Diameter } D_p = 0.6 \text{ in}$$

$$\text{Effective area } A_{pf} = 0.179 \text{ in}^2$$

$$\text{Elastic modulus } E_f = 22,480 \text{ ksi}$$

$$\text{Design tensile strength } f_{pu} = 341 \text{ ksi}$$

$$\text{Design tensile strain } \varepsilon_{pu} = 0.015 \text{ ksi}$$

$$\text{Jacking stress } f_{pj} = 239 \text{ ksi}$$

## GFRP rebar

$$\text{Bar size} = \# 5$$

$$\text{Diameter } d_{GFRP} = 0.625 \text{ in}$$

$$\text{Elastic modulus } E_{GFRP} = 6500 \text{ ksi}$$

$$\text{Design tensile strength}$$

$$f_{fu.GFRP} = 66.4 \text{ ksi}$$

$$\text{Bend } \phi_{bend} = 0.6$$

# DESIGN EXAMPLE: FIB

## Section Properties Non-Composite Section

$$h_g = 36 \text{ in}$$

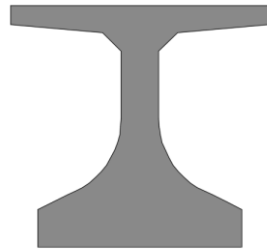
$$A_g = 807 \text{ in}^2$$

$$I_g = 1.275 \times 10^5 \text{ in}^4$$

$$y_{g,t} = 19.51 \text{ in} \quad y_{g,b} = 16.49 \text{ in}$$

$$S_{g,t} = 6537 \text{ in}^3 \quad S_{g,b} = 7735 \text{ in}^3$$

$$d_v = 38.80 \text{ in} \quad b_v = 7 \text{ in}$$



## Section Properties Composite Section

$$h_{g,c} = 45.5 \text{ in}$$

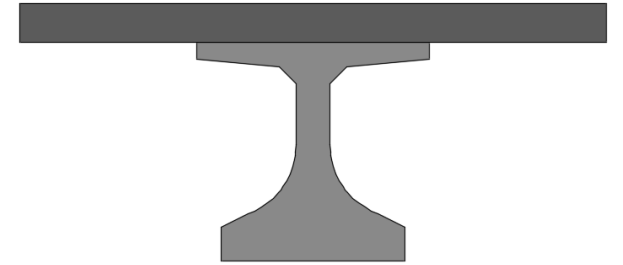
$$A_{g,c} = 1643 \text{ in}^2$$

$$I_{g,c} = 4.333 \times 10^5 \text{ in}^4$$

$$y_{g,c,t} = 16.52 \text{ in} \quad y_{g,c,b} = 28.98 \text{ in}$$

$$S_{g,c,t} = 2.622 \times 10^4 \text{ in}^3$$

$$S_{g,c,b} = 1.495 \times 10^4 \text{ in}^3$$



# DESIGN EXAMPLE: FIB

## Critical Section for Shear

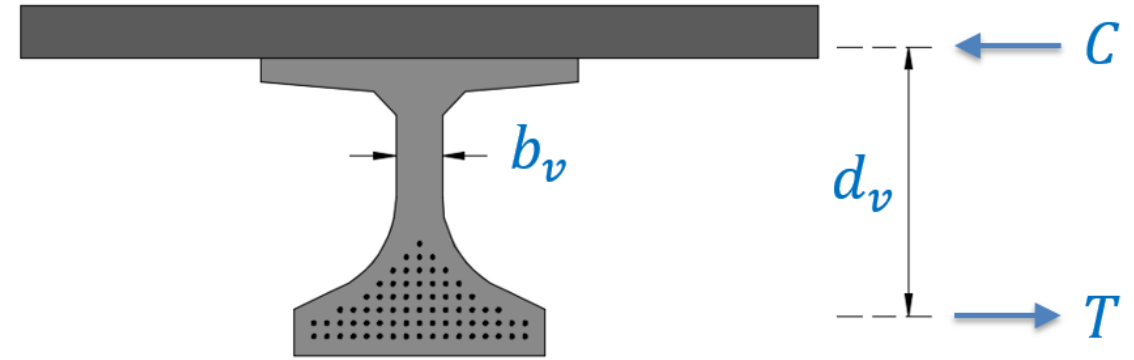
Critical Section is  $d_v$  from support

