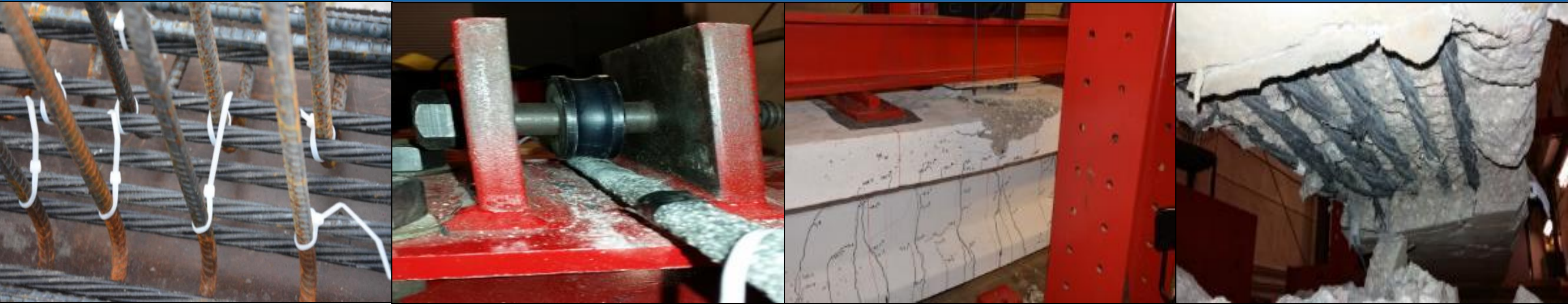
AASHTO CFRP-Prestressed Concrete Design Training Course



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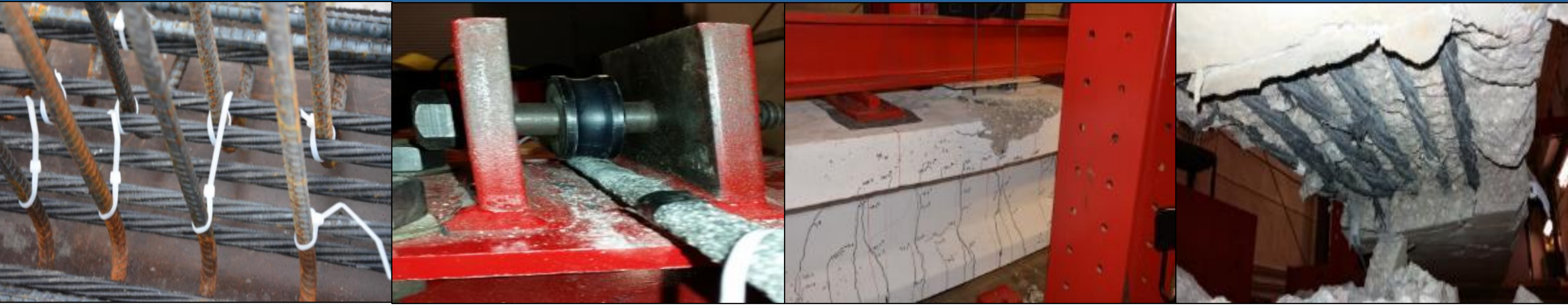
Department of Civil & Environmental Engineering

Design of Pretensioned Concrete Bridge Beams with Carbon Fiber-Reinforced Polymer (CFRP) Systems



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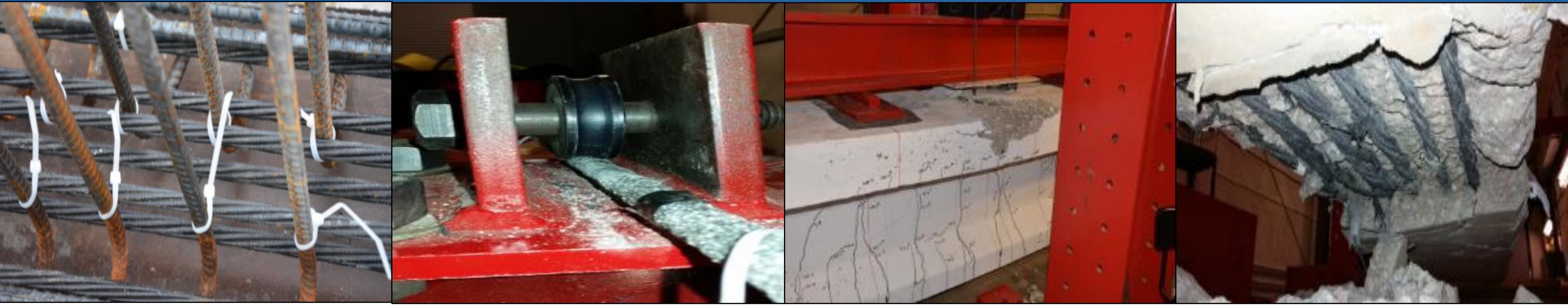
COURSE OUTLINE

1. Introduction & References
2. Prestressing CFRP
3. Flexural Design
4. Shear Design
5. Prestressed Piles
6. Design Examples

3. FLEXURAL DESIGN



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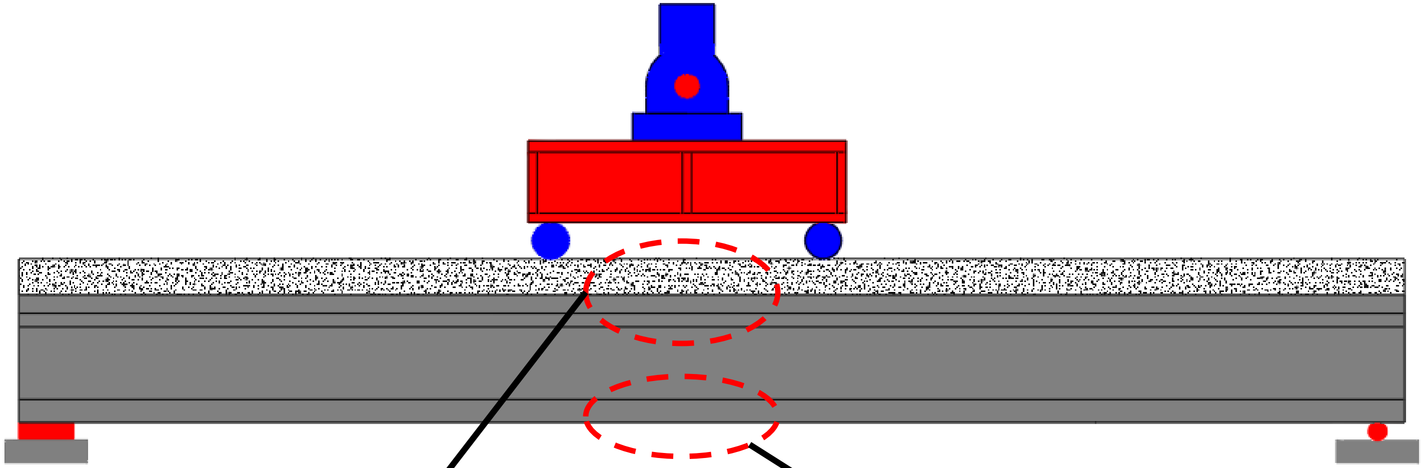
FLEXURAL DESIGN

- ✓ Failure Modes and Resistance Factor
- ✓ Assumptions for Design
- ✓ Stress Limits for Concrete
- ✓ Flexural Resistance
- ✓ Minimum Reinforcement
- ✓ Pretensioned Anchorage Zone
- ✓ Deflection and Camber

FLEXURAL DESIGN

Failure Modes



Concrete crushing
Compression failure

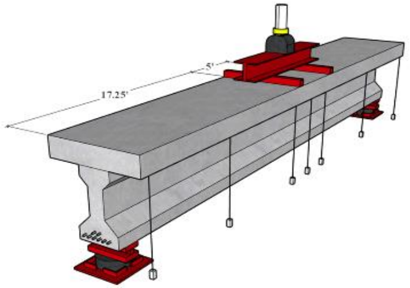
CFRP rupture
Tension failure

FLEXURAL DESIGN

Load-Deflection Behavior: CFRP vs. Steel Pretensioned Beam

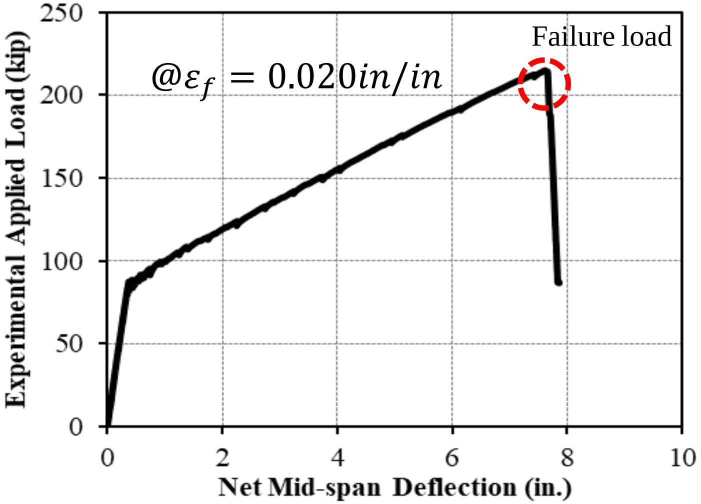
CFRP pretensioned beams

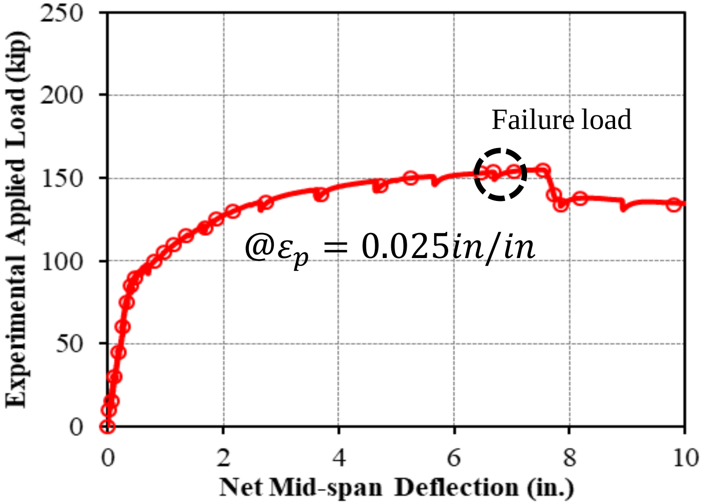
8 CFRP strands
(Ø = 0.6 in)
$A_{pf}f_{fpu}$= 625 kips

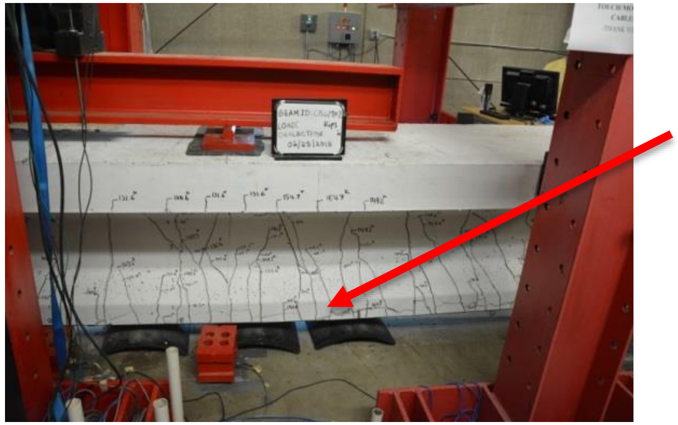


Steel pretensioned beams

8 steel strands
(Ø = 0.6 in)
A_sf_{pu}= 470 kips







CFRP Rupture

Concrete Crushing

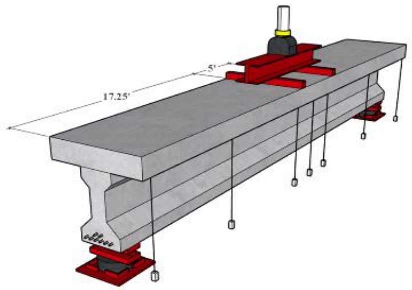
[NCHRP Report 907]

FLEXURAL DESIGN

Load-Deflection Behavior: CFRP vs. Steel Pretensioned Beam

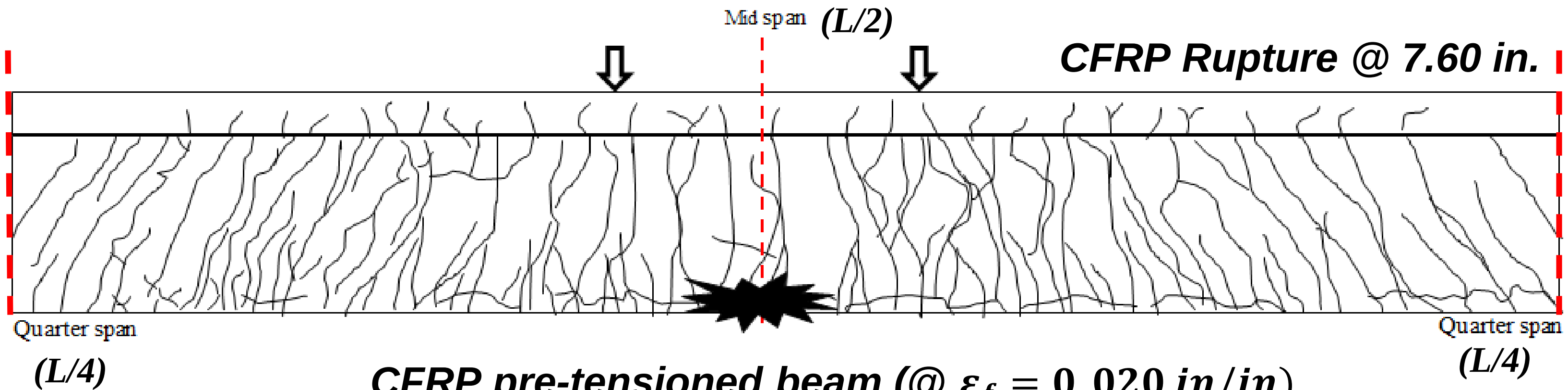
CFRP pretensioned beams

8 CFRP strands
(Ø = 0.6 in)
$A_{pf}f_{fpu}$= 625 kips

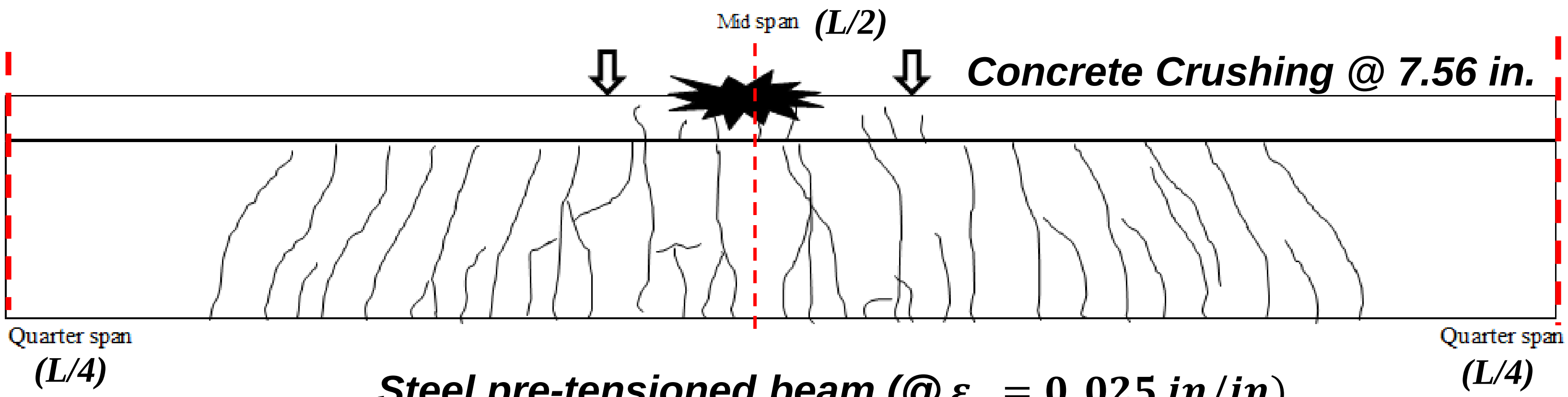


Steel pretensioned beams

8 steel strands
(Ø = 0.6 in)
A_sf_{pu}= 470 kips



CFRP pre-tensioned beam (@ $\varepsilon_f = 0.020\ in/in$)



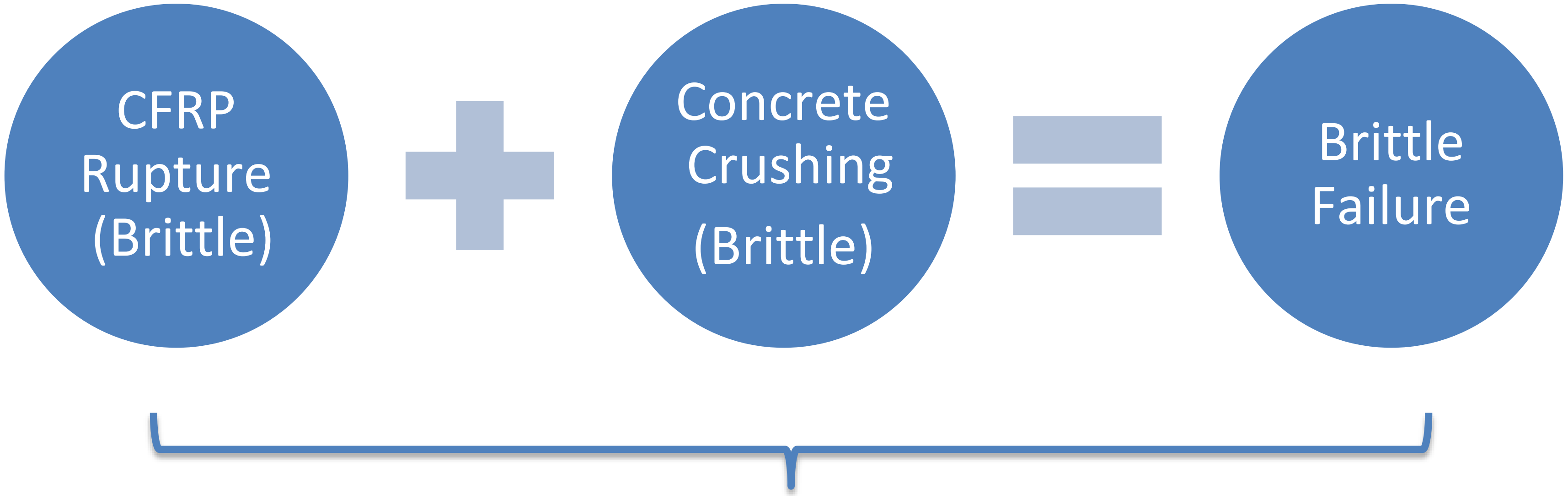
Steel pre-tensioned beam (@ $\varepsilon_p = 0.025\ in/in$)

[NCHRP Report 907]

FLEXURAL DESIGN

Resistance Factor

For flexural design, the resistance factor, ϕ, shall be taken as 0.75.



CFRP Rupture (Brittle)

\+

Concrete Crushing (Brittle)

=

Brittle Failure

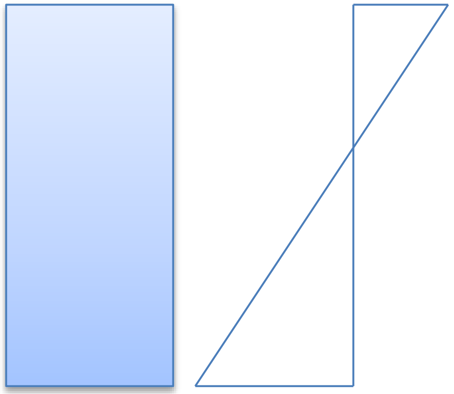
$\phi = 0.75$

[Same as concrete crushing]

[AASHTO CFRP-1 Specifications 1.5.3.2]

FLEXURAL DESIGN

Assumptions for Design: Service Limit State



Concrete strain

- The strains in the concrete vary linearly
- Where transformed section analysis is used to assess
  - elastic shortening at transfer
  - elastic response due to transient loads in prestressed components

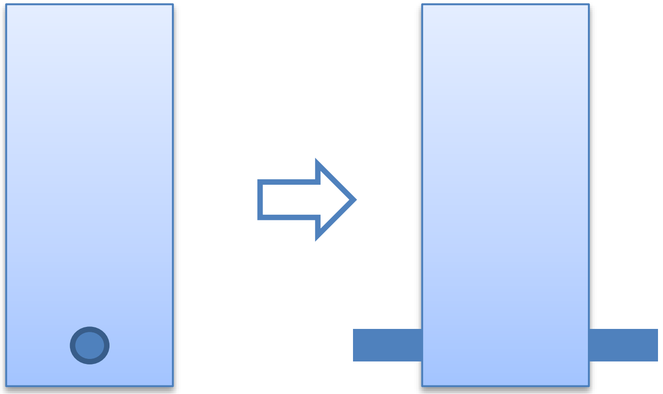
  transformed area properties → equivalent concrete area

$$A_{concrete,equivalent} = n \cdot A_{CFRP} \qquad n = \frac{E_f}{E_c}$$

- Where transformed section analysis is used to assess
  - time-dependent response to permanent load

$$A_{concrete,equivalent} = n \cdot A_{CFRP} \qquad n = \frac{E_f}{\boxed{E_c}}$$

Consider creep of concrete



equivalent concrete area

[AASHTO CFRP-1 Specifications 1.7.1]

FLEXURAL DESIGN

Assumptions for Design: Strength Limit State

- A rectangular stress block is used to model concrete behavior in compression zone

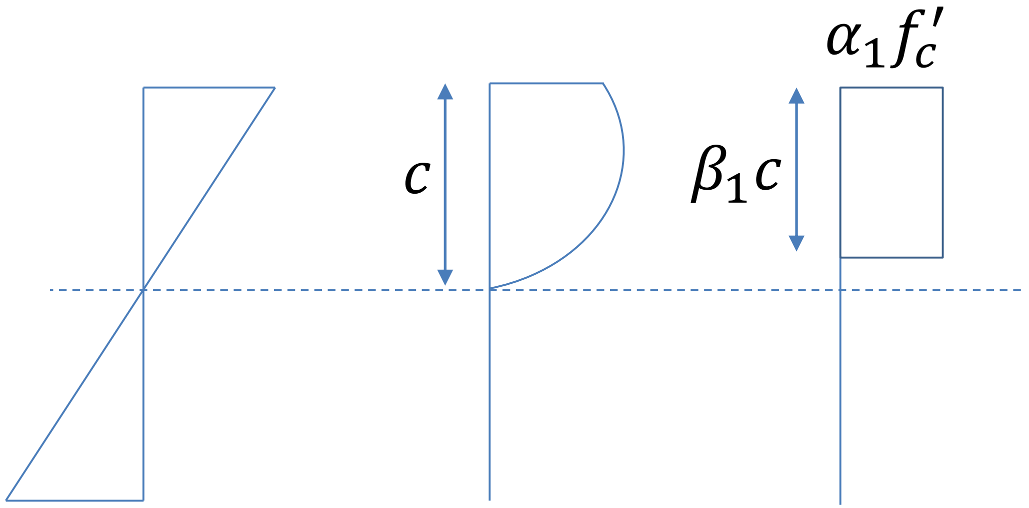
- Prestressing CFRP failure is defined to occur when

$$\varepsilon_p = \varepsilon_{pu}$$

- Balance strain condition

$$\varepsilon_p = \varepsilon_{pu} \text{ and } \varepsilon_{cc} = \varepsilon_{cu} = 0.003$$

- CFRP compression reinforcement shall be ignored in design for increasing capacity



[AASHTO CFRP-1 Specifications 1.7.2]

FLEXURAL DESIGN

Stress Limits for Concrete

- ➢ Initial Stresses, at release
- Compressive stress of concrete
- Tensile stress of concrete

- ➢ Final Stresses, at service
- Compressive stress of concrete
- Tensile stress of concrete

[AASHTO LRFD-8 Specifications 5.9.2.3]

FLEXURAL DESIGN

Compressive Stress Limits for Concrete, at Release

Compressive stress of concrete for temporary stress before losses

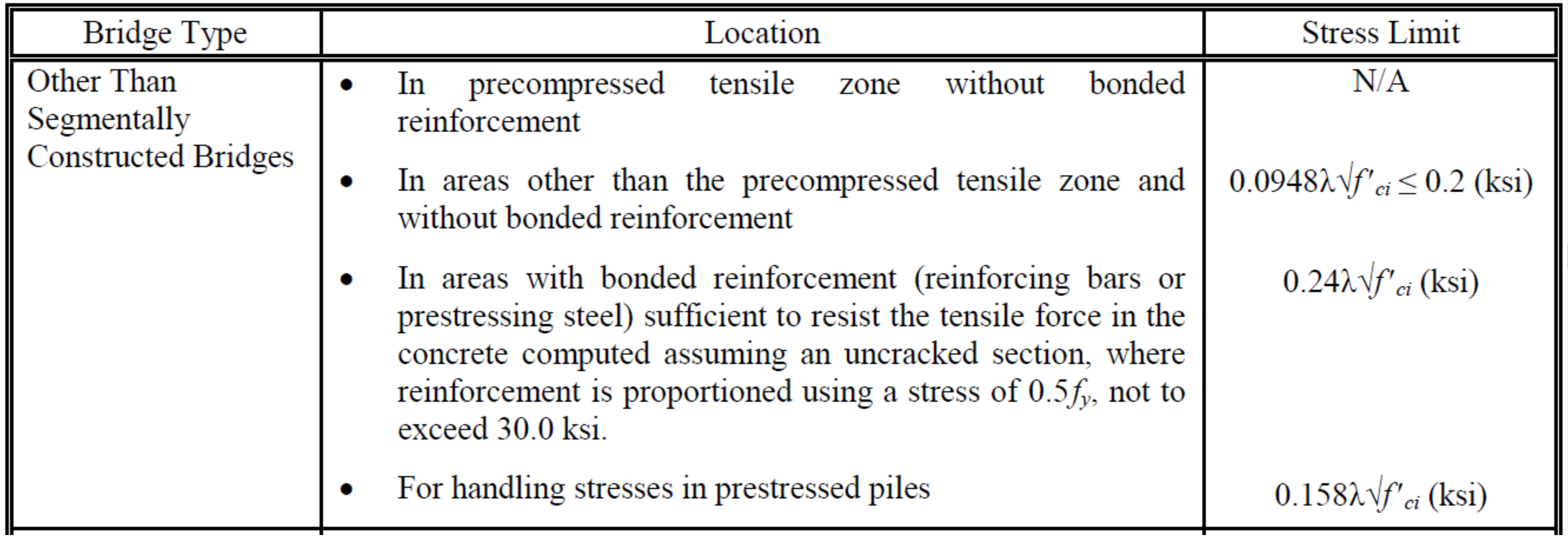
$$f_c \leq 0.65 f'_{ci}$$

[AASHTO LRFD-8 Specifications 5.9.2.3]

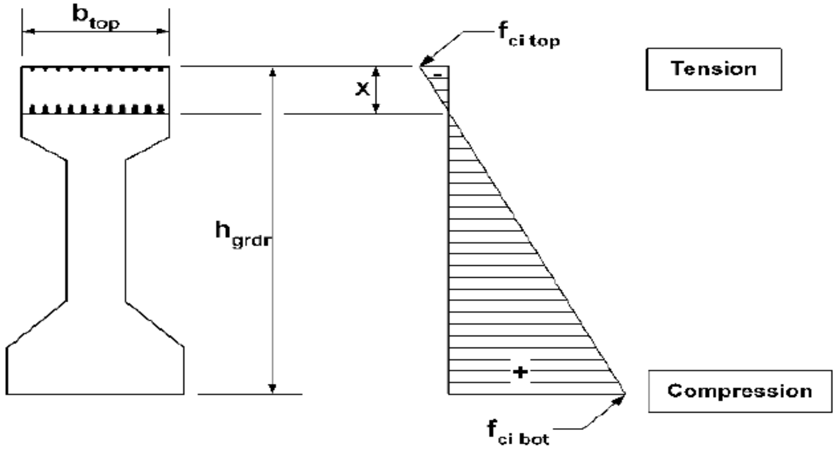
FLEXURAL DESIGN

Tensile Stress Limits for Concrete, at Release

Table 5.9.2.3.1b-1—Temporary Tensile Stress Limits in Prestressed Concrete before Losses



Bridge Type	Location	Stress Limit
Other Than Segmentally Constructed Bridges	• In precompressed tensile zone without bonded reinforcement	N/A
	• In areas other than the precompressed tensile zone and without bonded reinforcement	$0.0948\lambda\sqrt{f'_{ci}} \leq 0.2$ (ksi)
	• In areas with bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of $0.5f_y$, not to exceed 30.0 ksi.	$0.24\lambda\sqrt{f'_{ci}}$ (ksi)
	• For handling stresses in prestressed piles	$0.158\lambda\sqrt{f'_{ci}}$ (ksi)



$$T = \frac{f_{ci\ top}}{2} b_{top} x$$

$$A_s = \frac{T}{f_s}$$

where $f_s = 0.5 f_y \leq 30.0$ ksi

Figure C5.9.2.3.1b-1—Calculation of Tensile Force and Required Area of Reinforcement

$$0.0948\lambda\sqrt{f'_{ci}} \leq f_t \leq 0.2$$

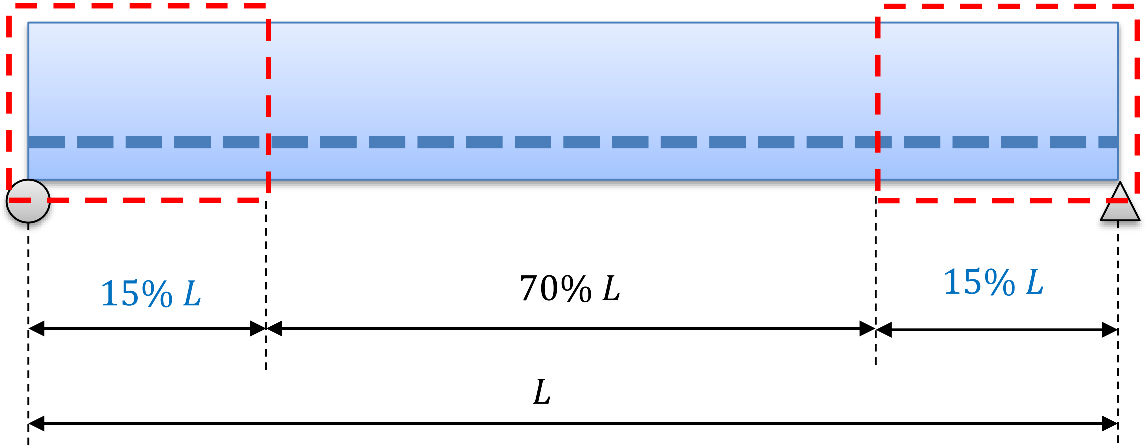
$$f_t \leq 0.24\lambda\sqrt{f'_{ci}}$$

[AASHTO LRFD-8 Specifications 5.9.2.3]

FLEXURAL DESIGN

Tensile Stress Limit For Concrete *[FDOT Modification]*

"…For the outer 15 percent of the design span of straight longitudinal beams, tensile stress at the top of beam at release may be taken as $0.24\lambda\sqrt{f'_{ci}}$ when the lesser of ***LRFD*** [C5.9.2.3.1b] or ***SDG*** Table 4.3.1-1 minimum tension reinforcement is developed in the section"



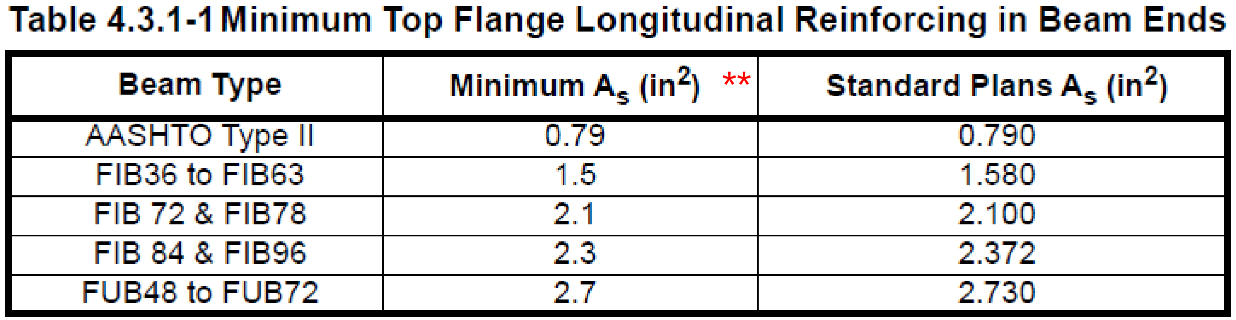


Table 4.3.1-1 Minimum Top Flange Longitudinal Reinforcing in Beam Ends

Beam Type	Minimum A_s (in^2) **	Standard Plans A_s (in^2)
AASHTO Type II	0.79	0.790
FIB36 to FIB63	1.5	1.580
FIB 72 & FIB78	2.1	2.100
FIB 84 & FIB96	2.3	2.372
FUB48 to FUB72	2.7	2.730

*** The minimum areas (A_s) in **SDG** Table 4.3.1-1, are based on the 30 ksi stress limit in **LRFD** [C5.9.2.3.1b] and some refined analysis with RESPONSE 2000. FRP longitudinal reinforcing may result in larger crack openings at the top flange surface due to lower stiffness.*

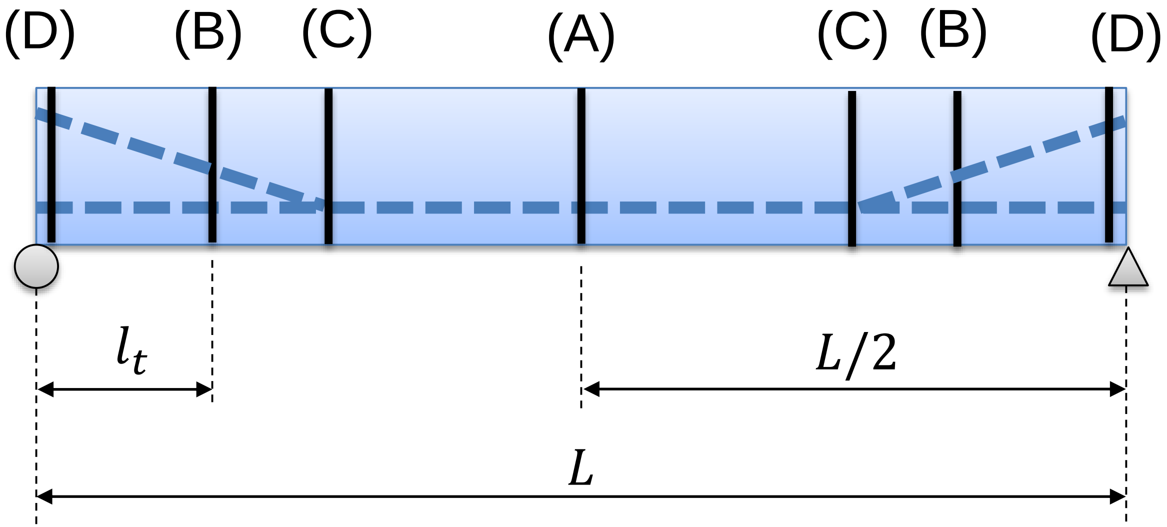
- Outer 15% regions, limitation of tensile stress at the top of beam according to FDOT ***SDG*** 4.3.1
- Middle 70% regions, limitation of tensile stress at the top of beam according to AASHTO ***LRFD*** 5.9.2.3

[FDOT SDG 4.3.1.C.3]

FLEXURAL DESIGN

Stress Limits for Concrete, at Release

e.g.: critical sections



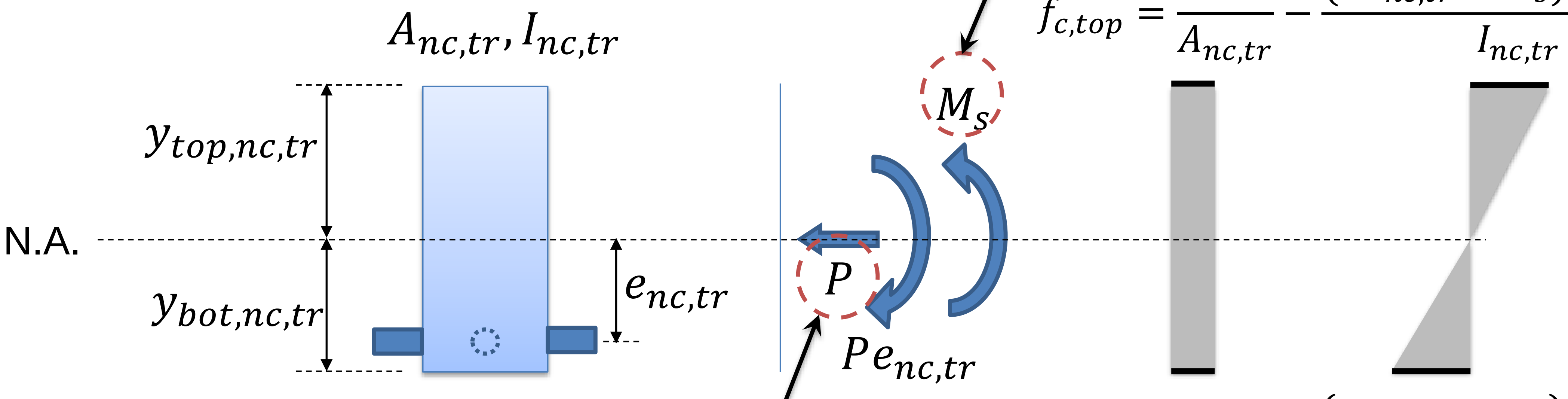
(A) Mid-span section
(B) At end of debond sections
(C) At the harp point section
(D) At support (transfer length) section

FLEXURAL DESIGN

e.g.: f_c at top and bottom fiber

(compression → positive)

Moment due to self-weight of beam



$$f_{c,top} = \frac{P}{A_{nc,tr}} - \frac{(Pe_{nc,tr} - M_s)y_{top,nc,tr}}{I_{nc,tr}}$$

$$f_{c,bot} = \frac{P}{A_{nc,tr}} + \frac{(Pe_{nc,tr} - M_s)y_{bot,nc,tr}}{I_{nc,tr}}$$

Transformed
non-composite
section

Prestressing force at release

$$P = A_{pf}f_{pj} = (n_{CFRP}A_{pf,each})f_{pj}$$

FLEXURAL DESIGN

Stress Limits for Concrete, at Release

Strands are partially debonded to satisfy the requirements for concrete stress at the time of release

- Symmetric
- Overall: Partially debonded strands $\leqslant$ 25% Total strands (30% in FDOT ***SDG*** 4.3.1); (*Note: AASHTO **LRFD 9th Ed**. relaxed the debonding requirements to a maximum of 45% under certain limitations. FDOT is implementing these requirements with **2021 Structures Manual**)*
- In each horizontal row: Partially debonded strands $\leqslant$ 40% Total strands
- Not more than 40% of the debonded strands, or 4 strands, whichever is greater, shall have the debonding terminated at any section

[AASHTO LRFD-8 Specifications 5.9.4.3.3]

FLEXURAL DESIGN

Compressive Stress Limits for Concrete, at Service

- Service limit state load combination: ***Service I***
- Check stresses in precast beam and deck slab

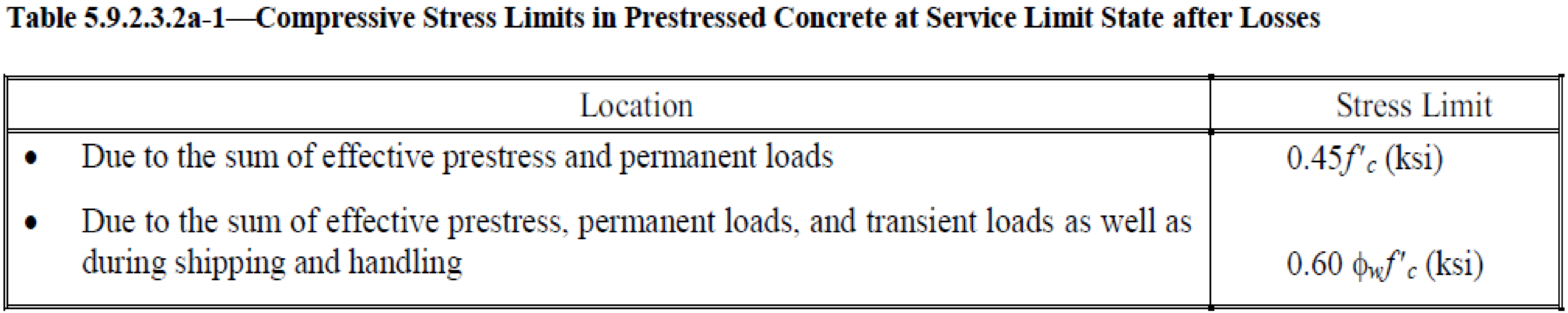


Table 5.9.2.3.2a-1—Compressive Stress Limits in Prestressed Concrete at Service Limit State after Losses

Location	Stress Limit
• Due to the sum of effective prestress and permanent loads	$0.45f'_c$ (ksi)
• Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling	$0.60\ \phi_w f'_c$ (ksi)

[AASHTO LRFD-8 Specifications 5.9.2.3.2]

FLEXURAL DESIGN

Tensile Stress Limits for Concrete, at Service

- Service limit state load combination: ***Service III***

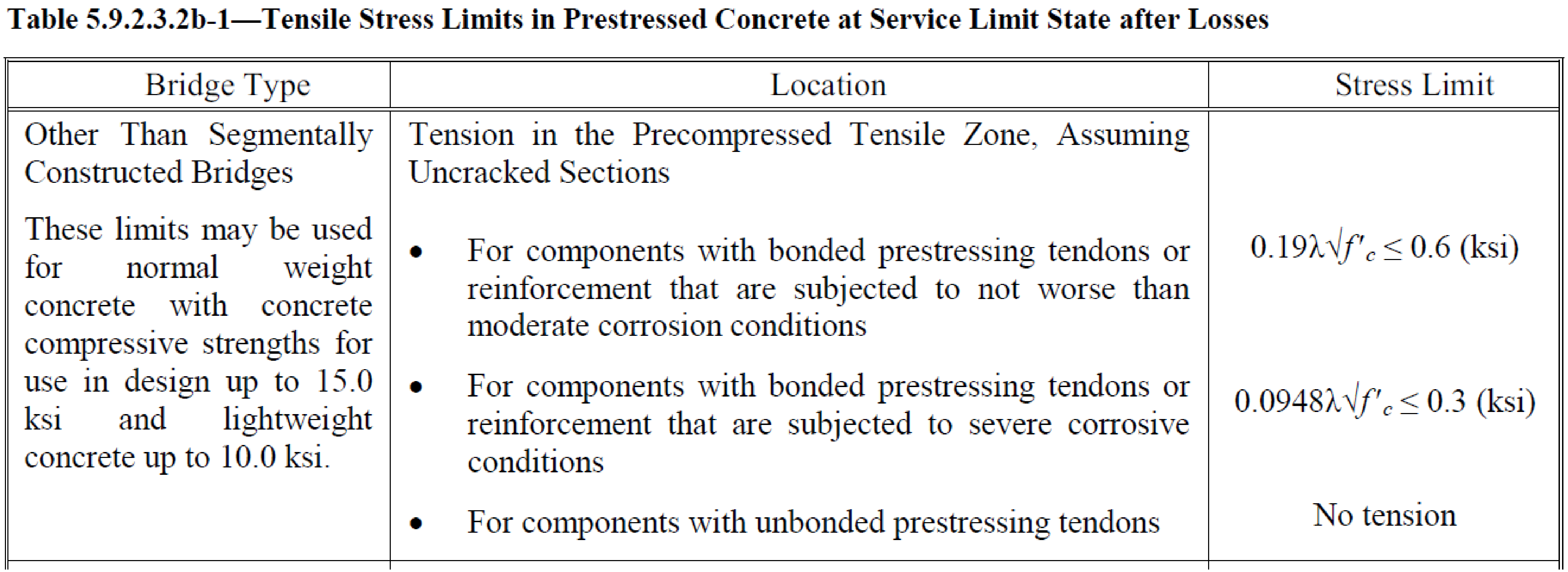


Table 5.9.2.3.2b-1—Tensile Stress Limits in Prestressed Concrete at Service Limit State after Losses

Bridge Type	Location	Stress Limit
Other Than Segmentally Constructed Bridges These limits may be used for normal weight concrete with concrete compressive strengths for use in design up to 15.0 ksi and lightweight concrete up to 10.0 ksi.	Tension in the Precompressed Tensile Zone, Assuming Uncracked Sections	
	• For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions	$0.19\lambda\sqrt{f'_c} \leq 0.6$ (ksi)
	• For components with bonded prestressing tendons or reinforcement that are subjected to severe corrosive conditions	$0.0948\lambda\sqrt{f'_c} \leq 0.3$ (ksi)
	• For components with unbonded prestressing tendons	No tension

[*AASHTO LRFD-8 Specifications* 5.9.2.3.2]