## Strength I

$$V_u = 272 \text{ kip}$$

$$M_u = 886 \text{ kip}$$

$$N_u = 0$$



$$d_v = \max(d_v, 0.72d_e, 0.72h_{g.c.}) = 38.80 \text{ in}$$

$$b_v = 7 \text{ in}$$

# DESIGN EXAMPLE: FIB

## Nominal Shear Strength

$$V_n = \min \begin{cases} V_n = V_c + V_f + V_p \\ V_n = 0.25f'_c b_v d_v + V_p \end{cases}$$

## Concrete Contribution

$$V_c = 0.316\beta\lambda\sqrt{f'_c} b_v d_v = 147 \text{ kip}$$

## Prestressing CFRP Contribution

$$V_p = 0 \quad (\text{Straight strands})$$

# DESIGN EXAMPLE: FIB

Need to compute the amount of FRP transverse reinforcement for shear?

No, if  $V_u \leq 0.5\phi(V_c + V_p)$   $\longrightarrow$  Provide  $\frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min}$ , check spacing  $s_{max}$

Yes, if  $V_u > 0.5\phi(V_c + V_p)$   $\longrightarrow$  Compute  $\left(\frac{A_v}{s}\right)_{req}$

$$V_n = V_c + V_f + V_p \leq 0.25f'_c b_v d_v + V_p$$

$$\left. \begin{array}{l} \phi V_n \geq V_u \\ V_n = V_c + V_f + V_p \leq 0.25f'_c b_v d_v + V_p \end{array} \right\} V_{f,req}$$

$$V_f = \frac{f_{fv} A_v d_v}{s}$$

$$\left. \begin{array}{l} V_{f,req} \\ \left(\frac{A_v}{s}\right)_{min} \end{array} \right\} \left(\frac{A_v}{s}\right)_{req} \longrightarrow \begin{array}{l} \text{Provide } \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min} \\ \text{Check spacing } s_{min}, s_{max} \end{array}$$

# DESIGN EXAMPLE: FIB

$$V_u = 272 \text{ kip} > 0.5\phi(V_c + V_p) = 66 \text{ kip}$$



Need to compute the amount of FRP transverse reinforcement for shear

$$V_{f,req} = 155 \text{ kip}$$



Assume double-leg #5 GFRP stirrups  $A_v = 0.614 \text{ in}^2$



$$S_{f,req} \leq 3.38 \text{ in}$$

# DESIGN EXAMPLE: FIB

Provide double-leg #5 GFRP stirrups @ 3"

$$A_v \geq A_{v,min} = 0.05 \frac{b_v s}{f_{fv}} \quad \text{OK}$$

$$s \leq s_{max} = \min(0.5d_v, 24 \text{ in}) \quad \text{OK}$$

# DESIGN EXAMPLE: FIB

**Concrete Contribution**  $V_c = 0.316\beta\lambda\sqrt{f'_c}b_vd_v = 147 \text{ kip}$

**Prestressing CFRP Contribution**  $V_p = 0$

**GFRP Stirrup Contribution**  $V_f = \frac{A_v f_{fv} d_v}{s} = 175 \text{ kip}$

## Nominal Shear Strength

$$V_n = \min \left\{ \begin{array}{l} V_n = V_c + V_f + V_p \\ V_n = 0.25f'_c b_v d_v + V_p \end{array} \right. = 322 \text{ kip}$$

$$\phi V_n > V_u = 272 \text{ kip} \quad \text{OK}$$



# DESIGN EXAMPLE: FIB

## Interface Shear

$$\phi V_{ni} \geq V_{ui}$$

$$v_{ni,req} = \frac{v_{hi}}{\phi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c)$$