FLEXURAL DESIGN

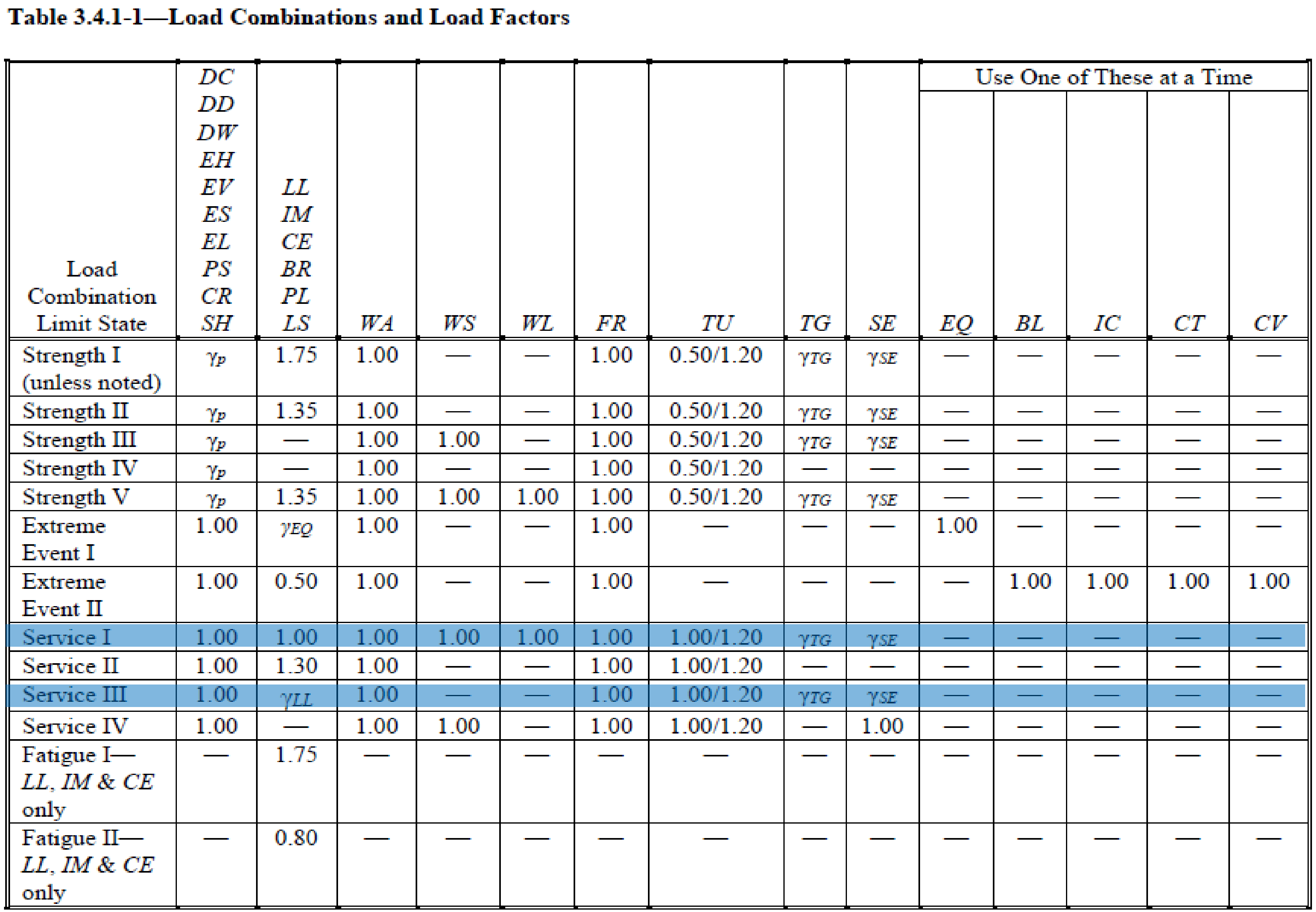


Table 3.4.1-1—Load Combinations and Load Factors

Load Combination Limit State	*DC* *DD* *DW* *EH* *EV* *ES* *EL* *PS* *CR* *SH*	*LL* *IM* *CE* *BR* *PL* *LS*	*WA*	*WS*	*WL*	*FR*	*TU*	*TG*	*SE*	Use One of These at a Time				
										EQ	*BL*	*IC*	*CT*	*CV*
Strength I (unless noted)	γ_p	1.75	1.00	—	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Strength II	γ_p	1.35	1.00	—	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Strength III	γ_p	—	1.00	1.00	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Strength IV	γ_p	—	1.00	—	—	1.00	0.50/1.20	—	—	—	—	—	—	—
Strength V	γ_p	1.35	1.00	1.00	1.00	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Extreme Event I	1.00	γ_{EQ}	1.00	—	—	1.00	—	—	—	1.00	—	—	—	—
Extreme Event II	1.00	0.50	1.00	—	—	1.00	—	—	—	—	1.00	1.00	1.00	1.00
Service I	1.00	1.00	1.00	1.00	1.00	1.00	1.00/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Service II	1.00	1.30	1.00	—	—	1.00	1.00/1.20	—	—	—	—	—	—	—
Service III	1.00	γ_{LL}	1.00	—	—	1.00	1.00/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Service IV	1.00	—	1.00	1.00	—	1.00	1.00/1.20	—	1.00	—	—	—	—	—
Fatigue I—*LL*, *IM* & *CE* only	—	1.75	—	—	—	—	—	—	—	—	—	—	—	—
Fatigue II—*LL*, *IM* & *CE* only	—	0.80	—	—	—	—	—	—	—	—	—	—	—	—

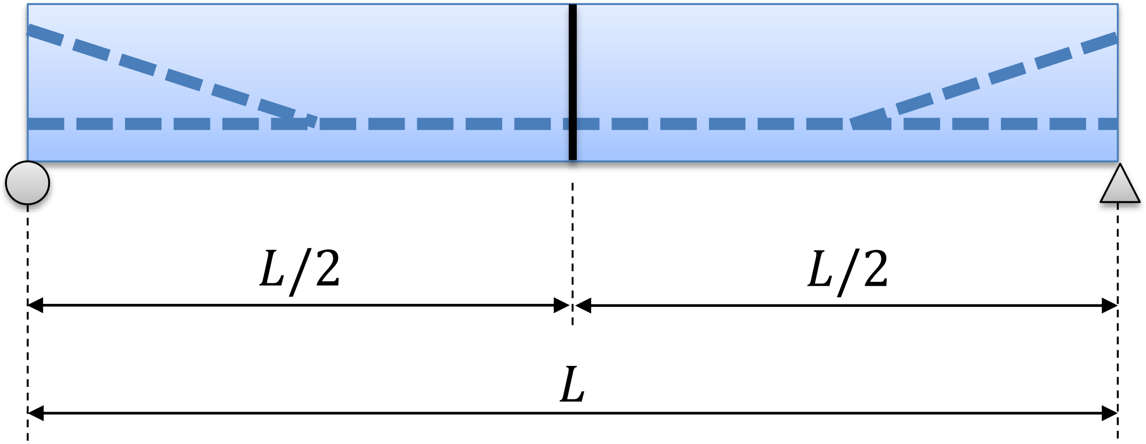
[AASHTO LRFD-8 Specifications 3.4.1]

FLEXURAL DESIGN

Stress Limits for Concrete, at Service

e.g.: critical sections

Mid-span section



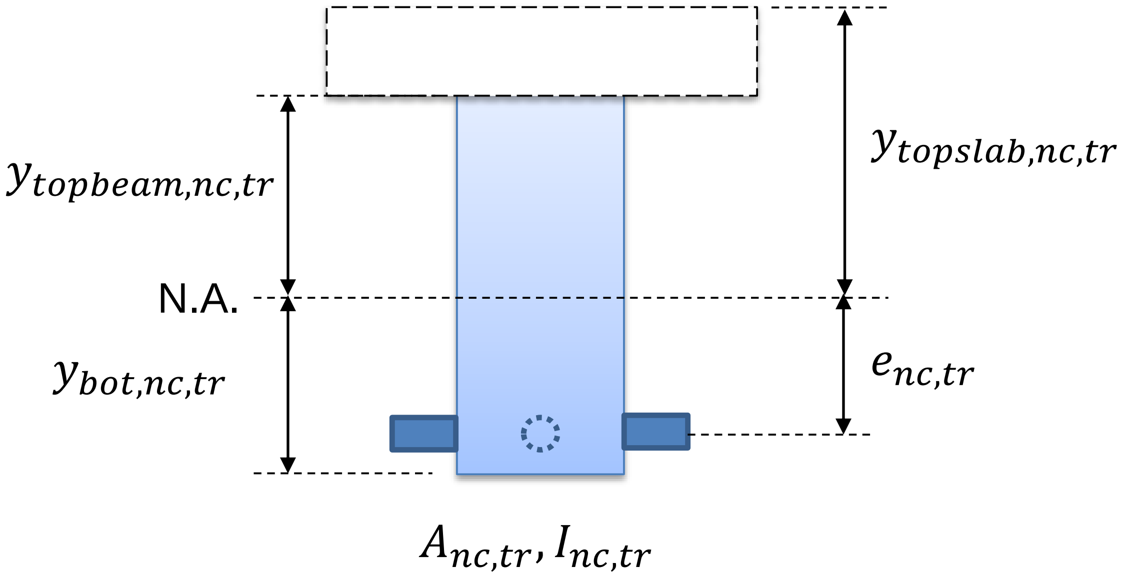
- Service I, under effective prestress and permanent loads
- ✓ Top beam (C)
- ✓ Top deck (C)

- Service I, under effective prestress, permanent loads, and transient loads
- ✓ Top beam (C)
- ✓ Top deck (C)

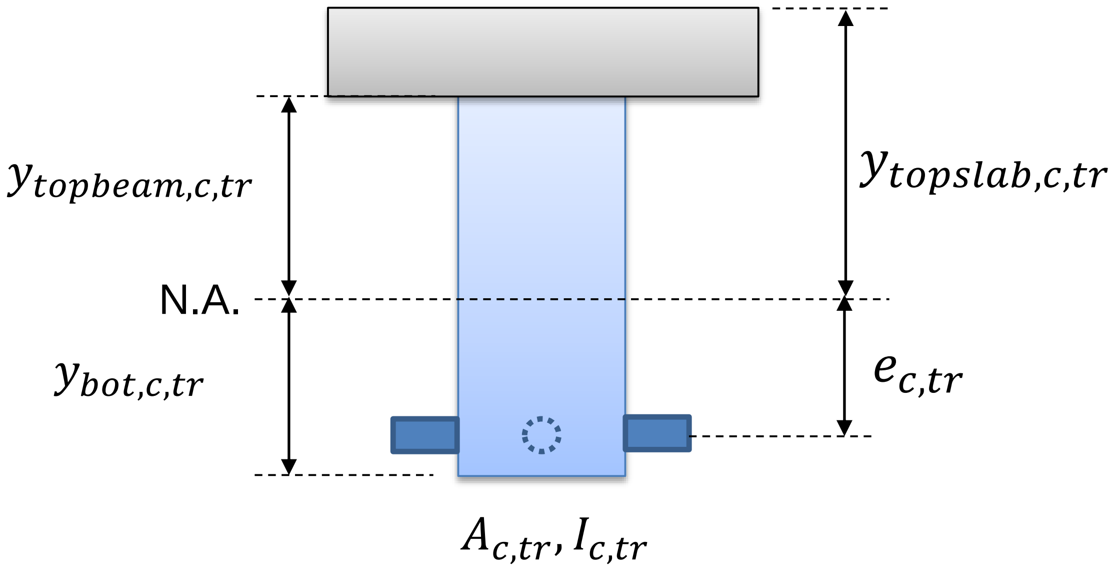
- Service III
- ✓ Bottom beam (T)

FLEXURAL DESIGN

e.g.: Transformed non-composite section and composite section



Transformed non-composite section



Transformed composite section

FLEXURAL DESIGN

e.g.

- Service I, under effective prestress and permanent loads

✓ Stress in the top slab, $f_{top,slab}$(C)

✓ Stress in the top beam $f_{top,beam}$ (C)

$$f_{top,slab}^{1} = \frac{M_{permanet,2} \cdot y_{topslab,c,tr}}{I_{c,tr}}$$

$$f_{top,beam}^{1} = \underbrace{\left(\frac{P}{A_{nc,tr}} - \frac{Pe_{nc,tr} \cdot y_{topbeam,nc,tr}}{I_{nc,tr}}\right)}_{\substack{\text{due to}\\ \text{prestress force}\\ \text{after all losses}}} + \underbrace{\frac{M_{permanet,1} \cdot y_{topbeam,nc,tr}}{I_{nc,tr}}}_{\substack{\text{due to}\\ \text{permanent loads 1}\\ \text{(e.g. beam self-weight,}\\ \text{deck self-weight,}\\ \text{permanent forms)}}} + \underbrace{\frac{M_{permanet,2} \cdot y_{topbeam,c,tr}}{I_{c,tr}}}_{\substack{\text{due to}\\ \text{permanent loads 2}\\ \text{(e.g. barriers, wearing}\\ \text{surface, utility)}}}$$

FLEXURAL DESIGN

e.g.

- Service I, under effective prestress, permanent loads, and transient loads
- ✓ Stress in the top slab, $f_{top,slab}$(C)
- ✓ Stress in the top beam $f_{top,beam}$ (C)

$$f_{top,slab}^{2} = f_{top,slab}^{1} + \frac{M_{transient} \cdot y_{topslab,c,tr}}{I_{c,tr}}$$

$$f_{top,beam}^{2} = f_{top,beam}^{1} + \underbrace{\frac{M_{transient} \cdot y_{topbeam,c,tr}}{I_{c,tr}}}_{\text{due to transient loads (e.g. truck load, lane live load)}}$$

FLEXURAL DESIGN

e.g.

- Service III

✓ Bottom beam (T)

$$f_{bot,beam}^{1} = \left(\overbrace{\frac{P}{A_{nc,tr}} + \frac{Pe_{nc,tr}\cdot y_{botbeam,nc,tr}}{I_{nc,tr}}}^{\text{due to prestress force after all losses}}\right) - \overbrace{\frac{M_{permanet,1}\cdot y_{botbeam,nc,tr}}{I_{nc,tr}}}^{\text{due to permanent loads 1 (e.g. beam self-weight, deck self-weight, permanent forms)}}$$

$$- \underbrace{\frac{M_{permanet,2}\cdot y_{botbeam,c,tr}}{I_{c,tr}}}_{\text{due to permanent loads 2 (e.g. barriers, wearing surface, utility)}} - \underbrace{\frac{\gamma_{LL}M_{transient}\cdot y_{botbeam,c,tr}}{I_{c,tr}}}_{\text{due to Live loads (e.g. truck load, lane live loads)}}$$

FLEXURAL DESIGN

Fatigue Limit State

- In regions of compressive stress due to unfactored loads and prestress in reinforced concrete components, fatigue shall be considered only if

Compressive stress due to
unfactored permanent loads and prestress

$$f_c < f_{t.max}$$

Maximum tensile stress
due to Fatigue I load combination

[AASHTO CFRP-1 Specifications 1.5.2]

FLEXURAL DESIGN

Fatigue Limit State

- Fatigue of reinforcement need not to be check for prestressed components designed to have

Extreme fiber tensile stress due to Service III Limit State

$$f_{t.max} \leq f_{t.allow}$$

Tensile stress limit specified in ***LRFD*** Table 5.9.2.3.2b-1 (Tensile Stress Limits for Concrete, at Service III Limit State)

[AASHTO CFRP-1 Specifications 1.5.2]

FLEXURAL DESIGN

Fatigue Limit State

- For fatigue considerations, concrete members shall satisfy:

Load factor specified in ***LRFD*** Table 3.4.1-1 for Fatigue I load combination

Force effect, live load stress range due to the passage of the fatigue load as specified in ***LRFD*** 3.6.1.4

$$\gamma(\Delta f) \leq (\Delta f)_{TH}$$

Constant-amplitude fatigue threshold for prestressing CFRP

- 9 ksi, for radii of curvature > 30.0 ft
- 5 ksi, for radii of curvature ⩽ 12.0 ft
- Linear interpolation for radii in between

[AASHTO CFRP-1 Specifications 1.5.2]

FLEXURAL DESIGN

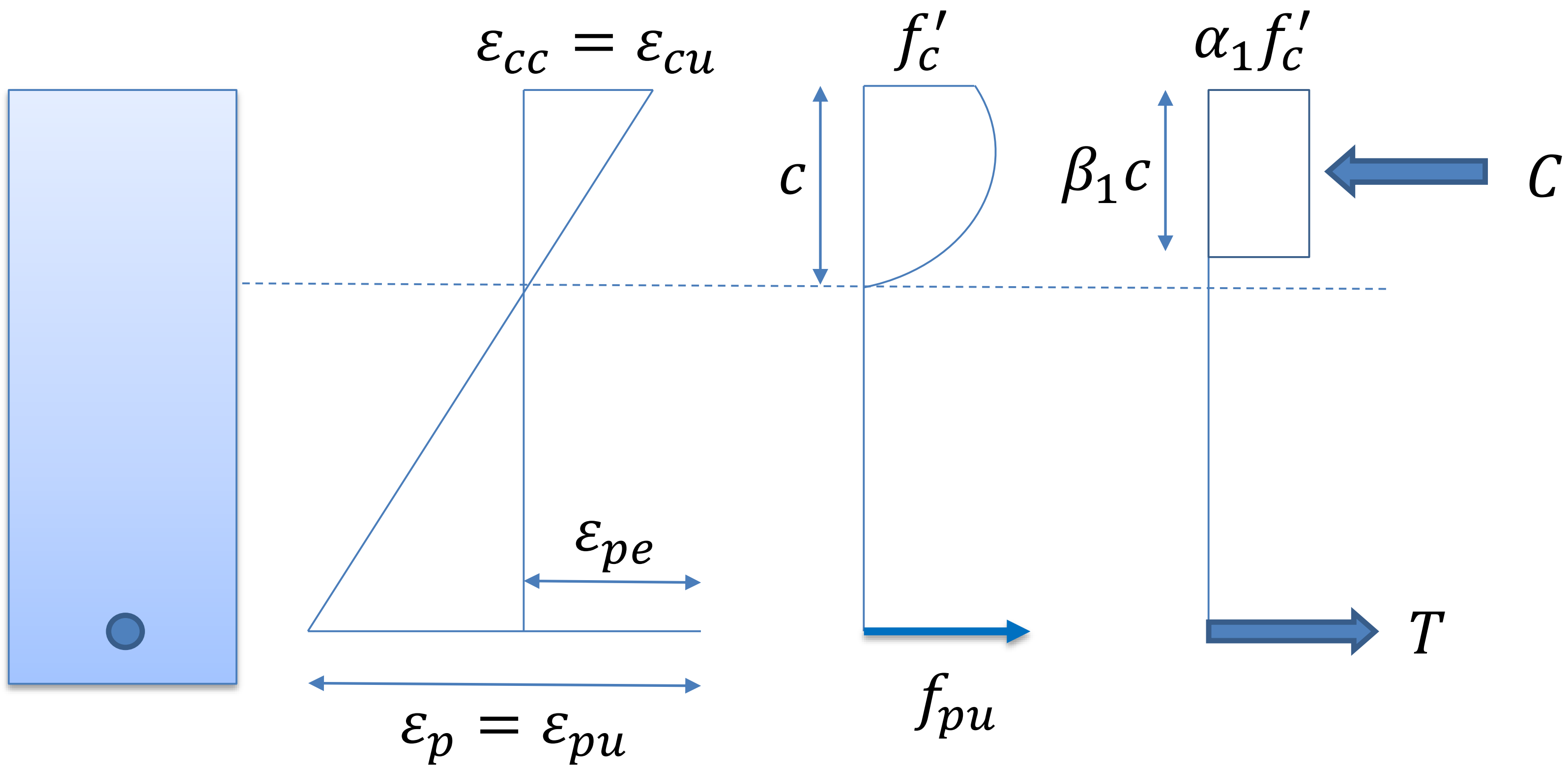
Strength Limit State

- Balanced Prestressed Reinforcement Ratio
- Tension-Controlled Section
- Compression-Controlled Section
- Flexural Resistance

[AASHTO LRFD-8 Specifications 5.9.2.3]

FLEXURAL DESIGN

Balanced Prestressed Reinforcement Ratio



$\rho > \rho_b$

Concrete crushing
Compression controlled

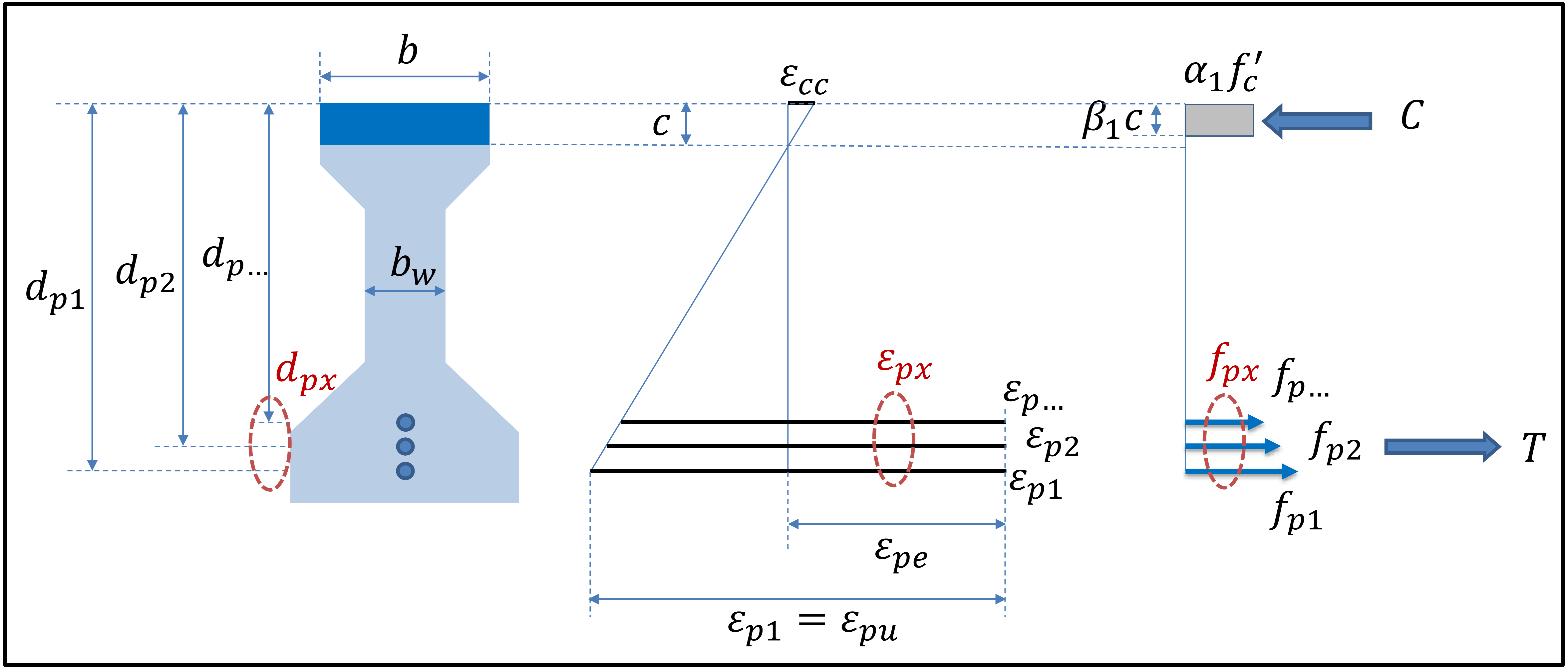
$\rho < \rho_b$

CFRP rupture
Tension controlled

$$\rho_b = \alpha_1 \beta_1 \frac{f_c'}{f_{pu}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{pu} - \varepsilon_{pe}}$$

FLEXURAL DESIGN

Tension-Controlled Section



[NCHRP 12-97]

FLEXURAL DESIGN

Tension-Controlled Section

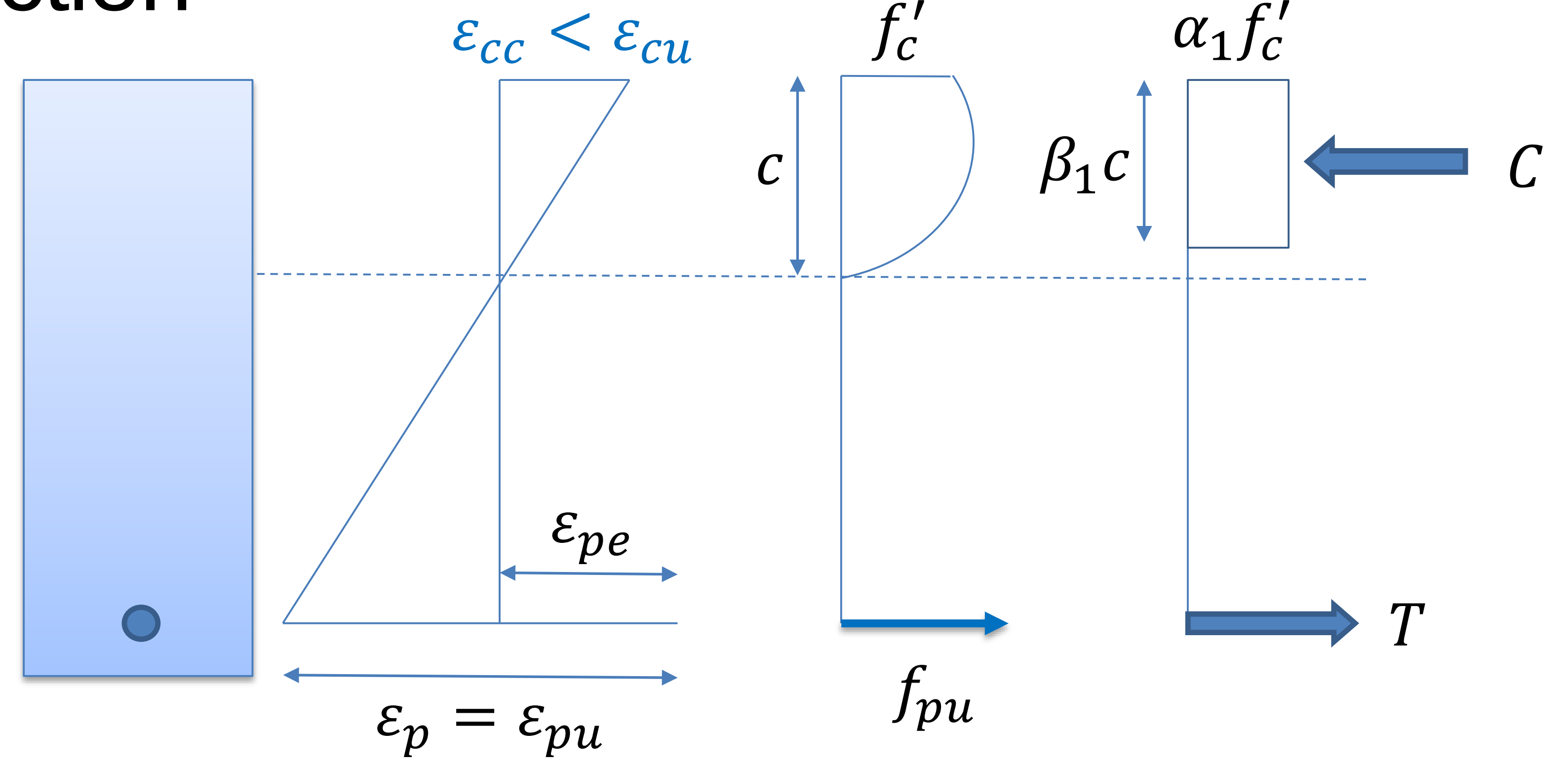
$$\varepsilon_p = \varepsilon_{pu} \qquad \varepsilon_{cc} < \varepsilon_{cu}$$

Concrete stress block factors

$$\alpha_1 = \left(\frac{1}{\beta_1}\right)\left(\frac{\varepsilon_{cc}}{\varepsilon_c'} - \frac{1}{3}\left(\frac{\varepsilon_{cc}}{\varepsilon_c'}\right)^2\right)\left(1 - \frac{f_c'}{60}\right)$$

$$\beta_1 = \frac{4 - \frac{\varepsilon_{cc}}{\varepsilon_c'}}{6 - 2\frac{\varepsilon_{cc}}{\varepsilon_c'}}\left(1.1 - \frac{f_c'}{50}\right) \geq 0.65$$

$$\varepsilon_c' = \left(1.6 + \frac{f_c'}{11}\right) \times 10^{-3}$$



← *Proposed by NCHRP Report 907*

Or can be estimated using AASHTO-LRFD approach
(The deviation is within 5 % with proposed value being conservative)

[AASHTO CFRP-1 Specifications C1.7.2.1]

FLEXURAL DESIGN

Tension-Controlled Section

Concrete stress block factors

For $f_c' \leq 10\ ksi \quad \alpha_1 = 0.85$

For $f_c' > 10\ ksi$

$$\alpha_1 = 0.85 - 0.02(f_c' - 10) \geq 0.75$$

For $f_c' \leq 4\ ksi \quad \beta_1 = 0.85$

For $f_c' > 4\ ksi$

$$\beta_1 = 0.85 - 0.05(f_c' - 4) \geq 0.65$$

Estimated using AASHTO-LRFD approach

OR

$$\alpha_1 = \left(\frac{1}{\beta_1}\right)\left(\frac{\varepsilon_{cc}}{\varepsilon_c'} - \frac{1}{3}\left(\frac{\varepsilon_{cc}}{\varepsilon_c'}\right)^2\right)\left(1 - \frac{f_c'}{60}\right)$$

$$\beta_1 = \frac{4 - \frac{\varepsilon_{cc}}{\varepsilon_c'}}{6 - 2\frac{\varepsilon_{cc}}{\varepsilon_c'}}\left(1.1 - \frac{f_c'}{50}\right) \geq 0.65$$

$$\varepsilon_c' = \left(1.6 + \frac{f_c'}{11}\right) \times 10^{-3}$$

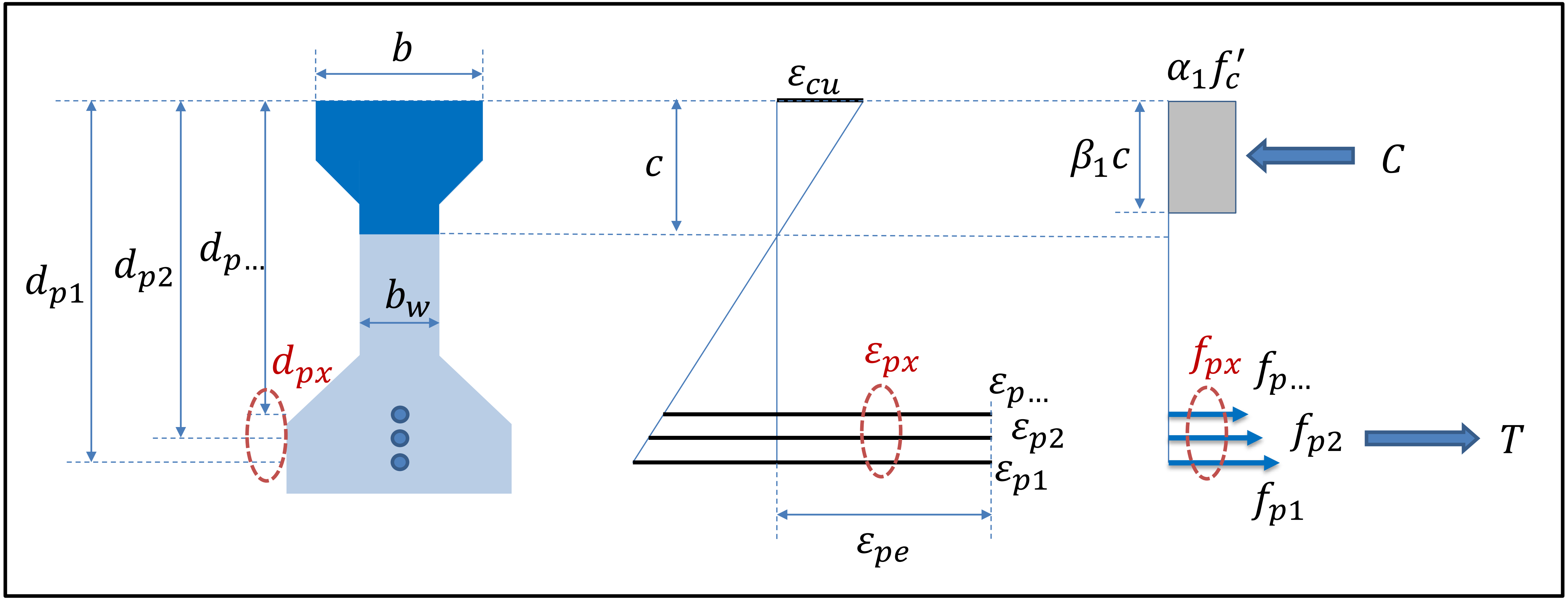
Proposed by NCHRP Report 907

(The deviation is within 5 % with proposed value being conservative)

[AASHTO CFRP-1 Specifications C1.7.2.1]   *[AASHTO LRFD-8 5.6.2.2]*

FLEXURAL DESIGN

Compression-Controlled Section



[NCHRP 12-97]

FLEXURAL DESIGN

Compression-Controlled Section

$$\varepsilon_p < \varepsilon_{pu}$$

$$\varepsilon_{cc} = \varepsilon_{cu}$$

Concrete stress block factors

For $f_c' \leq 10\ ksi$ $\alpha_1 = 0.85$

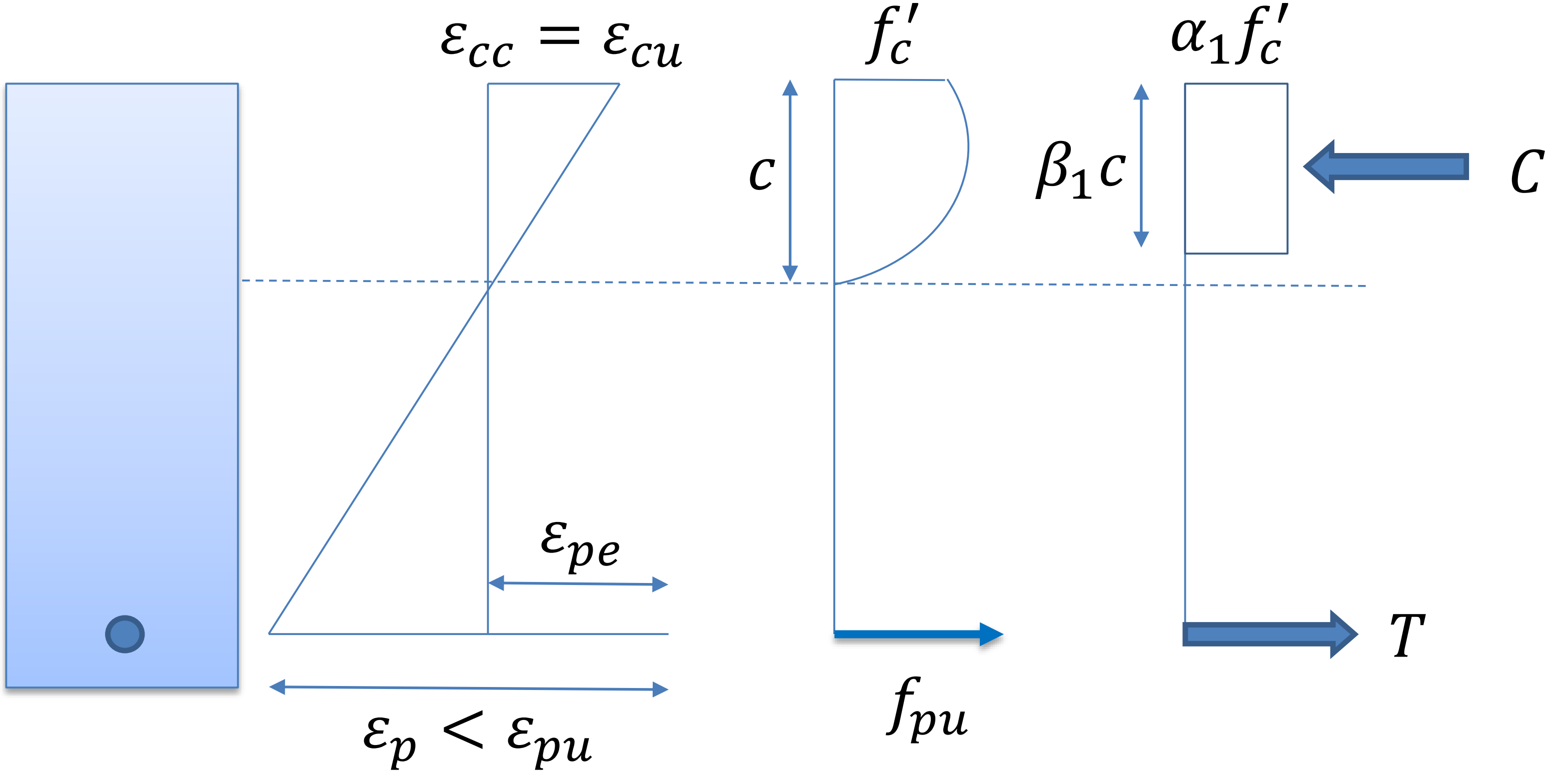
For $f_c' > 10\ ksi$

$$\alpha_1 = 0.85 - 0.02(f_c' - 10) \geq 0.75$$

For $f_c' \leq 4\ ksi$ $\beta_1 = 0.85$

For $f_c' > 4\ ksi$

$$\beta_1 = 0.85 - 0.05(f_c' - 4) \geq 0.65$$



FLEXURAL DESIGN

Stress in Prestressing CFRP at Nominal Flexural Resistance

Components with bonded prestressing CFRP

$$f_{px} = E_f \varepsilon_{px}$$

Iteration to find c

For T-section behavior

$$c = \frac{\sum_{x=1}^{n_p} A_{px} f_{px} - \alpha_1 f_c' (b - b_w) h_f}{\alpha_1 f_c' \beta_1 b_w}$$

For rectangular section behavior

$$c = \frac{\sum_{x=1}^{n_p} A_{px} f_{px}}{\alpha_1 f_c' \beta_1 b}$$

Equations of compatibility

$$\varepsilon_{px} = \varepsilon_{pe} + \varepsilon_{cc} \frac{d_{px} - c}{c}$$

Compression-controlled section:

$$\varepsilon_{cc} = \varepsilon_{cu}$$

Tension-controlled section:

$$\varepsilon_{p1} = \varepsilon_{pu}$$

$$\varepsilon_{cc} = \left(\varepsilon_{pu} - \varepsilon_{pe}\right) \frac{c}{d_{p1} - c}$$

[*AASHTO CFRP-1 Specifications* 1.7.3.1]

FLEXURAL DESIGN

Flexural Resistance

Resistance factor 0.75

$$M_r = \phi M_n$$

Factored Flexural Resistance

Nominal Flexural Resistance

[AASHTO CFRP-1 Specifications 1.7.3.2]

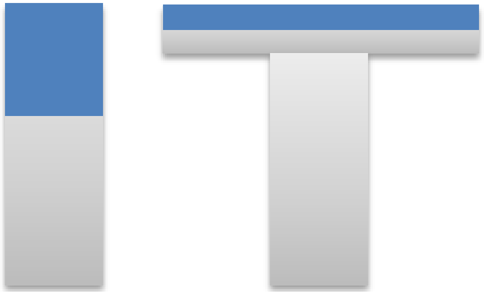
FLEXURAL DESIGN

Nominal Flexural Resistance

For flanged sections where the compression flange depth < $a = \beta_1 c$

$$M_n = \sum_{x=1}^{n_p} A_{px} f_{px} \left(d_{px} - \frac{a}{2} \right) + \alpha_1 f_c' (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2} \right)$$

For rectangular sections where the compression flange of flanged members > $a = \beta_1 c$



$$M_n = \sum_{x=1}^{n_p} A_{px} f_{px} \left(d_{px} - \frac{a}{2} \right) + \alpha_1 f_c' (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2} \right)$$

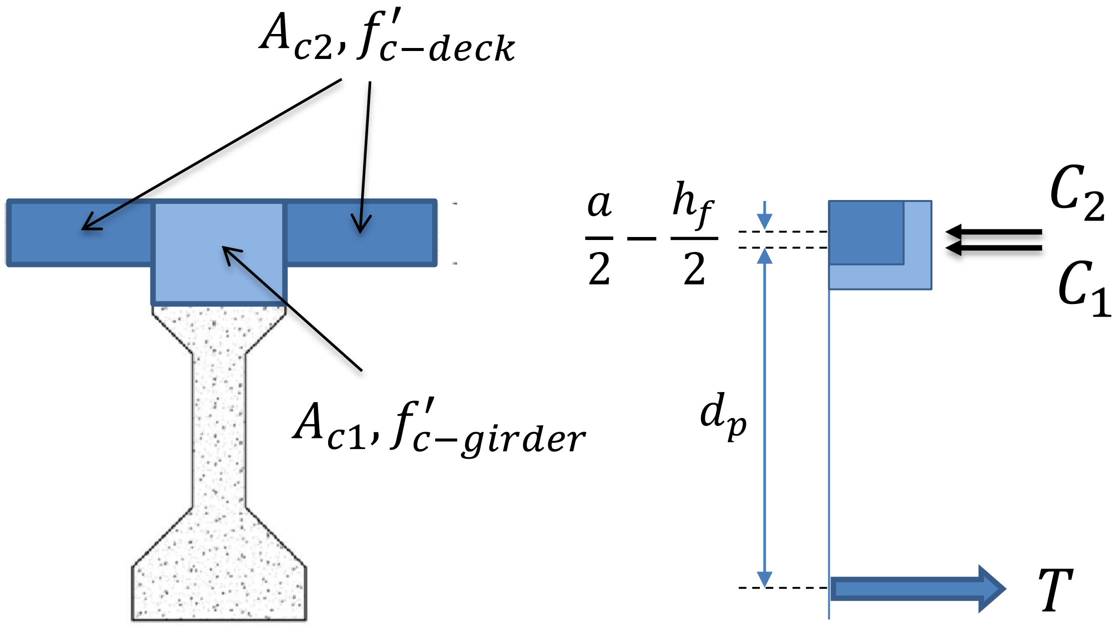
(Replace b_w with b)

[AASHTO CFRP-1 Specifications 1.7.3.2]

FLEXURAL DESIGN

Nominal Flexural Resistance

For composite girder sections in which the neutral axis is located below the deck and within the prestressed high strength concrete girders



$$M_n = \sum_{x=1}^{n_p} A_{px} f_{px} \left(d_{px} - \frac{a}{2}\right) + \alpha_1 f_c' (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

Concrete compressive strength of the deck

[AASHTO CFRP-1 Specifications 1.7.3.2]

FLEXURAL DESIGN

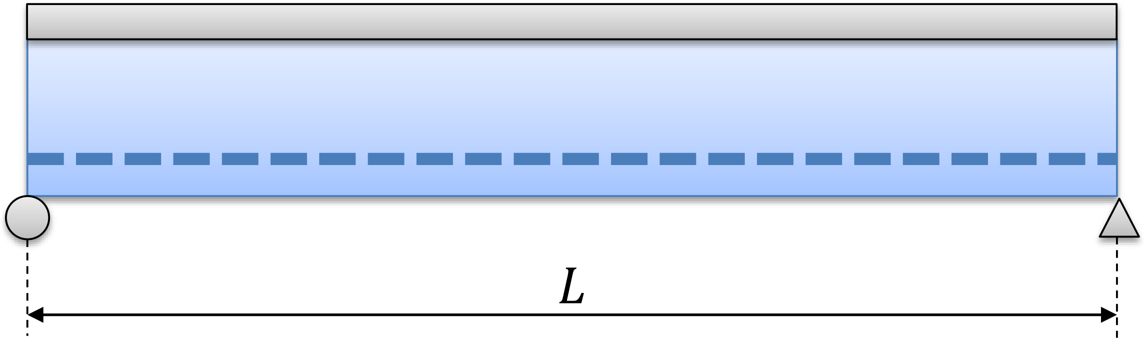
e.g.

- Load combination Strength I

HL-93 Truck load + Lane load

Dynamic Load Allowance (Truck load only)

$$M_u = 1.25DC + 1.5DW + 1.75(LL + IM)$$



Vehicle live load and dynamic allowance, e.g. truck load, lane live load

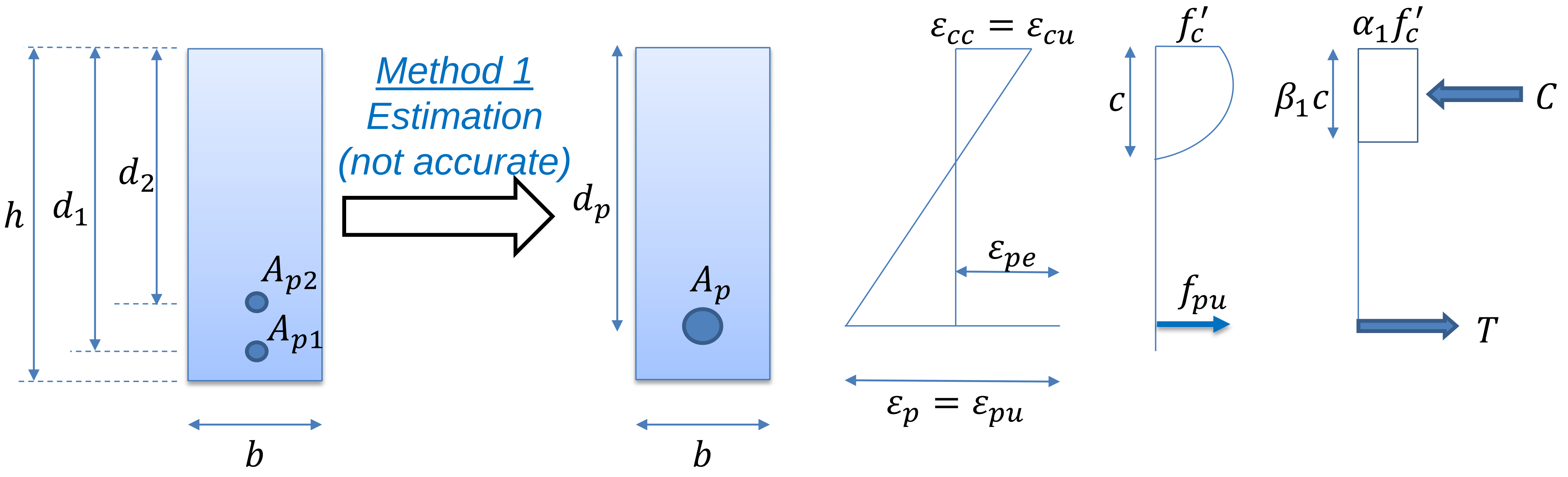
$$M_n \geq \frac{M_u}{\phi}$$

Wearing surfaces and Utilities

Component and attachments, e.g. beam self-weight, deck self-weight, barriers

FLEXURAL DESIGN

e.g. Compression/Tension Controlled?



Method 1

Estimation

(not accurate)

$$\rho_b = \alpha_1 \beta_1 \frac{f_c'}{f_{pu}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{pu} - \varepsilon_{pe}}$$

$$\rho = \frac{A_p}{b d_p}$$

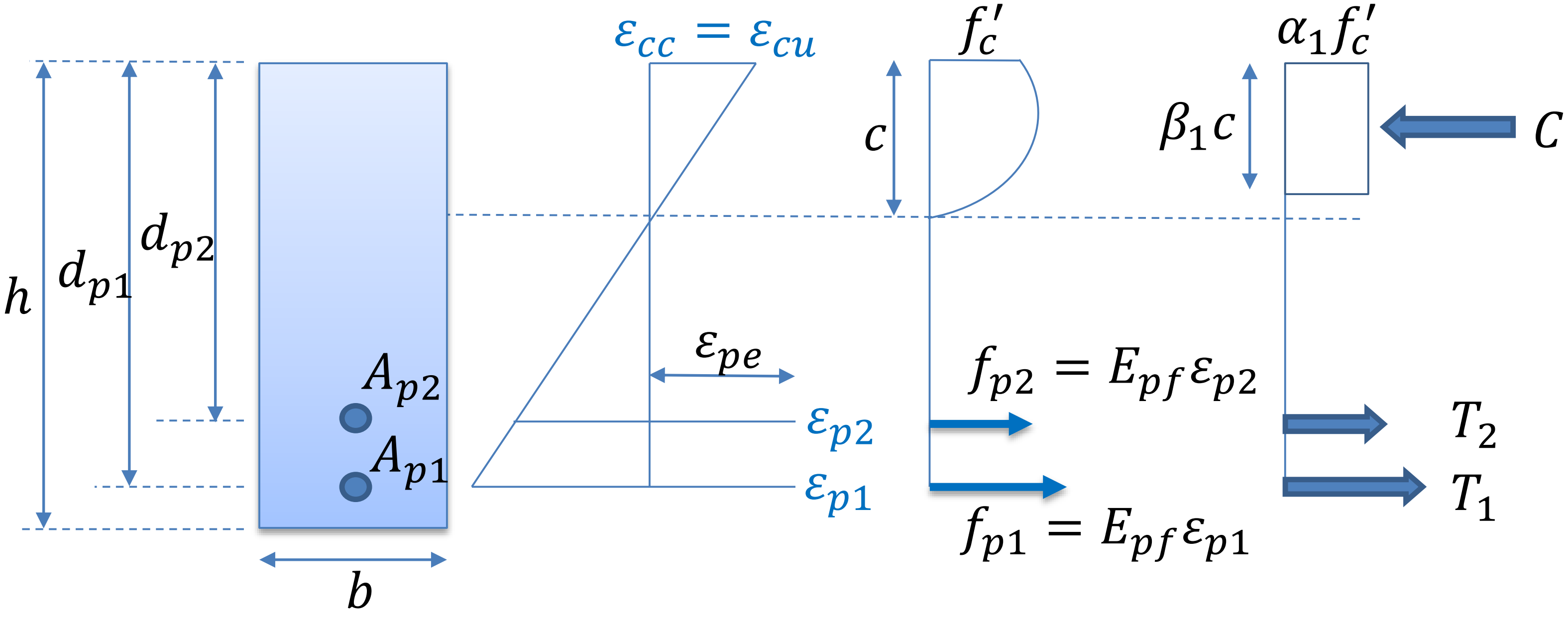
If $\rho > \rho_b$, Compression controlled

If $\rho < \rho_b$, Tension controlled

FLEXURAL DESIGN

e.g. Compression/Tension Controlled?