$$v_{ni} \leq K_1 f'_c A_{cv}$$

$$v_{ni} \leq K_2 A_{cv}$$

$$A_{cf,min}$$

$A_{cf,req}$



Provide

$$A_{cf} \geq A_{cf,req}$$

# DESIGN EXAMPLE: FIB

## Factored interface shear

$$v_{hi} = \frac{V_{ui}}{d_{vi}} = 89.0 \text{ kip/ft}$$

## Required nominal interface shear resistance

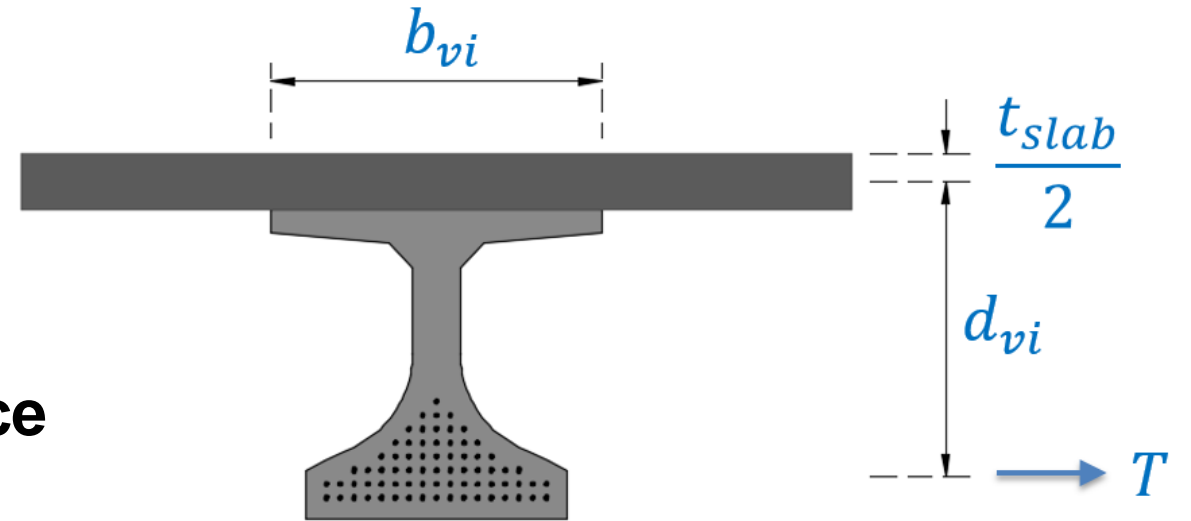
$$v_{ni,req} = \frac{v_{hi}}{\phi} = 98.9 \text{ kip/ft}$$

## Provided nominal interface shear resistance

$$\mu = 1$$

$$c = 0.28 \text{ ksi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c) = 215 \text{ kip/in} > v_{ni,req} \quad \text{OK}$$



# DESIGN EXAMPLE: FIB

## Provided nominal interface shear resistance

$$\mu = 1 \quad c = 0.28 \text{ ksi} \quad K_1 = 0.3 \quad K_2 = 1.8 \text{ ksi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c) = 215 \text{ kip/ft} > v_{ni,req} \quad \text{OK}$$

$$v_{ni} \leq K_1f'_cA_{cv} = 1469 \text{ kip/ft} \quad \text{OK}$$

$$v_{ni} \leq K_2A_{cv} = 1037 \text{ kip/ft} \quad \text{OK}$$

For cast-in-place concrete slab on clean concrete girder surfaces free of laitance, the minimum interface shear reinforcement, need not exceed the lesser of  $A_{vf,min}$  and the amount needed to resist  $1.33 \cdot V_{ui}/\phi$

$$v_{ni} > 1.33 \cdot v_{ui}/\phi \quad \text{NO Need to Satisfy} \quad A_{cf} > A_{vf,min} = \frac{0.05A_{cv}}{f_{fv}} = 1.1 \text{ in}^2/\text{ft}$$

# DESIGN EXAMPLE: FIB

## Longitudinal Reinforcement

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy

$$\sum_{x=1}^n A_{px} f_{px} \geq \frac{|M_u|}{d_v \phi_f} + 0.5 \frac{N_u}{\phi_c} + \left[ \left| \frac{V_u}{\phi_v} - V_p \right| - 0.5 V_f \right] \cot \theta$$