Method 2 (accurate):
Assume compression controlled, find c, then check if all $f_{pi} < f_{pu}$



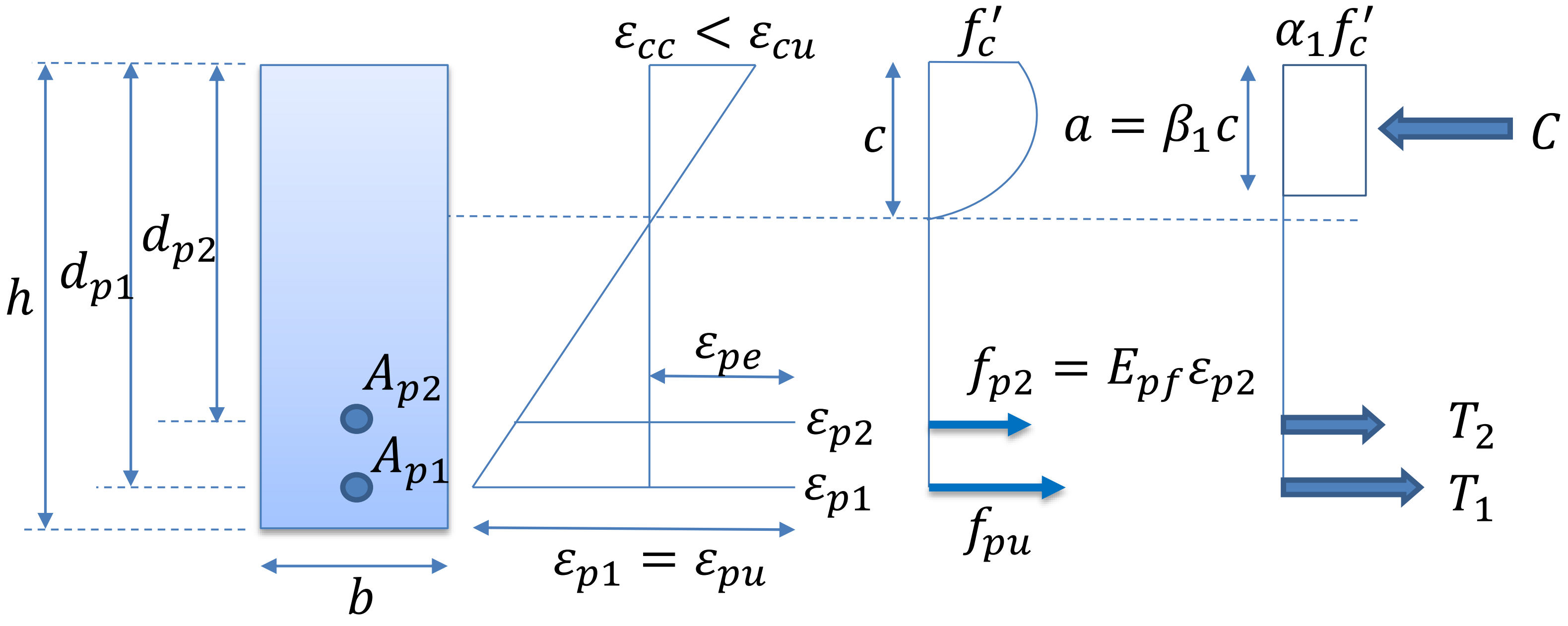
Equilibrium
Compatibility
Stress-strain relationship

$\longrightarrow$ Iteration to find c $\longrightarrow$ Check if $f_{pf} < f_{pu}$

Yes, compression controlled
No, tension controlled

FLEXURAL DESIGN

e.g. Tension-Controlled



$$c = \frac{\sum_{x=1}^{n_p} A_{px} f_{px}}{\alpha_1 f'_c \beta_1 b} = \frac{A_{p1} f_{pu} + A_{p2} f_{p2}}{\alpha_1 f'_c \beta_1 b}$$

Iteration to find c

$$M_n = \sum_{x=1}^{n_p} A_{px} f_{px} \left(d_{px} - \frac{a}{2}\right) = A_{p1} f_{p1} \left(d_{p1} - \frac{a}{2}\right) + A_{p2} f_{p2} \left(d_{p2} - \frac{a}{2}\right)$$

$$\varepsilon_{p1} = \varepsilon_{pu}$$

$$\varepsilon_{p2} = \varepsilon_{pe} + \left(\varepsilon_{pu} - \varepsilon_{pe}\right) \frac{d_{p2} - c}{d_{p1} - c}$$

$$\varepsilon_{cc} = \left(\varepsilon_{pu} - \varepsilon_{pe}\right) \frac{c}{d_{p1} - c}$$

$$\alpha_1 = \left(\frac{1}{\beta_1}\right)\left(\frac{\varepsilon_{cc}}{\varepsilon'_c} - \frac{1}{3}\left(\frac{\varepsilon_{cc}}{\varepsilon'_c}\right)^2\right)\left(1 - \frac{f'_c}{60}\right)$$

$$\beta_1 = \frac{4 - \frac{\varepsilon_{cc}}{\varepsilon'_c}}{6 - 2\frac{\varepsilon_{cc}}{\varepsilon'_c}}\left(1.1 - \frac{f'_c}{50}\right) \geq 0.65$$

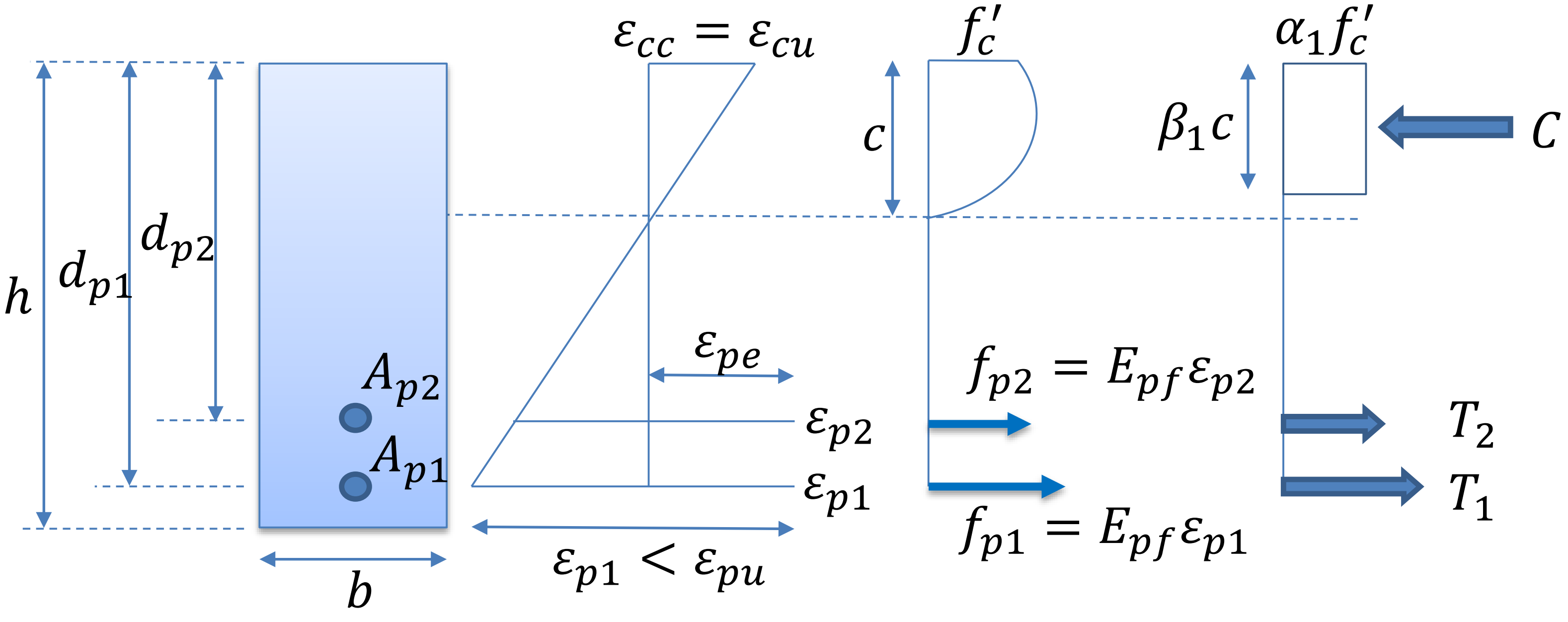
$$\varepsilon'_c = \left(1.6 + \frac{f'_c}{11}\right) \times 10^{-3}$$

Or using AASHTO-LRFD estimation

[AASHTO CFRP-1 Specifications 1.7.2.1]

FLEXURAL DESIGN

e.g. Compression-Controlled



$$\varepsilon_{cc} = \varepsilon_{cu}$$

$$\varepsilon_{p1} = \varepsilon_{pe} + \varepsilon_{cc}\frac{d_{p1} - c}{c}$$

$$\varepsilon_{p2} = \varepsilon_{pe} + \varepsilon_{cc}\frac{d_{p2} - c}{c}$$

For $f_c' \leq 10\ ksi$ $\alpha_1 = 0.85$

For $f_c' > 10\ ksi$

$$\alpha_1 = 0.85 - 0.02(f_c' - 10) \geq 0.75$$

For $f_c' \leq 4\ ksi$ $\beta_1 = 0.85$

For $f_c' > 4\ ksi$

$$\beta_1 = 0.85 - 0.05(f_c' - 4) \geq 0.65$$

$$c = \frac{\sum_{x=1}^{n_p} A_{px} f_{px}}{\alpha_1 f_c' \beta_1 b} = \frac{A_{p1} f_{pu} + A_{p2} f_{p2}}{\alpha_1 f_c' \beta_1 b}$$

Iteration to find c

$$M_n = \sum_{x=1}^{n_p} A_{px} f_{px}\left(d_{px} - \frac{a}{2}\right) = A_{p1} f_{p1}\left(d_{p1} - \frac{a}{2}\right) + A_{p2} f_{p2}\left(d_{p2} - \frac{a}{2}\right)$$

[AASHTO CFRP-1 Specifications 1.7.2.1]

FLEXURAL DESIGN

Minimum Reinforcement - Prestressing CFRP

For tension-controlled flexural members

the amount of prestressing CFRP shall be adequate to develop M_r

$$M_r \geq \min \begin{cases} 1.33\,M_u & \text{(applicable strength load combination)} \\ \text{Cracking moment} & M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\left(\dfrac{S_c}{S_{nc}} - 1\right)\right] \end{cases}$$

[*AASHTO CFRP-1 Specifications 1.7.3.3*]

FLEXURAL DESIGN

Minimum Reinforcement - Prestressing CFRP

Recommendation by NCHRP Report 906 (2019)

$$M_r \geq \min \begin{cases} \text{Cracking moment } M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right] \\ \alpha\, M_u \text{ (applicable strength load combination)} \end{cases}$$

$$1.0 \leq \alpha = 1.0 + \frac{0.33(\varepsilon_t - \varepsilon_{cl})}{(\varepsilon_{tl} - \varepsilon_{cl})} \leq 1.33$$

ε_t Net tensile strain in the extreme tension steel at nominal resistance, per AASHTO ***LRFD*** 5.5.4.2

ε_{cl} Compression-controlled strain limit in the extreme tension steel

ε_{tl} Tension-controlled strain limit in the extreme tension steel

[NCHRP Report 906]

FLEXURAL DESIGN

modulus of rupture of beam concrete $f_r = 0.24\lambda\sqrt{f_c'}$

compressive stress in concrete (bottom) due to effective prestress force (after all losses) only

$$f_{cpe} = \frac{P_e}{A_{nc,tr}} + \frac{P_e e_{nc,tr}}{S_{bot,nc,tr}}$$

total unfactored dead load (e.g. beam and slab self weight, permanent forms) moment acting on the non-composite section

$$M_{cr} = \gamma_3 \left[(\gamma_1 f_r + \gamma_2 f_{cpe}) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right]$$

prestress variability factor
1.1 for bonded prestressing CFRP
1.0 for unbonded prestressing CFRP

section modulus (bottom, tension side) for composite and non-composite (transformed) sections, $S_{bot,c,tr}$, $S_{bot,nc,tr}$

flexural cracking variability factor
1.6 for concrete structures

1.0 for prestressed concrete structures

[AASHTO CFRP-1 Specifications 1.7.3.3]

FLEXURAL DESIGN

Minimum Reinforcement – FRP Reinforcement *(Mainly for Slab)*

Shrinkage and temperature reinforcement (perpendicular to span)

$$\rho_{f,ts} = 0.0018 \times \frac{60{,}000}{f_{fu}} \frac{E_s}{E_f}$$

For shrinkage and temperature FRP reinforcement

- It is recommended $0.0014 \leq \rho_{f,ts} \leq 0.0036$
- Spacing = min (3 times of slab thickness, 12 inches)

[ACI 440.1R-15 9.1]

FLEXURAL DESIGN

Pretensioned Anchorage Zone

- Splitting Resistance

$$P_r = f_s A_s \geq 0.04 \cdot P_i$$

Splitting Resistance

Stress in steel
$\leq$ 20 ksi

For Steel Bar
NOT for FRP

Total prestressing force at transfer

$$P_i = n_p f_{pi} A_{pf}$$

For pretensioned I-girder or bulb tees, A_s is the total area of the vertical reinforcement located with a distance of $h/4$ from the end of the member

[AASHTO LRFD-8 Specifications 5.9.4.4]

FLEXURAL DESIGN

Pretensioned Anchorage Zone

- Splitting Resistance

$$P_r = f_f A_f \geq \boxed{0.04 \cdot P_i}$$

Total prestressing force at transfer

$$P_i = n_p f_{pi} A_{pf}$$

Splitting Resistance

Stress in FRP bar ≤ 20 ksi

Note: In ***LRFD,*** f_s for steel is limited to 20 ksi for crack control. However, for FRP, it may depend on the “effective” stiffness of the anchorage reinforcing

LRFD:

- 4% P_i from the end of the beam to $h/4$

FDOT ***SDG*** 4.3.1.D *(more restrictive)*

- 3% P_j from the end of the beam to $h/8$, but $\geq 10''$
- 5% P_j from the end of the beam to $h/4$, but $\geq 10''$
- 6% P_j from the end of the beam to $3h/8$, but $\geq 10''$

For pretensioned I-girder or bulb tees, A_f is the total area of the vertical reinforcement located with a distance of $h/4$ from the end of the member

[AASHTO LRFD-8 Specifications 5.9.4.4] *[FDOT SDG 4.3.1.D]*

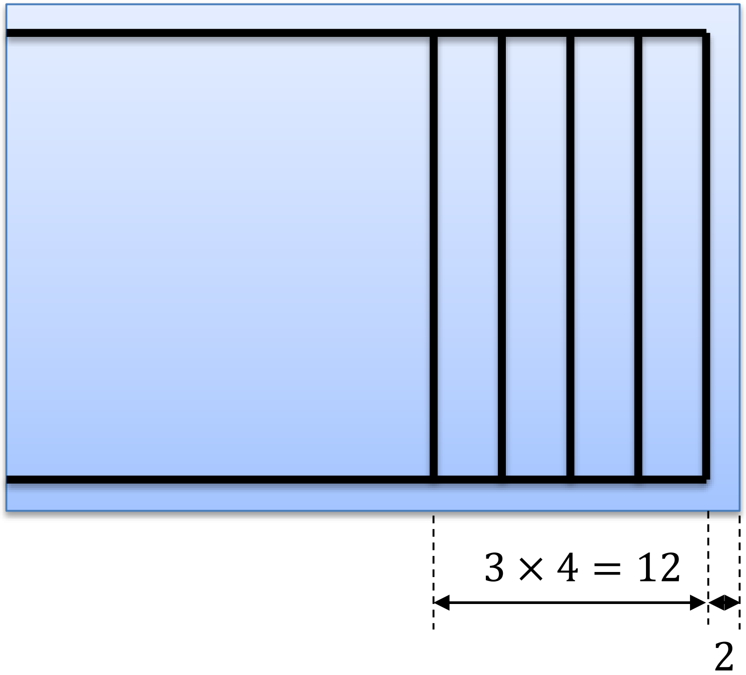
FLEXURAL DESIGN

e.g. $P_r = f_s A_s \geq 0.04 \cdot P_i$

Steel Bar Example
NOT for FRP

$P_i = 1500\ kips; f_s = 20\ ksi; h = 60\ in$

$$A_s \geq \frac{0.04 \cdot P_i}{f_s} = 3\ in^2$$



Try No.5 closed ties (2 legs); each tie $A_s = 2 \times 0.31 = 0.62\ in^2$

Number of ties $= 3/0.62 = 4.8 \rightarrow 5$

Distance within which anchorage reinforcement has to be provided $h/4 = 15\ in$

Use **5 No.5 closed ties (2 legs) at 3 *in.* center-to-center**, with the first tie starting at 2 in from beam end

Check, $A_{s,provide} = 5 \times 2 \times 0.31 = 3.1\ in^2 > A_{s,req}$

FLEXURAL DESIGN

FDOT ***SDG*** 4.3.1.D *(more restrictive)*

- 3% P_j from the end of the beam to $h/8$, but $\geq 10''$
- 5% P_j from the end of the beam to $h/4$, but $\geq 10''$
- 6% P_j from the end of the beam to $3h/8$, but $\geq 10''$

e.g. $P_r = f_f A_f \geq 0.05 \cdot P_j$

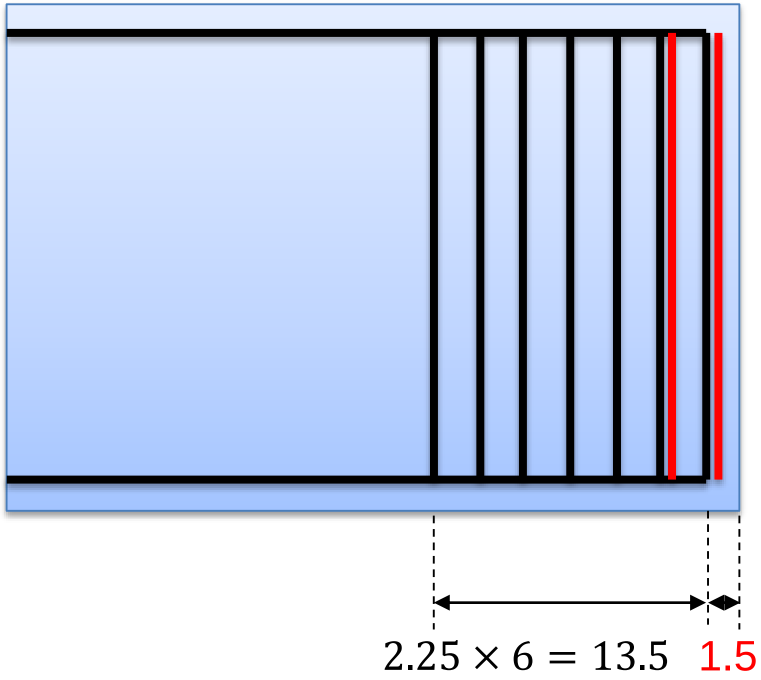
$$P_j = 1500\ kips; f_f = 15\ ksi; h = 60\ in$$

$$A_f \geq \frac{0.05 \cdot P_j}{f_f} = 5\ in^2$$

Try No.5 closed GFRP ties (2 legs); each tie $A_f = 2 \times 0.31 = 0.62\ in^2$

Number of ties $= 5/0.62 = 8.1 \rightarrow 9$

Within $h/4 = 15\ in$



Use **<u>9~No.5 closed ties (2 legs) GFRP bars at 2.25 *in.* center-to-center</u>**, bundle the first two ties with the first tie starting at 1.5-in. from beam end

Check, $A_{f,provide} = 9 \times 2 \times 0.31 = 5.6\ in^2 > A_{s,req}$

FLEXURAL DESIGN

Pretensioned Anchorage Zone

- Confinement Reinforcement

  ✓ For the distance of 1.5d from the end of the beams other than box beams, reinforcement shall be placed to confine the prestressing CFRP in the bottom flange. The reinforcement shall not be less than No. 3 deformed bars, with spacing not exceeding 6.0 inch and shaped to enclose the strands

  ✓ For box beams, transverse reinforcement shall be provided and anchored by extending the leg of stirrup into the web of the girder

[AASHTO LRFD-8 Specifications 5.9.4.4]

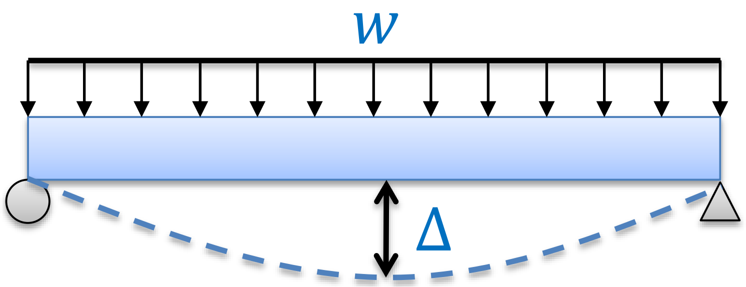
FLEXURAL DESIGN

Deflection and Camber

- ✓ Dead load
- ✓ Live load
- ✓ Prestressing
- ✓ Concrete creep
- ✓ Concrete shrinkage

- Instantaneous deflection calculation

e.g.



$$\Delta = \frac{5}{384}\frac{wl^4}{EI}$$

E = modulus of elasticity of concrete
I = gross moment of inertia I_g
or effective moment of inertia I_e

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \times \beta_d I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr} \leq I_g$$

[AASHTO CFRP-1 Specifications 1.7.3.4]

FLEXURAL DESIGN

Deflection and Camber

- Instantaneous deflection estimation

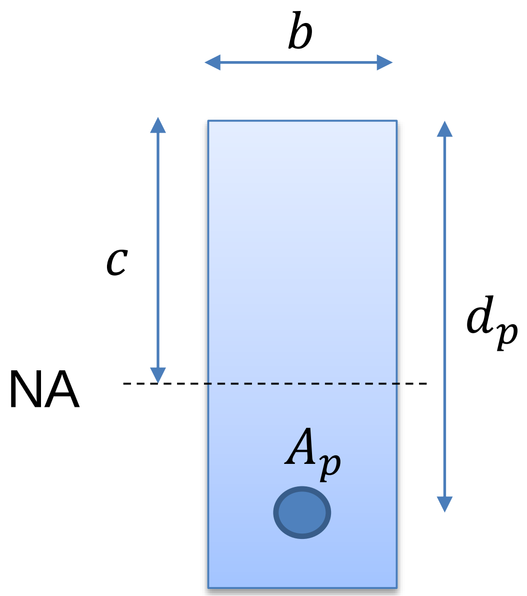
Cracking moment

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \times \beta_d I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr} \le I_g$$

Max. moment

$$\beta_d = 0.5\left(\frac{E_f}{E_s} + 1\right)$$

$$I_{cr} = \frac{bc^3}{3} + nA_{pf}\left(d_p - c\right)^2 \qquad n = \frac{E_f}{E_c}$$



[AASHTO CFRP-1 Specifications 1.7.3.4]

FLEXURAL DESIGN

Deflection and Camber

- In AASHTO ***CFRP-1*** Specifications [1.7.3.4]

Long term deflection = 4 × instantaneous deflection

- In *AASHTO* ***LRFD-8*** Specifications [C5.6.3.5.2]

"In prestressed concrete, the long-term deflection is usually based on mix-specific data, possibly in combination with the calculation procedures in Article 5.4.2.3. Other methods of calculating deflections which consider the different types of loads and the sections to which they are applied, such as that found in PCI handbook, may also be used."

[AASHTO CFRP-1 Specifications 1.7.3.4] *[AASHTO LRFD-8 C5.6.3.5.2]*

FLEXURAL DESIGN

Deflection and Camber Due to Dead Load

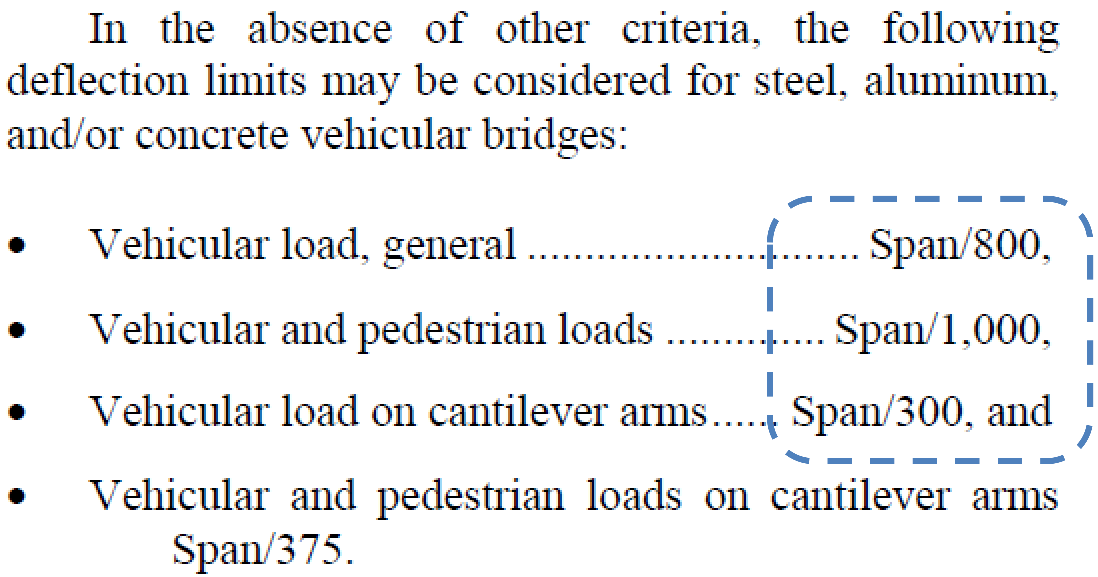
e.g.

- Due to prestressing at transfer $\Delta_{pt}=\dfrac{(Pe)l^2}{8E_{ci}I_g}$ *(after elastic shortening prestress loss)*
- Due to beam self-weight $\Delta_{gb}=\dfrac{5w_{beam}l^4}{384E_{ci}I_g}$
- Due to slab, haunch, and permanent form weights $\Delta_{gshf}=\dfrac{5\left(w_{slab}+w_{haunch}+w_{form}\right)l^4}{384E_{ci}I_g}$
- Due to barrier, wearing surface, and utilities $\Delta_{gvwsu}=\dfrac{5(w_b+w_{vs}+w_u)l^4}{384E_cI_{composite}}$

FLEXURAL DESIGN

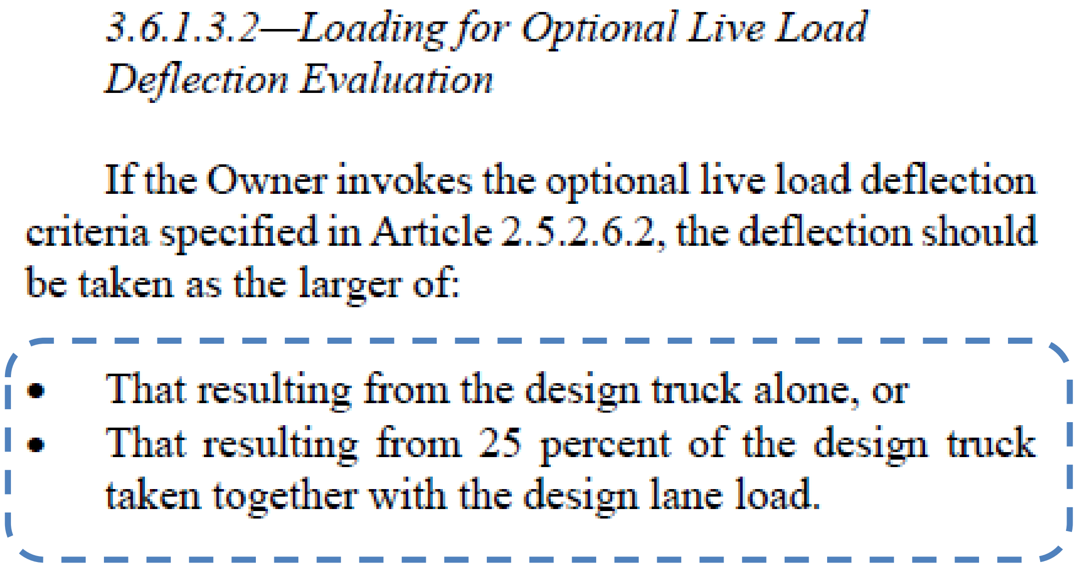
Deflection Limit Due to Live Load

Deflection due to live load and impact (optional in ***LRFD***, mandatory in FDOT ***SDG*** or meet Span/Depth ratio)



In the absence of other criteria, the following deflection limits may be considered for steel, aluminum, and/or concrete vehicular bridges:

- Vehicular load, general Span/800,
- Vehicular and pedestrian loads Span/1,000,
- Vehicular load on cantilever arms..... Span/300, and
- Vehicular and pedestrian loads on cantilever arms Span/375.



3.6.1.3.2—Loading for Optional Live Load Deflection Evaluation

If the Owner invokes the optional live load deflection criteria specified in Article 2.5.2.6.2, the deflection should be taken as the larger of:

- That resulting from the design truck alone, or
- That resulting from 25 percent of the design truck taken together with the design lane load.

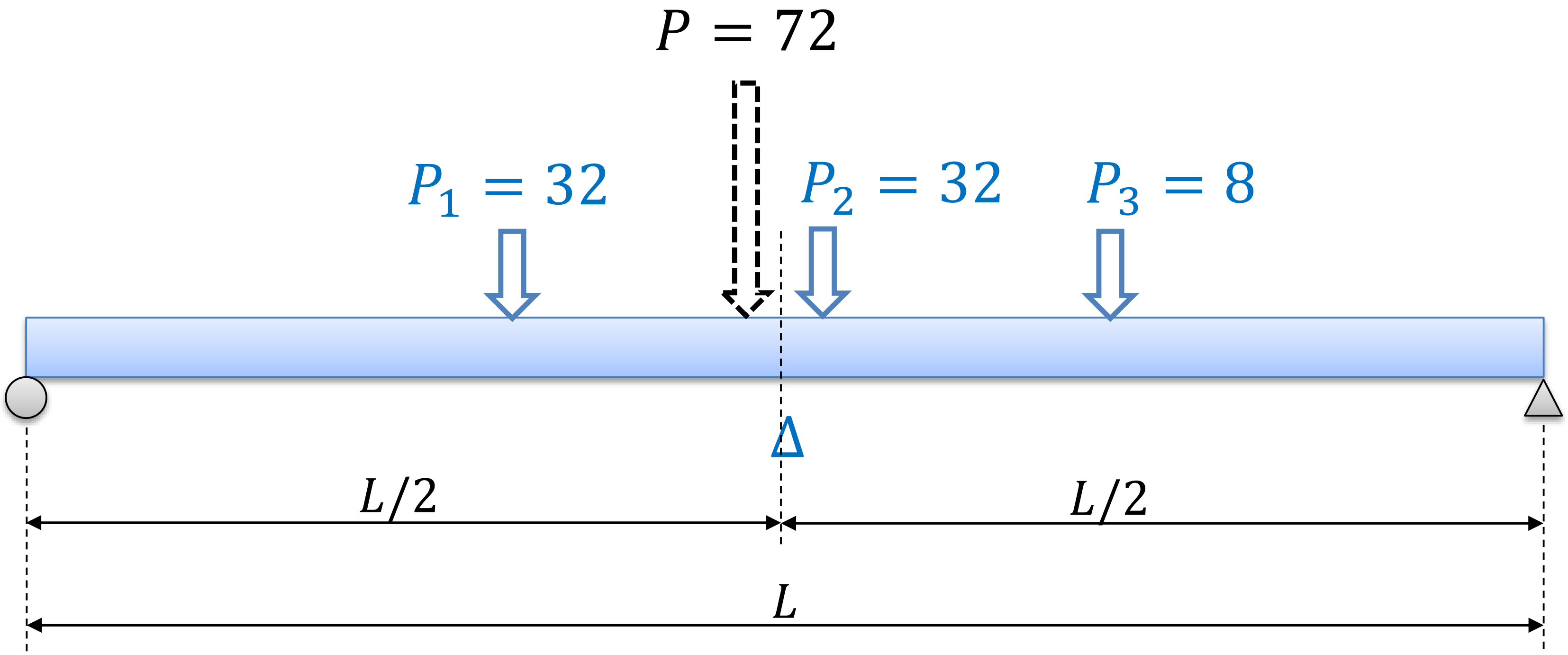
[*AASHTO LRFD-8 Specifications* 2.5.2.6.2 [*FDOT SDG* 1.2]

FLEXURAL DESIGN

Deflection Limit Due to Live Load

Deflection due to live load and impact: truck load

e.g.



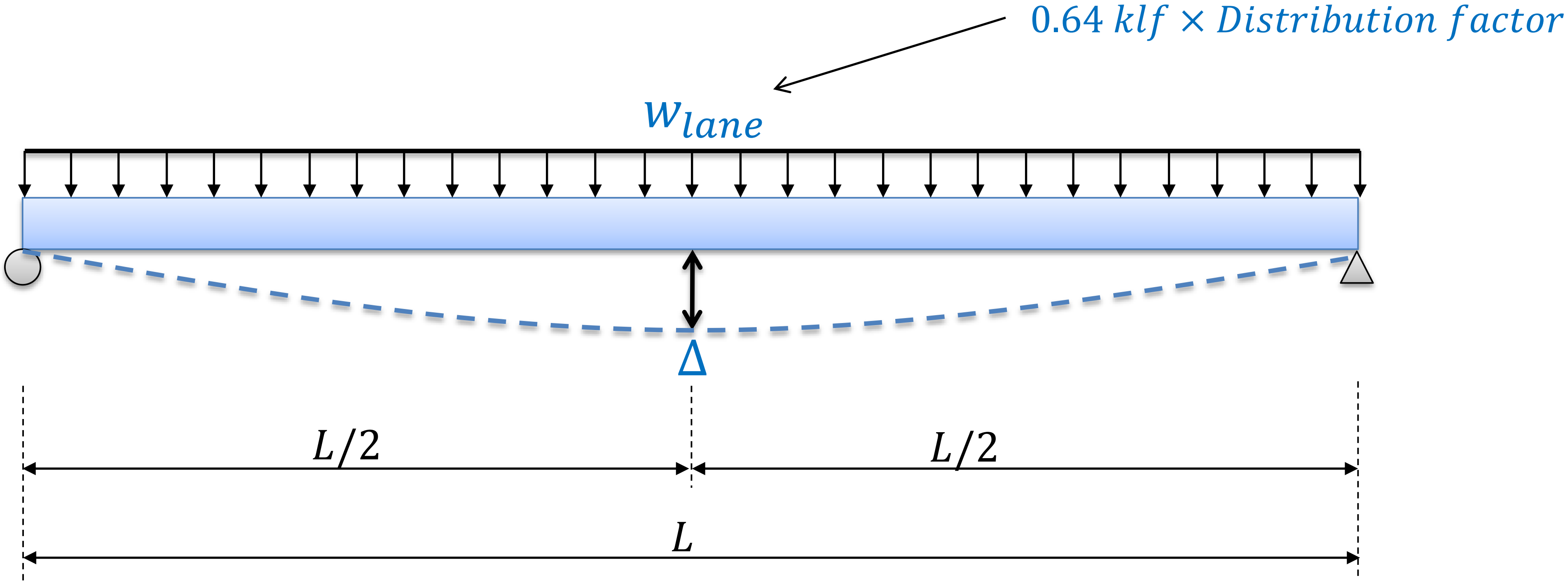
$$\Delta = \Delta_{P1} + \Delta_{P2} + \Delta_{P3}$$

FLEXURAL DESIGN

Deflection Limit Due to Live Load

Deflection due to live load and impact: lane live load

e.g.



$$\Delta = \frac{5 w_{lane} L^4}{384 EI}$$

Questions?

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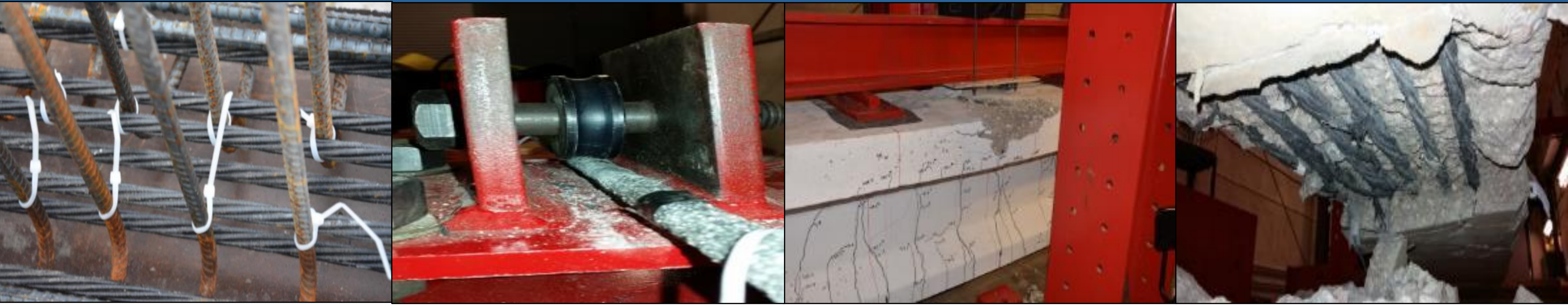
3. FLEXURAL DESIGN

3.1 Review Questions: Fundamentals



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REVIEW QUESTIONS

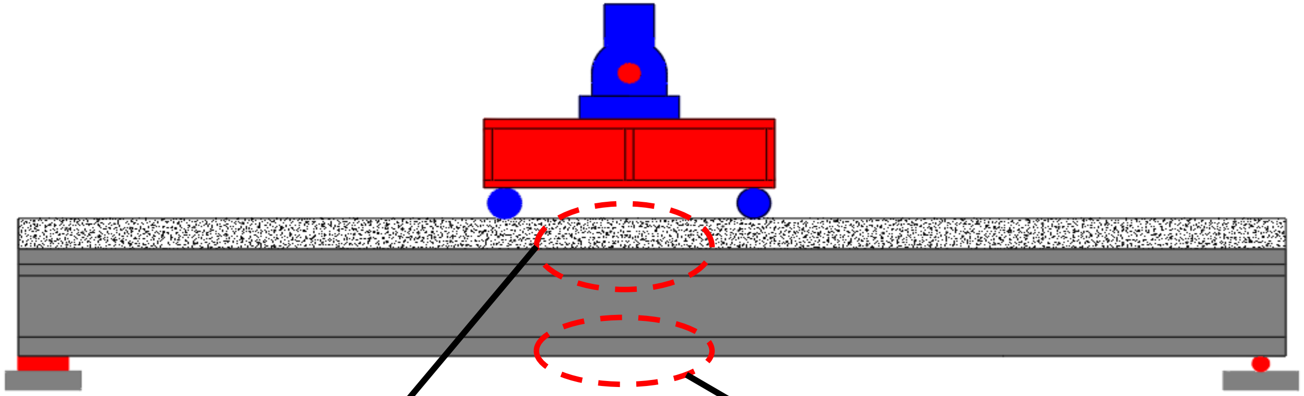
3.1.1) When designing pretensioned beam with prestressing CFRP, the preferred failure mode in flexure is ______?

a. CFRP rupture

b. Concrete crushing

c. None – it is not safe to design with CFRP strands

REVIEW QUESTIONS



Concrete crushing
Compression failure

CFRP rupture
Tension failure

✓

Selected Mode of Failure

REVIEW QUESTIONS

3.1.1) When designing pretensioned beam with prestressing CFRP, the preferred failure mode in flexure is _____?

a. **CFRP rupture**

b. Concrete crushing

c. None – it is not safe to design with CFRP strands

REVIEW QUESTIONS

3.1.2) In CFRP-PC flexural design, the strength resistance factor (Φ) is ____ (than) Steel-PC for *Tension-Controlled* section?

a. Lower

b. Higher

c. The same as

REVIEW QUESTIONS

3.1.3) In CFRP-PC flexural design, the strength resistance factor (Φ) is ___ (than) Steel-PC for *Compression-Controlled* section?

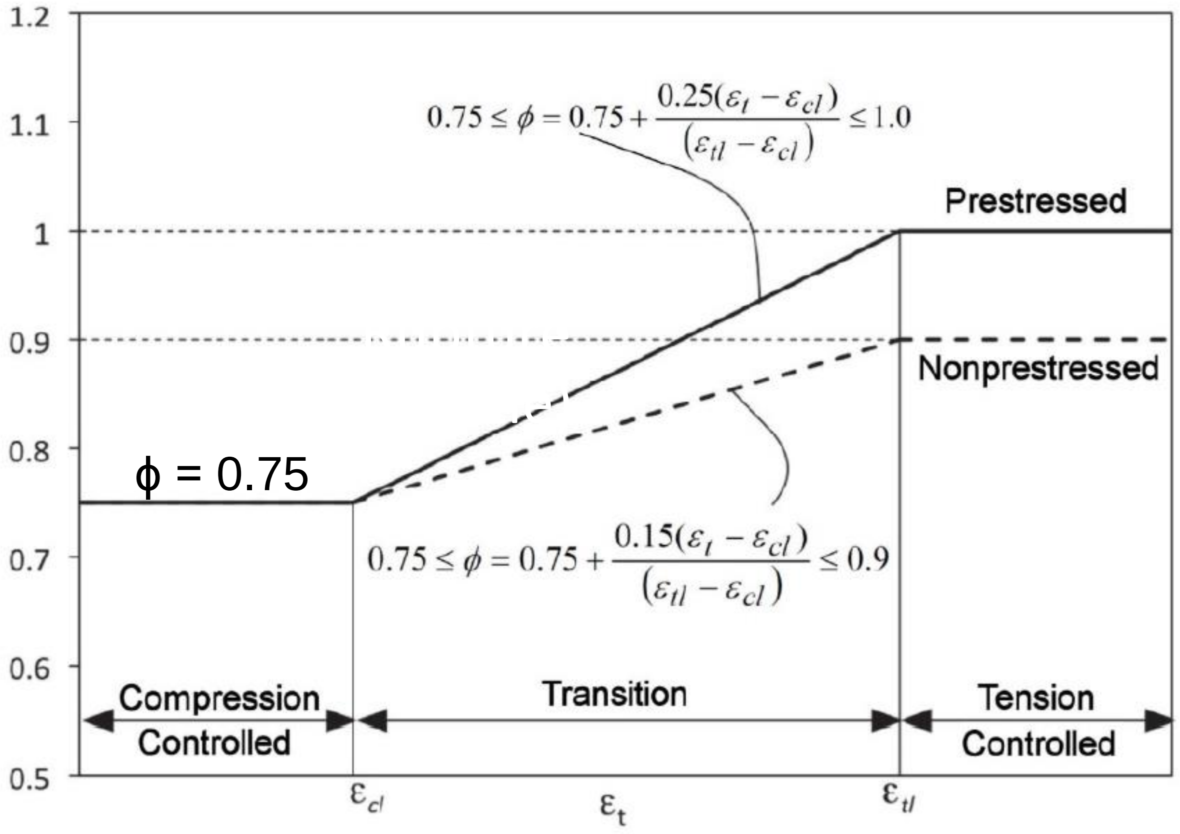
a. Lower

b. Higher

c. The same as

REVIEW QUESTIONS

Resistance Factor for flexural design



Steel-PC

Compression failure
Concrete crushing
Brittle failure

Tension failure
CFRP rupture
Brittle failure

ϕ = 0.75

CFRP-PC

REVIEW QUESTIONS

3.1.2) In CFRP-PC flexural design, the strength resistance factor (Φ) is ___ (than) Steel-PC for *Tension-Controlled* section?

a. **Lower**

b. Higher

c. The same as

REVIEW QUESTIONS

3.1.3) In CFRP-PC flexural design, the strength resistance factor (Φ) is ____ (than) Steel-PC for *Compression-Controlled* section?

a. Lower

b. Higher

c. The same as

REVIEW QUESTIONS

3.1.4) In CFRP-PC flexural design, which of the following assumptions is FALSE?

a. Plane sections remain plane after deformation

b. Tensile strength of concrete is not neglected

c. Stress strain of prestressing CFRP is linear until failure

d. A rectangular stress block is used to model concrete behavior in compression zone

REVIEW QUESTIONS

Assumptions for Design: Strength Limit State

- A rectangular stress block is used to model concrete behavior in compression zone

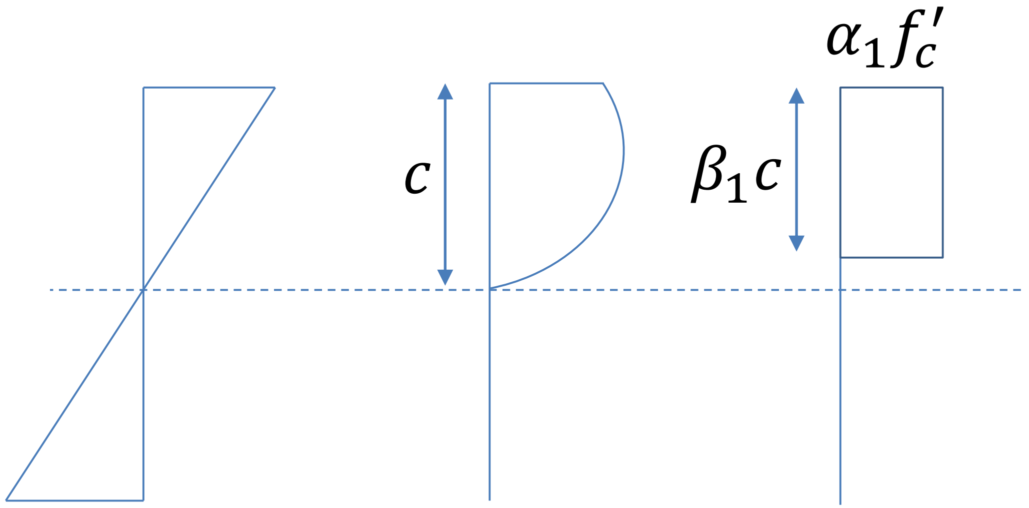
- Prestressing CFRP failure is defined to occur when

$$\varepsilon_p = \varepsilon_{pu}$$

- Balance strain condition

$$\varepsilon_p = \varepsilon_{pu} \text{ and } \varepsilon_{cc} = \varepsilon_{cu} = 0.003$$