For example, at support location

563 kip  $\geq$  442 kip      **OK**

**No Additional Longitudinal Reinforcement is Required**

# 4. SHEAR DESIGN

## 4.3 Design Example: FSB 12x57





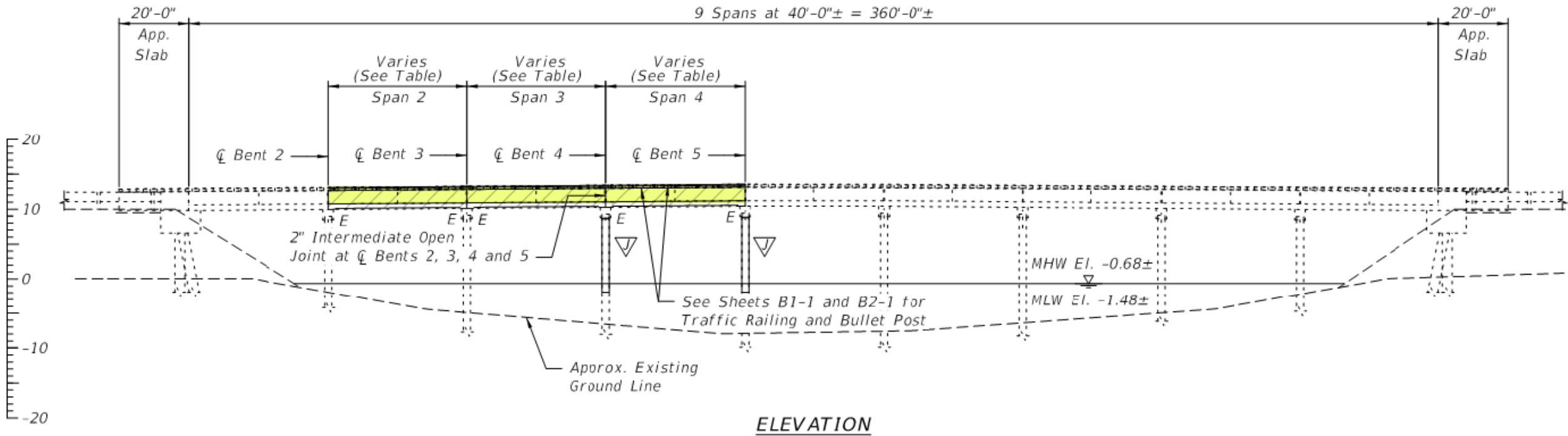
# DESIGN EXAMPLE: FSB

## US 1 Over Cow Key Channel, Key West, Florida



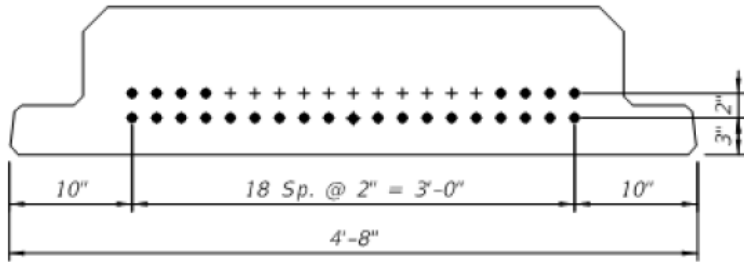
# DESIGN EXAMPLE: FSB

## Geometry, from US 1 Over Cow Key Channel, Key West, Florida

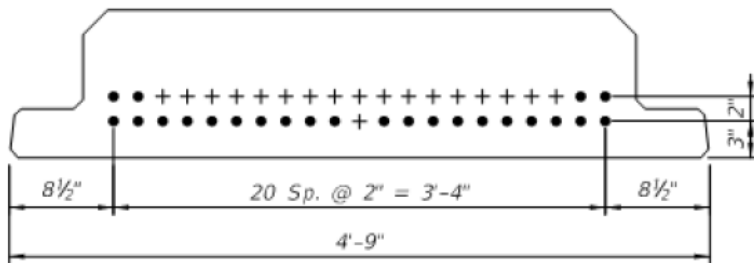


# DESIGN EXAMPLE: FSB

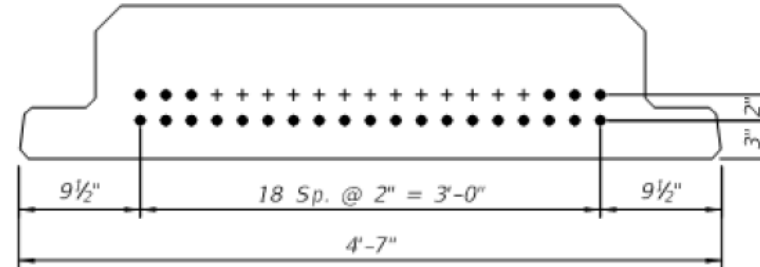
## US 1 Over Cow Key Channel, Key West, Florida



TYPE ① 27 STRANDS



TYPE ② 24 STRANDS



TYPE ① 25 STRANDS

### FIBER-REINFORCED POLYMER REINFORCING:

1. FSB precast panels shall be reinforced using Glass-Fiber Reinforced Polymer (GFRP) and shall be prestressed using Carbon-Fiber Reinforced Polymer (CFRP).
2. All FRP reinforcing shall be per Specification Section 932 and 933.
3. FRP reinforcing detail dimensions are out-to-out of bars.
4. For standard bar bending detail for FRP, see FDOT Developmental Design Index D21310.

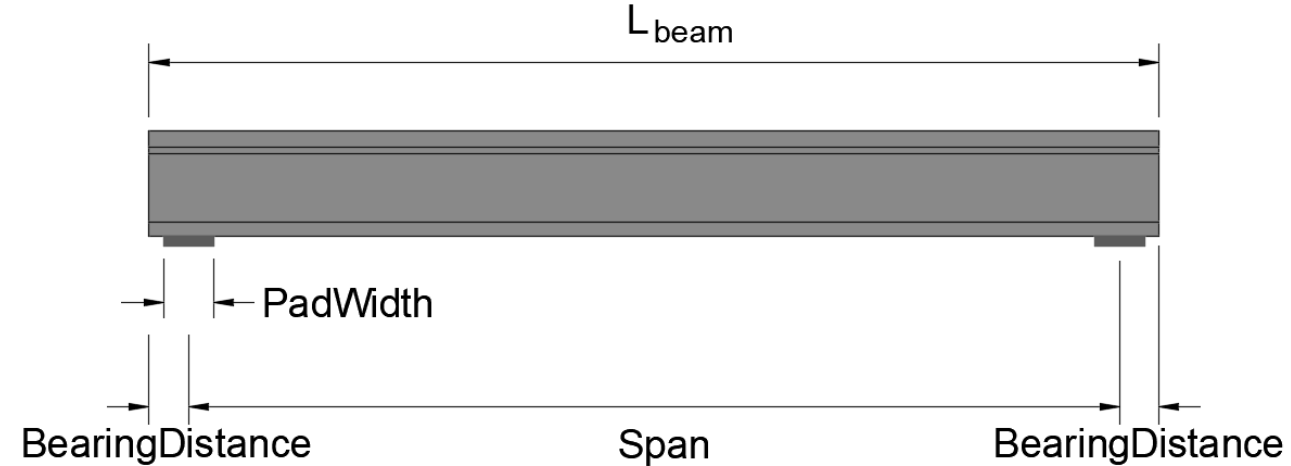
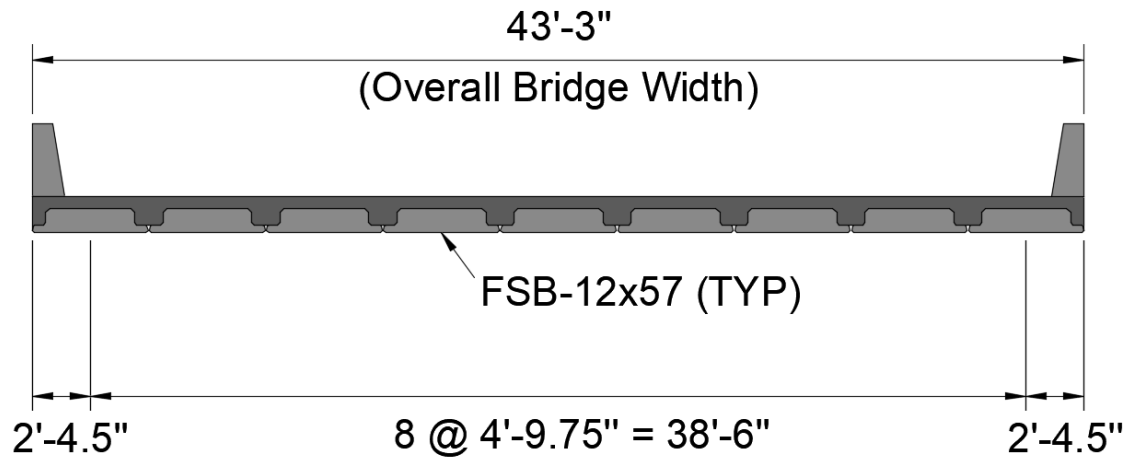
ITEM NUMBER	ITEM DESCRIPTION	UNIT	44174015201	44174015201 BR# 900086	44174015201 BR# 900125	QUANTITY TOTAL
0110- 3-	REMOVAL OF EXISTING STRUCTURES/BRIDGES 44174015201 900086	(LS)		1.000		1.000
0450- 8- 13	PRESTRESSED BEAM: FLORIDA SLAB BEAM, BEAM DEPTH 12", WIDTH 55-57"	LF		1046.000	1395.000	2441.000