- CFRP compression reinforcement shall be ignored in design for increasing capacity



[AASHTO CFRP-1 Specifications 1.7.2]

REVIEW QUESTIONS

3.1.4) In CFRP-PC flexural design, which of the following assumptions is FALSE?

a. Plane sections remain plane after deformation

b. <u>Tensile strength of concrete is not neglected</u>

c. Stress strain of prestressing CFRP is linear until failure

d. A rectangular stress block is used to model concrete behavior in compression zone

3. FLEXURAL DESIGN

3.2 Design Example: FIB 36

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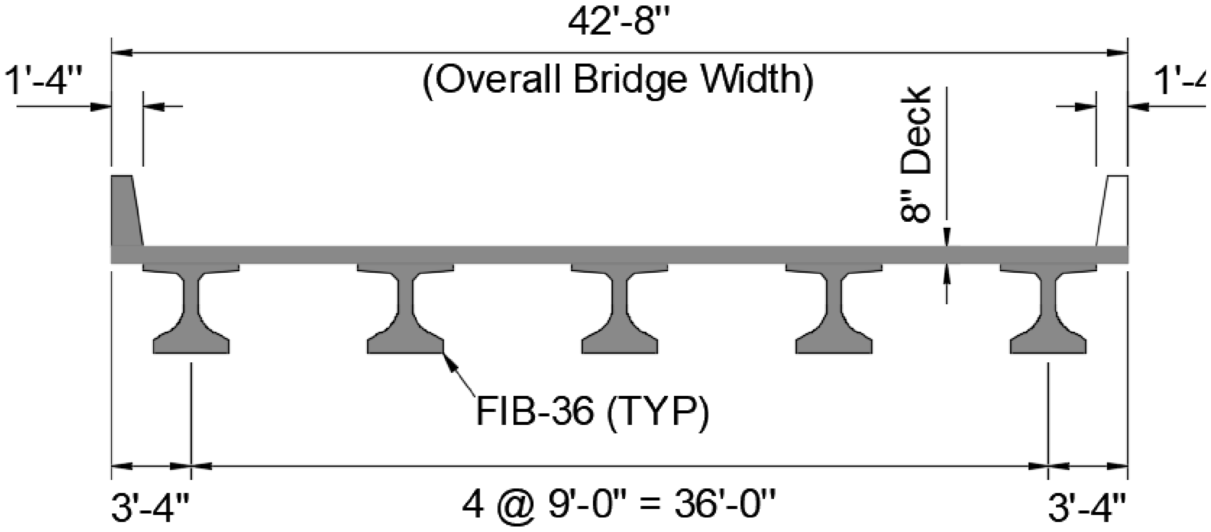
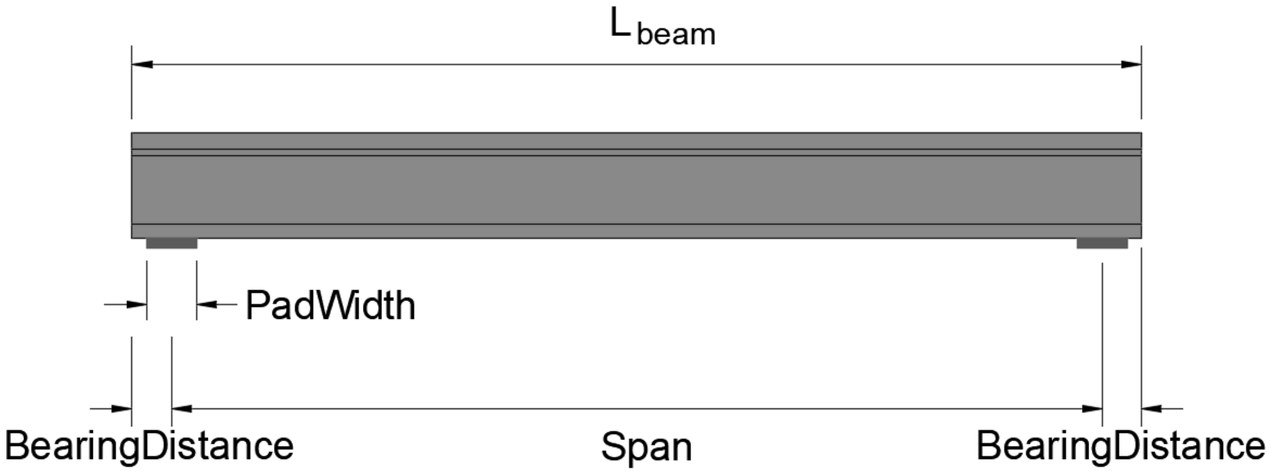
DESIGN EXAMPLE: FIB

Geometry ***This example is simplified from "SR 687/4th Street, NB bridge" project***





$\text{Beam Span} = 87.667\ \text{ft}$

$\text{BeamSpacing} = 9\ \text{ft}$

$\text{BridgeWidth} = 42.667\ \text{ft}$

$\text{Slab thickness } t_{slab} = 8.5\ \text{in}$

$\text{Beam depth } h_{beam} = 36\ \text{in}$

DESIGN EXAMPLE: FIB

Concrete

Beam $f'_{c.beam} = 8.5$ ksi

Beam $f'_{ci.beam} = 6$ ksi

Beam $E_{c.beam} = 5112$ ksi

Beam $E_{ci.beam} = 4557$ ksi

Slab $f'_{c.slab} = 5.5$ ksi

Slab $E_{c.slab} = 4428$ ksi

Unit weight $\gamma_c = \boxed{150 \text{ pcf}}$

Note: 145 pcf is permitted

CFRP strand

Diameter $D_p = 0.6$ in

Effective area $A_{pf} = 0.179$ in^2

Elastic modulus $E_p = 22{,}480$ ksi

Design tensile strength $f_{pu} = 341$ ksi

Design tensile strain $\varepsilon_{pu} = 0.015$ ksi

Jacking stress $f_{pj} = 239$ ksi

GFRP rebar

Bar size $= \# 5$

Diameter $d_{GFRP} = 0.625$ in

Elastic modulus $E_{GFRP} = 6500$ ksi

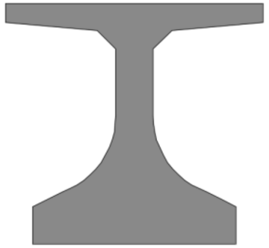
Design tensile strength

$f_{fu.GFRP} = 66.4$ ksi

Bend $\varphi_{bend} = 0.6$

DESIGN EXAMPLE: FIB

Section Properties
Non-Composite Section



$h_g = 36 \text{ in}$

$A_g = 807 \text{ in}^2$

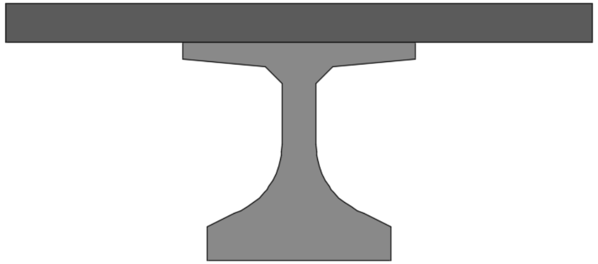
$I_g = 1.275 \times 10^5 \text{ in}^4$

$y_{g.t} = 19.51 \text{ in}$ $y_{g.b} = 16.49 \text{ in}$

$S_{g,t} = 6537 \text{ in}^3$

$S_{g,b} = 7735 \text{ in}^3$

Section Properties
Composite Section



$h_{g.c} = 45.5 \text{ in}$

$A_{g.c} = 1643 \text{ in}^2$

$I_{g.c} = 4.333 \times 10^5 \text{ in}^4$

$y_{g.c.t} = 16.52 \text{ in}$ $y_{g.c.b} = 28.98 \text{ in}$

$S_{g,c.t} = 2.622 \times 10^4 \text{ in}^3$

$S_{g,c.b} = 1.495 \times 10^4 \text{in}^3$

DESIGN EXAMPLE: FIB

Loads and Load Distributions

At Mid Span

$M_{DC,mid} = 2025 \text{ kip} \cdot \text{ft}$

$M_{DW,mid} = 0 \text{ kip} \cdot \text{ft}$

$M_{LL+IM,mid} = 1628 \text{ kip} \cdot \text{ft}$

$M_{Str1,mid} = 5381 \text{ kip} \cdot \text{ft}$

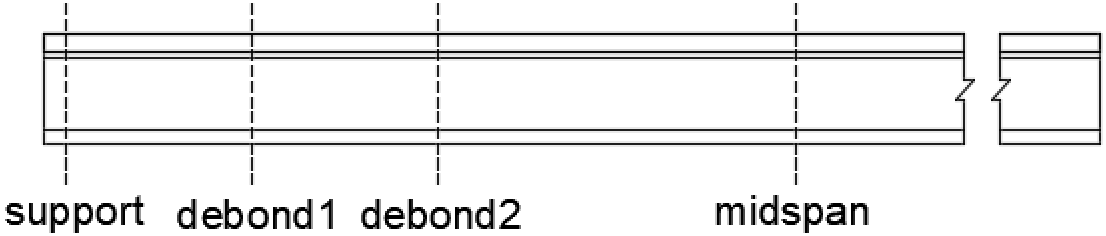
$M_{Srv1,mid} = 3650 \text{ kip} \cdot \text{ft}$

$M_{Srv3,mid} = 3328 \text{ kip} \cdot \text{ft}$

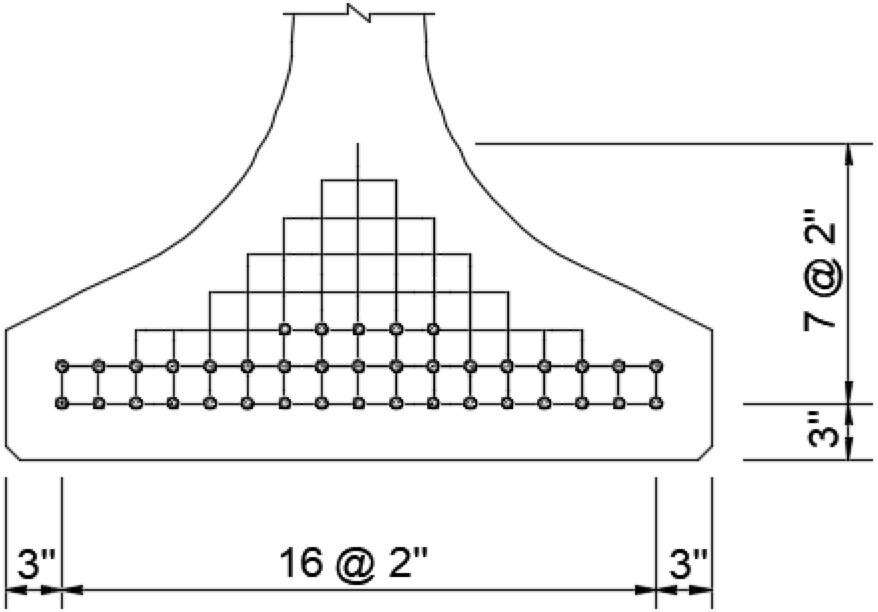
DESIGN EXAMPLE: FIB

Strand Patten

Maximum 8 rows of strands for FIB-36



$$n_{max} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \\ 13 \\ 17 \\ 17 \end{bmatrix} \qquad n_{provide} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 17 \\ 17 \end{bmatrix}$$



DESIGN EXAMPLE: FIB

Transformed Section Properties

Non-Composite Section

$h_g = 36\text{ in}$

$A_{g.tr} = 831\text{ in}^2$

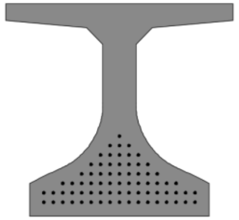
$I_{g.tr} = 1.310 \times 10^5\text{ in}^4$

$y_{g.tr.t} = 19.86\text{ in}$ $y_{g.tr.b} = 16.14\text{ in}$

$S_{g,tr.t} = 6596\text{ in}^3$

$S_{g,tr.b} = 8112\text{ in}^3$

$e_{g,tr} = 11.76\text{ in}$



Transformed Section Properties

Composite Section

$h_{g.c} = 45.5\text{ in}$

Assume #5 GFRP bar @12”, both top and bottom of deck

$A_{g.c.tr} = 1669\text{ in}^2$

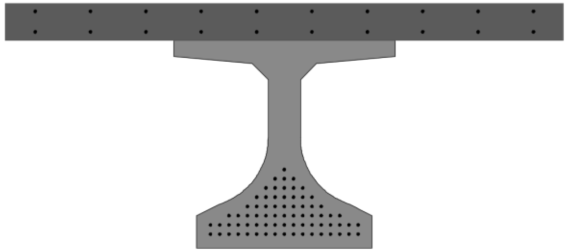
$I_{g.c.tr} = 4.389 \times 10^5\text{ in}^4$

$y_{g.c.tr.t} = 16.73\text{ in}$ $y_{g.c.tr.b} = 28.77\text{ in}$

$S_{g,c.tr.t} = 2.624 \times 10^4\text{ in}^3$

$S_{g,c.tr.b} = 1.525 \times 10^4\text{in}^3$

$e_{g,c.tr} = 24.39\text{ in}$



DESIGN EXAMPLE: FIB

Total Prestress Losses

Elastic shortening $\Delta f_{pES} = 14.277\text{ ksi}$

Long-Term Losses $\Delta f_{pLT} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)_{id} + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)_{df} = 39.245\text{ ksi}$

Loss due to temperature change (Assume 0 in this example) $\Delta f_{pTH} = 0$

Total prestress loss $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pTH} = 53.522\text{ ksi} \quad (22.4\%)$

Effective prestress after all losses

$f_{pe} = 185\text{ ksi} < f_{pe.limit} = 0.65 f_{pu} = 222\text{ ksi} \qquad \mathbf{OK}$

DESIGN EXAMPLE: FIB

- ➢ **Tensile and Compressive Stress Limits for Concrete, <u>at Release</u>**

- ➢ **Tensile and Compressive Stress Limits for Concrete, <u>at Service</u>**

- ***Service I limit state, under effective prestress and permanents loads***

- ***Service I limit state, under effective prestress, permanents loads, and transient loads***

- ***Service III limit state***

- ➢ **Strength Limit State: Strength I**

- ➢ **Minimum Reinforcement – Prestressing CFRP**

DESIGN EXAMPLE: FIB

Tensile and Compressive Stress Limits for Concrete, at Release

At Mid-Span

$$f_{c.beam,t} = \frac{P_{pj}}{A_{g.tr}} + \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = 0.505\text{ ksi} > f_{t.limit} = -0.588\text{ ksi} \quad \textbf{OK}$$

$$f_{c.beam,b} = \frac{P_{pj}}{A_{g.tr}} - \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = 3.226\text{ ksi} < f_{c.limit} = 3.900\text{ ksi} \quad \textbf{OK}$$

DESIGN EXAMPLE: FIB

Tensile and Compressive Stress Limits for Concrete, at Release

At Support (Transfer length from support)

$$f_{c.beam,t} = \frac{P_{pj}}{A_{g.tr}} + \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = -0.801\ \text{ksi} < f_{t.limit} = -0.588\ \text{ksi} \quad \textbf{NG}$$

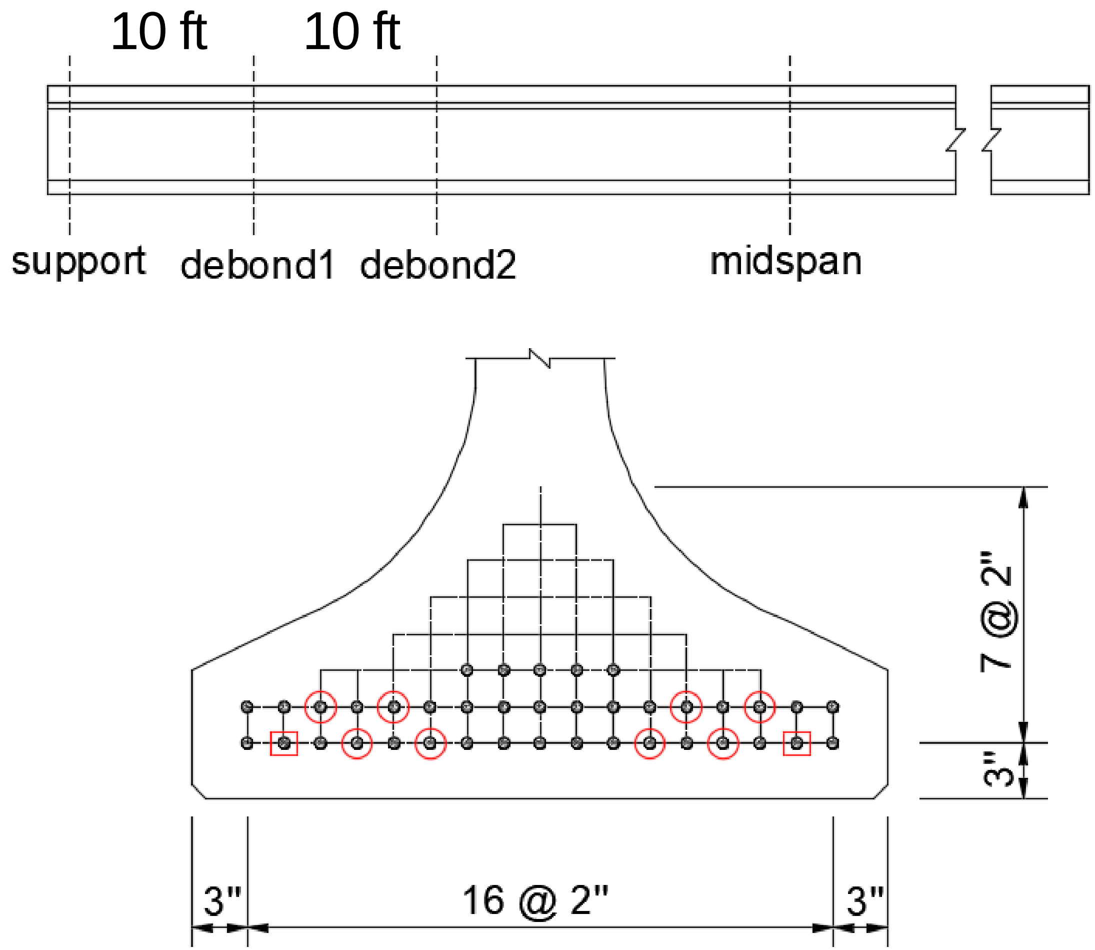
$$f_{c.beam,b} = \frac{P_{pj}}{A_{g.tr}} - \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = 4.287\ \text{ksi} > f_{c.limit} = 3.900\ \text{ksi} \quad \textbf{NG}$$

Need to debond strands!!

DESIGN EXAMPLE: FIB

Strand Patten after Debonding

$$\begin{array}{ccc}
\text{After debond Support} & \text{After debond 1 (10 ft)} & \text{No debond (20 ft)} \\
\begin{bmatrix}0\\0\\0\\0\\0\\5\\13\\11\end{bmatrix} &
\begin{bmatrix}0\\0\\0\\0\\0\\5\\17\\15\end{bmatrix} &
\begin{bmatrix}0\\0\\0\\0\\0\\5\\17\\17\end{bmatrix}
\end{array}$$



☐ Strands debonded 20’ from end of beam

○ Strands debonded 10’ from end of beam

DESIGN EXAMPLE: FIB

Tensile and Compressive Stress Limits for Concrete, at Release

At Support (Transfer length from support)

$f_{c.beam,t} = -0.527 \text{ ksi} > f_{t.limit} = -0.588 \text{ ksi} \quad \textbf{OK}$

$f_{c.beam,b} = 3.170 \text{ ksi} < f_{c.limit} = 3.900 \text{ ksi} \quad \textbf{OK}$

At debond 1 (10 ft)

$f_{c.beam,t} = -0.306 \text{ ksi} > f_{t.limit} = -0.588 \text{ ksi} \quad \textbf{OK}$

$f_{c.beam,b} = 3.707 \text{ ksi} < f_{c.limit} = 3.900 \text{ ksi} \quad \textbf{OK}$

At debond 2 (20 ft)

$f_{c.beam,t} = 0.071 \text{ ksi} > f_{t.limit} = -0.588 \text{ ksi} \quad \textbf{OK}$

$f_{c.beam,b} = 3.455 \text{ ksi} < f_{c.limit} = 3.900 \text{ ksi} \quad \textbf{OK}$

DESIGN EXAMPLE: FIB

Tensile and Compressive Stress Limits for Concrete, at Service (Mid Span)

Service I limit state, under effective prestress and permanents loads

Slab, top $f_{c.slab,t} = 0.076 \text{ ksi} < f_{c.limit} = 2.475 \text{ ksi}$ **OK**

Beam, top $f_{c.beam,t} = 2.669 \text{ ksi} < f_{c.limit} = 3.825 \text{ ksi}$ **OK**

Service I limit state, under effective prestress, permanents loads, and transient loads

Slab, top $f_{c.slab,t} = 0.820 \text{ ksi} < f_{c.limit} = 3.300 \text{ ksi}$ **OK**

Beam, top $f_{c.beam,t} = 2.991 \text{ ksi} < f_{c.limit} = 5.100 \text{ ksi}$ **OK**

DESIGN EXAMPLE: FIB

Tensile and Compressive Stress Limits for Concrete, at Service (Mid Span)

Service III limit state

Beam, bottom $f_{c.beam,b} = -0.478 \text{ ksi} > f_{t.limit} = -0.588 \text{ ksi} \quad \mathbf{OK}$

Tensile and Compressive Stress Limits for Concrete, at Release ✓

Tensile and Compressive Stress Limits for Concrete, at Service ✓

DESIGN EXAMPLE: FIB

Strength limit state: Strength I

(Mid Span)

Assume Compression-Controlled Section $\varepsilon_{cc} = \varepsilon_{cu} = 0.003$ $f_{pf,bottom\,row} < f_{pu}$

$$\left.\begin{array}{l}\text{Equilibrium: Compression = Tension}\\ \\ \text{Strain compatibility}\end{array}\right\}\ \text{Iteration, find compression depth c}$$

⇨ Bottom row CFRP $f_{pf} = 435\text{ ksi} > f_{pu} = 341\text{ ksi}$ **NG**

⇨ **Assumption NOT Satisfied**

⇨ **Tension-Controlled Section**

DESIGN EXAMPLE: FIB

Strength limit state: Strength I

(Mid Span)

Tension-Controlled Section $f_{pf,bottom\,row} = f_{pu} = 341\ \text{ksi}$ $\varepsilon_{cc} < \varepsilon_{cu}$

Equilibrium: Compression = Tension

Strain compatibility

} Iteration, find compression depth c

⇨ Concrete, top fiber $\varepsilon_{cc} = 0.0014 < \varepsilon_{cu} = 0.003$ **Assumption Satisfied**