# DESIGN EXAMPLE: FSB

## Geometry

*This Example is simplified from "US 1 Over Cow Key Channel" bridge*



Beam Span = 38.917 ft

BeamSpacing = 4.813 ft

BridgeWidth = 43.25 ft

Slab thickness  $t_{slab} = 6$  in

Beam depth  $h_{beam} = 12$  in

# DESIGN EXAMPLE: FSB

## Concrete

$$\text{Beam } f'_{c.beam} = 8.5 \text{ ksi}$$

$$\text{Beam } f'_{ci.beam} = 6 \text{ ksi}$$

$$\text{Beam } E_{c.beam} = 5112 \text{ ksi}$$

$$\text{Beam } E_{ci.beam} = 4557 \text{ ksi}$$

$$\text{Slab } f'_{c.slab} = 5.5 \text{ ksi}$$

$$\text{Slab } E_{c.slab} = 4428 \text{ ksi}$$

$$\text{Unit weight } \gamma_c = 150 \text{ pcf}$$

*Note: 145 pcf is permitted*

## CFRP strand

$$\text{Diameter } D_p = 0.6 \text{ in}$$

$$\text{Effective area } A_{pf} = 0.179 \text{ in}^2$$

$$\text{Elastic modulus } E_p = 22,480 \text{ ksi}$$

$$\text{Design tensile strength } f_{pu} = 341 \text{ ksi}$$

$$\text{Design tensile strain } \varepsilon_{pu} = 0.015 \text{ ksi}$$

$$\text{Jacking stress } f_{pj} = 239 \text{ ksi}$$

## GFRP rebar

$$\text{Bar size} = \# 4$$

$$\text{Diameter } d_{GFRP} = 0.5 \text{ in}$$

$$\text{Elastic modulus } E_{GFRP} = 6500 \text{ ksi}$$

$$\text{Design tensile strength}$$

$$f_{fu.GFRP} = 77.0 \text{ ksi}$$

$$\text{Bend } \phi_{bend} = 0.6$$

# DESIGN EXAMPLE: FSB

## Section Properties Non-Composite Section

$$h_g = 12 \text{ in}$$

$$A_g = 582 \text{ in}^2$$

$$I_g = 7084 \text{ in}^4$$

$$y_{g,t} = 6.35 \text{ in} \quad y_{g,b} = 5.65 \text{ in}$$

$$S_{g,t} = 1116 \text{ in}^3$$

$$S_{g,b} = 1254 \text{ in}^3$$



## Section Properties Composite Section

$$h_{g,c} = 18 \text{ in}$$

$$A_{g,c} = 974 \text{ in}^2$$

$$I_{g,c} = 2.598 \times 10^4 \text{ in}^4$$

$$y_{g,c,t} = 9.22 \text{ in} \quad y_{g,c,b} = 8.78 \text{ in}$$

$$S_{g,c,t} = 2819 \text{ in}^3$$

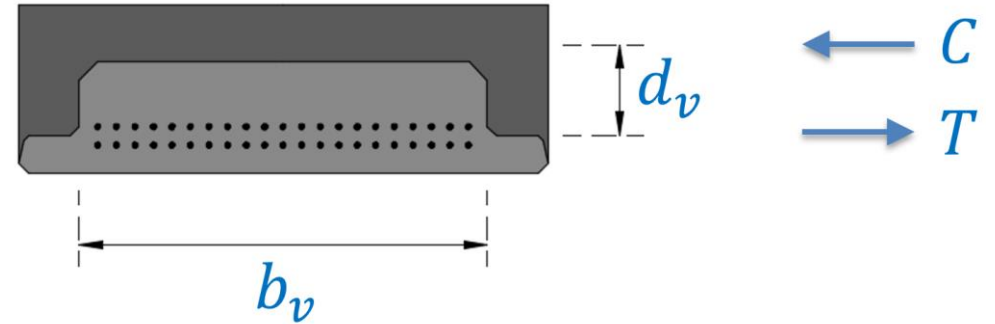
$$S_{g,c,b} = 2959 \text{ in}^3$$



# DESIGN EXAMPLE: FSB

## Critical Section for Shear

Critical Section is  $d_v$  from support



## Strength I

$$V_u = 103 \text{ kip}$$

$$M_u = 121 \text{ kip}$$

$$N_u = 0$$

$$d_v = \max(d_v, 0.72d_e, 0.72h_{g.c.}) = 13.50 \text{ in}$$

$$b_v = 45 \text{ in}$$

# DESIGN EXAMPLE: FSB

## Nominal Shear Strength

$$V_n = \min \begin{cases} V_n = V_c + V_f + V_p \\ V_n = 0.25f'_c b_v d_v + V_p \end{cases}$$

## Concrete Contribution

$$V_c = 0.316\beta\lambda\sqrt{f'_c} b_v d_v = 350 \text{ kip}$$

## Prestressing CFRP Contribution

$$V_p = 0 \quad (\text{Straight strands})$$

# DESIGN EXAMPLE: FSB

Need to compute the amount of FRP transverse reinforcement for shear?

No, if  $V_u \leq 0.5\phi(V_c + V_p)$   $\longrightarrow$  Provide  $\frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min}$ , check spacing  $s_{max}$

Yes, if  $V_u > 0.5\phi(V_c + V_p)$   $\longrightarrow$  Compute  $\left(\frac{A_v}{s}\right)_{req}$

$$V_n = V_c + V_f + V_p \leq 0.25f'_c b_v d_v + V_p$$

$$V_n \geq \phi V_u$$

$$V_f = \frac{f_{fv} A_v d_v}{s}$$

$$\left(\frac{A_v}{s}\right)_{min}$$

$$V_{f,req}$$

$$\left(\frac{A_v}{s}\right)_{req}$$

$$\longrightarrow$$

$$\text{Provide } \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min}$$

$$\text{Check spacing } s_{min}, s_{max}$$

# DESIGN EXAMPLE: FSB

$$V_u = 103 \text{ kip} < 0.5\phi(V_c + V_p) = 157 \text{ kip}$$



NO need to compute the amount of FRP transverse reinforcement for shear

Provide minimum transverse shear reinforcement



Provide 4-leg #5 GFRP stirrups @ 9"

$$A_v \geq A_{v,min} = 0.05 \frac{b_v s}{f_{fv}} \quad \text{OK}$$

$$s \leq s_{max} \quad \text{OK}$$

# DESIGN EXAMPLE: FSB

**Concrete Contribution**  $V_c = 0.316\beta\lambda\sqrt{f'_c}b_vd_v = 350 \text{ kip}$

**Prestressing CFRP Contribution**  $V_p = 0$

**GFRP Stirrup Contribution**  $V_f = \frac{A_vf_{fv}d_v}{s} = 33 \text{ kip}$

## Nominal Shear Strength

$$V_n = \min \left\{ \begin{array}{l} V_n = V_c + V_f + V_p \\ V_n = 0.25f'_cb_vd_v + V_p \end{array} \right. = 383 \text{ kip}$$