⇨ Nominal Flexural Capacity $M_n = 7560\ \text{kip} \cdot \text{ft}$

$\phi M_n = 5670\ \text{kip} \cdot \text{ft} > M_{Str1} = 5381\ \text{kip} \cdot \text{ft}$ **OK**

DESIGN EXAMPLE: FIB

Minimum Reinforcement – Prestressing CFRP

Tension-Controlled Section $\phi M_n \geq \min\left(1.33 M_u, M_{cr}\right)??$

$$M_{cr} = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right) S_{g.c.tr.b} - M_{dnc}\left(\frac{S_{g.c.tr.b}}{S_{g.tr.b}} - 1\right)\right] = 4579\ \mathrm{kip \cdot ft}$$

$$\phi M_n = 5670\ \mathrm{kip \cdot ft} > \min\left(1.33 M_u, M_{cr}\right) = 4579\ \mathrm{kip \cdot ft}$$

OK

3. FLEXURAL DESIGN

3.3 Design Example: FSB 12-57

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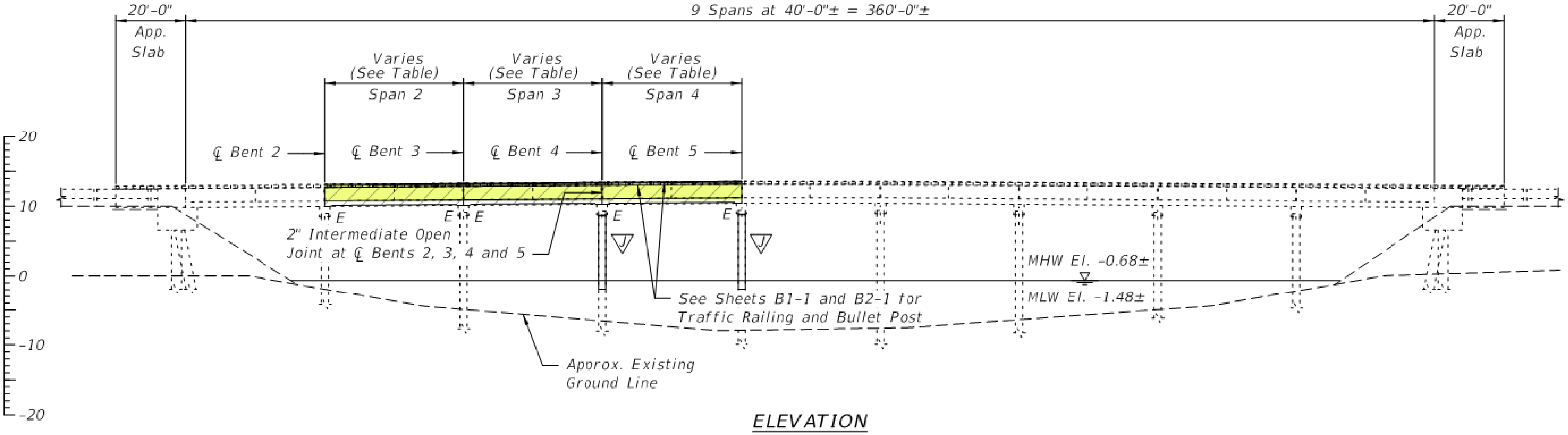
Department of Civil & Environmental Engineering

DESIGN EXAMPLE: FSB

US 1 Over Cow Key Channel, Key West, Florida

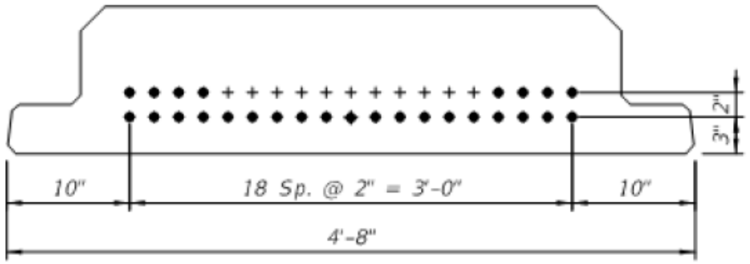
DESIGN EXAMPLE: FSB

Geometry, from US 1 Over Cow Key Channel, Key West, Florida



DESIGN EXAMPLE: FSB

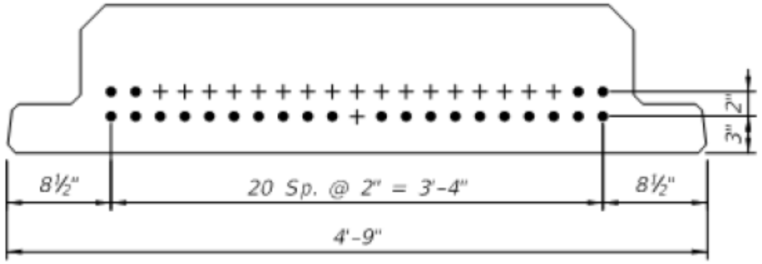
US 1 Over Cow Key Channel, Key West, Florida



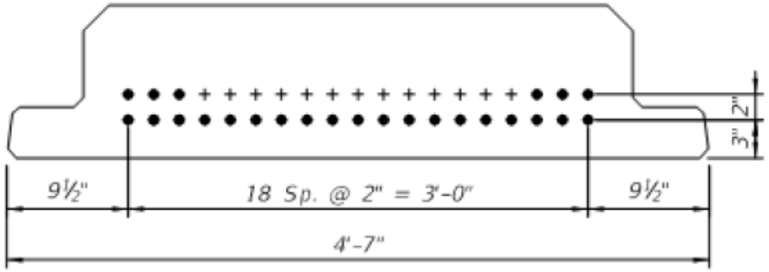
TYPE ① 27 STRANDS

FIBER-REINFORCED POLYMER REINFORCING:

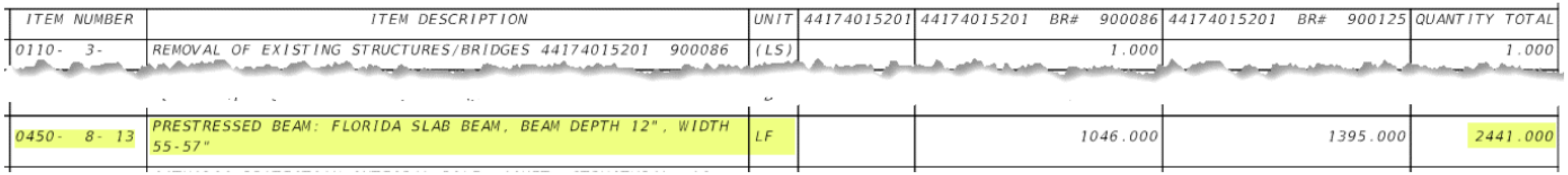
1. FSB precast panels shall be reinforced using Glass-Fiber Reinforced Polymer (GFRP) and shall be prestressed using Carbon-Fiber Reinforced Polymer (CFRP).
2. All FRP reinforcing shall be per Specification Section 932 and 933.
3. FRP reinforcing detail dimensions are out-to-out of bars.
4. For standard bar bending detail for FRP, see FDOT Developmental Design Index D21310.



TYPE ② 24 STRANDS



TYPE ① 25 STRANDS

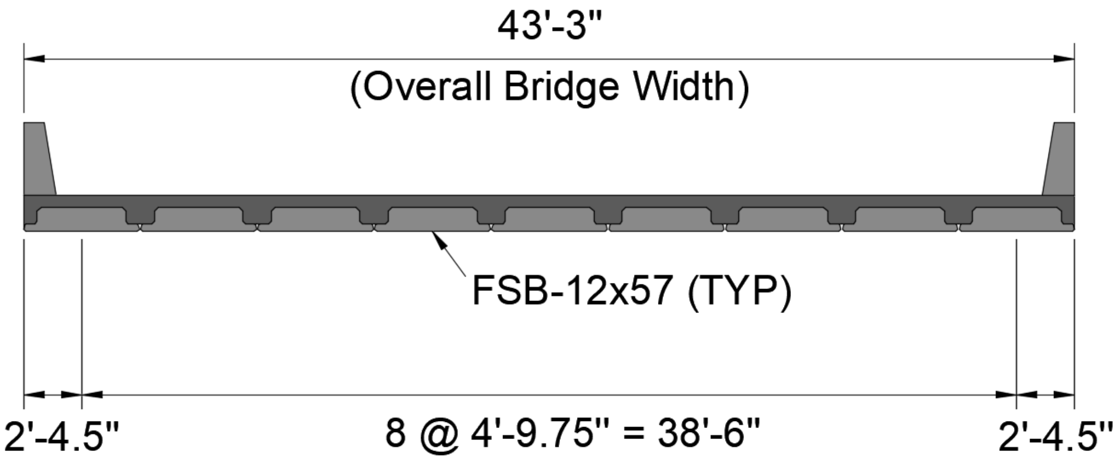


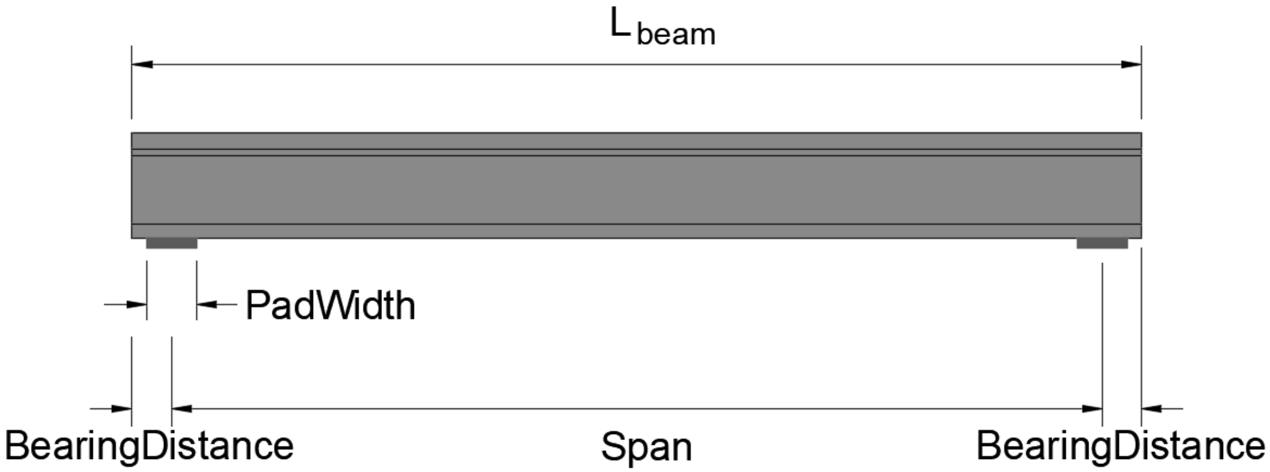
ITEM NUMBER	ITEM DESCRIPTION	UNIT	44174015201	44174015201 BR# 900086	44174015201 BR# 900125	QUANTITY TOTAL
0110- 3-	REMOVAL OF EXISTING STRUCTURES/BRIDGES 44174015201 900086	(LS)		1.000		1.000
0450- 8- 13	PRESTRESSED BEAM: FLORIDA SLAB BEAM, BEAM DEPTH 12", WIDTH 55-57"	LF		1046.000	1395.000	2441.000

DESIGN EXAMPLE: FSB

Geometry

This Example is simplified from "US 1 Over Cow Key Channel" bridge





$\text{Beam Span} = 38.917\ \text{ft}$

$\text{BeamSpacing} = 4.813\ \text{ft}$

$\text{BridgeWidth} = 43.25\ \text{ft}$

$\text{Slab thickness } t_{slab} = 6\ \text{in}$

$\text{Beam depth } h_{beam} = 12\ \text{in}$

DESIGN EXAMPLE: FSB

Concrete

Beam $f'_{c.beam} = 8.5\text{ ksi}$

Beam $f'_{ci.beam} = 6\text{ ksi}$

Beam $E_{c.beam} = 5112\text{ ksi}$

Beam $E_{ci.beam} = 4557\text{ ksi}$

Slab $f'_{c.slab} = 5.5\text{ ksi}$

Slab $E_{c.slab} = 4428\text{ ksi}$

Unit weight $\gamma_c = \boxed{150\text{ pcf}}$

Note: 145 pcf is permitted

CFRP strand

Diameter $D_p = 0.6\text{ in}$

Effective area $A_{pf} = 0.179\text{ in}^2$

Elastic modulus $E_p = 22{,}480\text{ ksi}$

Design tensile strength $f_{pu} = 341\text{ ksi}$

Design tensile strain $\varepsilon_{pu} = 0.015\text{ ksi}$

Jacking stress $f_{pj} = 239\text{ ksi}$

GFRP rebar

Bar size $= \#\ 4$

Diameter $d_{GFRP} = 0.5\text{ in}$

Elastic modulus $E_{GFRP} = 6500\text{ ksi}$

Design tensile strength

$f_{fu.GFRP} = 77.0\text{ ksi}$

Bend $\varphi_{bend} = 0.6$

DESIGN EXAMPLE: FSB

Section Properties
Non-Composite Section

$h_g = 12$ in

$A_g = 582 \text{ in}^2$

$I_g = 7084 \text{ in}^4$

$y_{g.t} = 6.35$ in $\qquad y_{g.b} = 5.65$ in

$S_{g,t} = 1116 \text{ in}^3$

$S_{g,b} = 1254 \text{ in}^3$

Section Properties
Composite Section

$h_{g.c} = 18$ in

$A_{g.c} = 974 \text{ in}^2$

$I_{g.c} = 2.598 \times 10^4 \text{ in}^4$

$y_{g.c.t} = 9.22$ in $\qquad y_{g.c.b} = 8.78$ in

$S_{g,c.t} = 2819 \text{ in}^3$

$S_{g,c.b} = 2959 \text{ in}^3$

DESIGN EXAMPLE: FSB

Loads and Load Combinations

At Mid Span

$M_{DC,mid} = 221 \text{ kip} \cdot \text{ft}$

$M_{DW,mid} = 29.7 \text{ kip} \cdot \text{ft}$

$M_{LL+IM,mid} = 245 \text{ kip} \cdot \text{ft}$

$M_{Str1,mid} = 750 \text{ kip} \cdot \text{ft}$

$M_{Srv1,mid} = 496 \text{ kip} \cdot \text{ft}$

$M_{Srv3,mid} = 447 \text{ kip} \cdot \text{ft}$

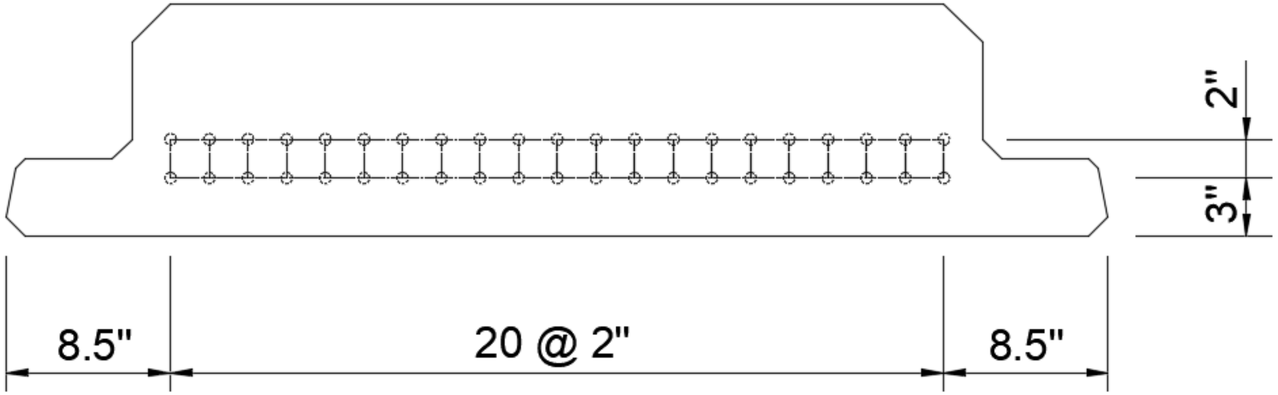
DESIGN EXAMPLE: FSB

Strand Patten

Maximum 2 rows of strands for FSB 12x57

$$n_{max} = \begin{bmatrix} 21 \\ 21 \end{bmatrix}$$

$$n_{provide} = \begin{bmatrix} 0 \\ 18 \end{bmatrix}$$



DESIGN EXAMPLE: FSB

Transformed Section Properties

Non-Composite Section

$h_g = 36 \text{ in}$

$A_{g.tr} = 593 \text{ in}^2$

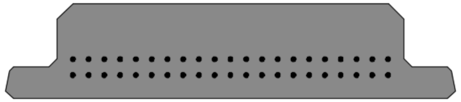
$I_{g.tr} = 7159 \text{ in}^4$

$y_{g.tr.t} = 6.40 \text{ in}$ $y_{g.tr.b} = 5.60 \text{ in}$

$S_{g,tr.t} = 1119 \text{ in}^3$

$S_{g,tr.b} = 1278 \text{ in}^3$

$e_{g,tr} = 2.60 \text{ in}$



Transformed Section Properties

Composite Section

Assume #4 GFRP bar @12", In deck

$h_{g.c} = 45.5 \text{ in}$

$A_{g.c.tr} = 985 \text{ in}^2$

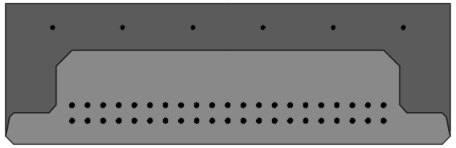
$I_{g.c.tr} = 2.600 \times 10^4 \text{ in}^4$

$y_{g.c.tr.t} = 9.21 \text{ in}$ $y_{g.c.tr.b} = 8.79 \text{ in}$

$S_{g,c.tr.t} = 2821 \text{ in}^3$

$S_{g,c.tr.b} = 2959 \text{ in}^3$

$e_{g,c.tr} = 5.79 \text{ in}$



DESIGN EXAMPLE: FSB

Total Prestress Losses

Elastic shortening $\Delta f_{pES} = 7.51 \text{ ksi}$

Long-Term Losses $\Delta f_{pLT} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)_{id} + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)_{df} = 36.39 \text{ ksi}$

Loss due to temperature change (Assume 0 in this example) $\Delta f_{pTH} = 0$

Total prestress loss $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} + \Delta f_{pTH} = 43.90 \text{ ksi} \quad (18.4\%)$

Effective prestress after all losses

$f_{pe} = 195 \text{ ksi} < f_{pe.limit} = 0.65 f_{pu} = 222 \text{ ksi}$ **OK**

DESIGN EXAMPLE: FSB

- **Tensile and Compressive Stress Limits for Concrete, at Release**

- **Tensile and Compressive Stress Limits for Concrete, at Service**

- ***Service I limit state, under effective prestress and permanents loads***

- ***Service I limit state, under effective prestress, permanents loads, and transient loads***

- ***Service III limit state***

- **Strength Limit State: Strength I**

- **Minimum Reinforcement – Prestressing CFRP**

DESIGN EXAMPLE: FSB

Tensile and Compressive Stress Limits for Concrete, at Release

At Mid-Span

$$f_{c.beam,t} = \frac{P_{pj}}{A_{g.tr}} + \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = 0.740\ \text{ksi} > f_{t.limit} = -0.588\ \text{ksi} \qquad \mathbf{OK}$$

$$f_{c.beam,b} = \frac{P_{pj}}{A_{g.tr}} - \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = 1.784\ \text{ksi} < f_{c.limit} = 3.900\ \text{ksi} \qquad \mathbf{OK}$$

DESIGN EXAMPLE: FSB

Tensile and Compressive Stress Limits for Concrete, at Release

At Support (Transfer length from support)

$$f_{c.beam,t} = \frac{P_{pj}}{A_{g.tr}} + \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = -0.194 \text{ ksi} > f_{t.limit} = -0.588 \text{ ksi} \quad \textbf{OK}$$

$$f_{c.beam,b} = \frac{P_{pj}}{A_{g.tr}} - \frac{M_{w.beam} - P_{pj}e_{g.tr}}{S_{g.tr.t}} = 2.602 \text{ ksi} < f_{c.limit} = 3.900 \text{ ksi} \quad \textbf{OK}$$

DESIGN EXAMPLE: FSB

Tensile and Compressive Stress Limits for Concrete, at Service (Mid Span)

Service I limit state, under effective prestress and permanents loads

Slab, top $f_{c.slab,t} = 0.199 \text{ ksi} < f_{c.limit} = 2.475 \text{ ksi} \quad \mathbf{OK}$

Beam, top $f_{c.beam,t} = 1.858 \text{ ksi} < f_{c.limit} = 3.825 \text{ ksi} \quad \mathbf{OK}$

Service I limit state, under effective prestress, permanents loads, and transient loads

Slab, top $f_{c.slab,t} = 1.242 \text{ ksi} < f_{c.limit} = 3.300 \text{ ksi} \quad \mathbf{OK}$

Beam, top $f_{c.beam,t} = 2.222 \text{ ksi} < f_{c.limit} = 5.100 \text{ ksi} \quad \mathbf{OK}$

Service III limit state

Beam, bottom $f_{c.beam,b} = -0.567 \text{ ksi} > f_{t.limit} = -0.588 \text{ ksi} \quad \mathbf{OK}$

DESIGN EXAMPLE: FSB

Strength limit state: Strength I

(Mid Span)

Assume Compression-Controlled Section $\varepsilon_{cc} = \varepsilon_{cu} = 0.003$ $f_{pf,bottom\,row} < f_{pu}$

Equilibrium: Compression = Tension

Strain compatibility

$\}$ Iteration, find compression depth c

⇨ Bottom row CFRP $f_{pf} = 308 \text{ ksi} < f_{pu} = 341 \text{ ksi}$ **Assumption Satisfied**

⇨ Nominal Flexural Capacity $M_n = 1091 \text{ kip} \cdot \text{ft}$

$$\phi M_n = 818 \text{ kip} \cdot \text{ft} > M_{Str1} = 750 \text{ kip} \cdot \text{ft}$$

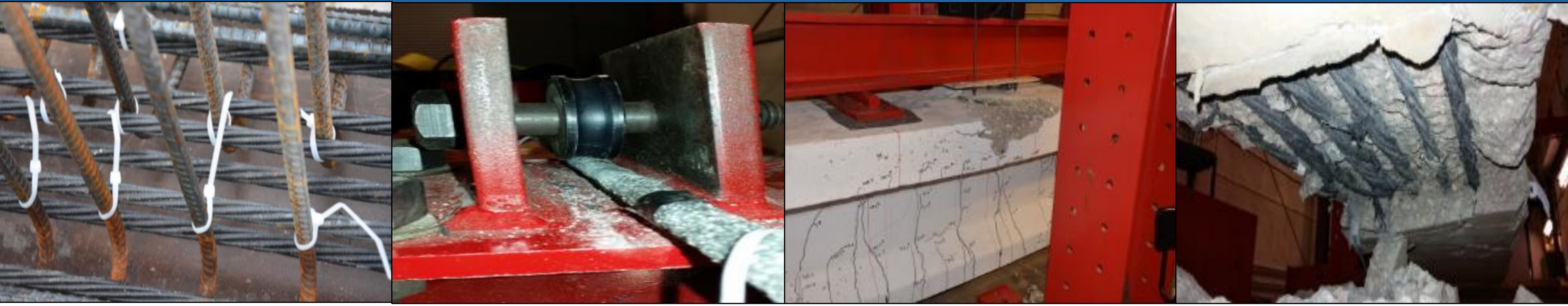
OK

DESIGN EXAMPLE: FSB

Minimum Reinforcement – Prestressing CFRP

Compression-Controlled Section, **NO NEED to check**

AASHTO CFRP-Prestressed Concrete Design Training Course



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