$$\phi V_n > V_u = 103 \text{ kip} \quad \text{OK}$$



# DESIGN EXAMPLE: FSB

## Interface Shear

$$\phi V_{ni} \geq V_{ui}$$

$$v_{ni,req} = \frac{v_{hi}}{\phi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c)$$

$$v_{ni} \leq K_1 f'_c A_{cv}$$

$$v_{ni} \leq K_2 A_{cv}$$

$$A_{cf,min}$$

$A_{cf,req}$



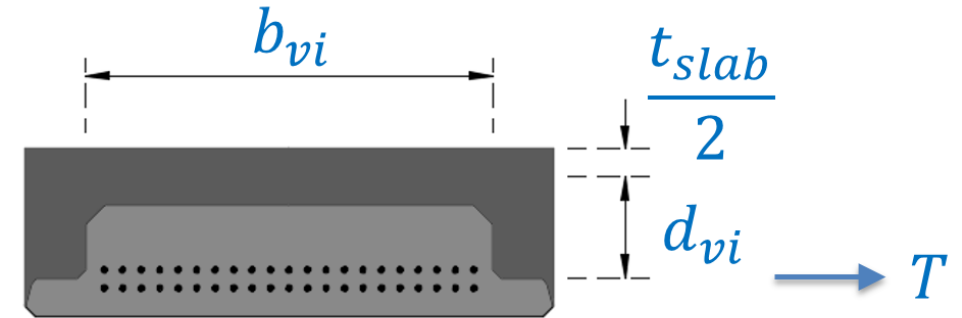
Provide

$$A_{cf} \geq A_{cf,req}$$

# DESIGN EXAMPLE: FSB

## Factored interface shear

$$v_{hi} = \frac{V_{ui}}{d_{vi}} = 103 \text{ kip/ft}$$



## Required nominal interface shear resistance

$$v_{ni,req} = \frac{v_{hi}}{\phi} = 114 \text{ kip/ft}$$

## Provided nominal interface shear resistance

$$\mu = 1$$

$$c = 0.28 \text{ ksi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c) = 226 \text{ kip/ft} > v_{ni,req} \quad \text{OK}$$

# DESIGN EXAMPLE: FSB

## Provided nominal interface shear resistance

$$\mu = 1 \quad c = 0.28 \text{ ksi} \quad K_1 = 0.3 \quad K_2 = 1.8 \text{ ksi}$$

$$v_{ni} = cA_{cv} + \mu(A_{cf}f_{fv} + P_c) = 226 \text{ kip/ft} > v_{ni,req} \quad \text{OK}$$

$$v_{ni} \leq K_1 f'_c A_{cv} = 1790 \text{ kip/ft} \quad \text{OK}$$

$$v_{ni} \leq K_2 A_{cv} = 1264 \text{ kip/ft} \quad \text{OK}$$

For cast-in-place concrete slab on clean concrete girder surfaces free of laitance, the minimum interface shear reinforcement, need not exceed the lesser of  $A_{vf.min}$  and the amount needed to resist  $1.33 \cdot V_{ui}/\phi$

$$v_{ni} > 1.33 \cdot V_{ui}/\phi \quad \text{NO need to satisfy} \quad A_{cf} > A_{vf.min} = \frac{0.05A_{cv}}{f_{fv}} = 1.35 \text{ in}^2/\text{ft}$$

# DESIGN EXAMPLE: FSB

## Longitudinal Reinforcement

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy

$$\sum_{x=1}^n A_{px} f_{px} \geq \frac{|M_u|}{d_v \phi_f} + 0.5 \frac{N_u}{\phi_c} + \left[ \left| \frac{V_u}{\phi_v} - V_p \right| - 0.5 V_f \right] \cot \theta$$

For example, at support location

254 kip  $\geq$  197 kip      **OK**

**No Additional Longitudinal Reinforcement is Required**

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