



## FRP-RC Design - Part 2

Steve Nolan , Raphael Kampmann, Marco Rossini

### Adapted from...

Composites Australia, December 5, 2018

### Design of concrete structures internally reinforced with FRP bars

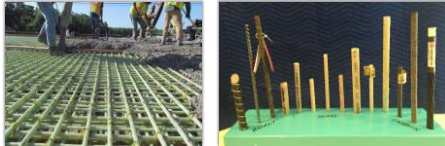
Canada Research Chair in Advanced Composite Materials for Civil Structures  
NSERC/Industrial Research Chair in Innovative FRP Reinforcement for Concrete  
Director, The University of Sherbrooke Research Centre on FRP Composites  
Department of Civil Engineering  
University of Sherbrooke, Sherbrooke, QC, Canada  
E-mail: [brahim.benmokrane@usherbrooke.ca](mailto:brahim.benmokrane@usherbrooke.ca)

2



## Course Description

Fiber-reinforced polymer (FRP) materials have emerged as an alternative for producing reinforcing bars for concrete structures. Due to other differences in the physical and mechanical behavior of FRP materials versus steel, unique guidance on the engineering and construction of concrete structures reinforced with FRP bars is necessary.



3



## Learning Objectives

- Understand the mechanical properties of FRP bars
- Describe the behavior of FRP bars
- Describe the design assumptions
- Describe the flexural/shear/compression design procedures of concrete members internally reinforced with FRP bars
- Describe the use of internal FRP bars for serviceability & durability design including long-term deflection
- Review the procedure for determining the development and splice length of FRP bars.

4



## Content of the Course

### FRP-RC Design - Part 1, (50 min.)

This session will introduce concepts for reinforced concrete design with FRP rebar. Topics will address:

- Materials & Design Specifications;
- Design & Typical Applications;
- FRP Rebar Properties;
- New Developments and Solutions;

### FRP-RC Design - Part 2, (50 min.)

This session will introduce Basalt FRP rebar that is being standardized under FHWA funded project **ST1C-0004-00A** with extended FDOT research under BE694, and provide training on the flexural design of beams, slabs, and columns for:

- Design Assumptions and Material Properties
- Ultimate capacity and rebar development length under strength limit states;
- Crack width, sustained load resistance, and deflection under service limit state;

5



## Content of the Complete Course

### BFRP-RC Design - Part 3, (50 min.)

This session continues with Basalt FRP rebar from Part 2, covering shear and axial design of columns at the strength limit states for:

- Fatigue resistance under the Fatigue limit state;
- Shear resistance of beams and slabs;
- Axial Resistance of columns;
- Combined axial and flexure loading.

### FRP-RC Design - Part 4 (*Not included at FTS - for future training*):

This session continues with FRP rebar from Part 3, covering detailing and plans preparation:

- Minimum Shrinkage and Temperature Reinforcing
- Bar Bends and Splicing
- Reinforcing Bar Lists
- General Notes & Specifications

6



## Session 2: Design Assumptions and Material Properties

- FRP bar is anisotropic
  - High strength only in the fiber direction
  - Anisotropic behavior affects shear strength, dowel action and bond performance
- FRP bar does not exhibit yielding: is elastic until failure
  - Design accounts for lack of ductility

## Session 2: Design Assumptions and Material Properties

- AASHTO-G82:** LRFD Bridge Design Guide Specifications for GFRP-Reinforced Concrete Bridges 2<sup>nd</sup> Edition (2018)
- FDOT Construction and Materials Specifications:** Section 932-3 "Fiber Reinforced Polymer (FRP) Reinforcing Bars" (2019) (**FRP-2020**)
- ASTM D7957-17: "Standard Specification for Solid Round Glass Fiber Reinforced Polymer Bars for Concrete Reinforcement", 1<sup>st</sup> Edition (2017)



## Session 2: Design Assumptions and Material Properties

- CAN/CSA-S6:** Canadian Highway Bridge Design Code, Section 16 "Fibre Reinforced Polymers (FRP) Structures".
- CAN/CSA-S806:** Design and Construction of Building Components with FRP.
- CAN/CSA-S807:** Specifications for Fibre Reinforced Polymers.



## Session 2: Design Assumptions and Material Properties

### General Standard Philosophy

These are limit states-based standards

- They follow the same basic procedures as other **AASHTO** or **CSA** structural design standards (concrete structures reinforced with steel bars);
- They are intended primarily for design of concrete structures reinforced internally with FRP bars (and/or grids or externally with sheets and laminates - for repair and retrofit under **CSA**);
- The standards cover areas for which adequate theoretical and experimental evidence is available to justify the relevant provisions
- The design provisions are intended to be on the conservative side.

## Session 2: Design Assumptions and Materials

### Load Factors and Load Combinations

- AASHTO-GS2** uses the same load factors as in **AASHTO-LRFD BDS (Chapter 3)**
- CSA-S806** uses the same load factors as in **CSA A23.3-14** code. Load combinations are also the same as in **CSA A23.3-14**, which are based on the National Building Code of Canada
- CSA-S6 – Section 16** on FRP Structures - uses the same load factors and load combinations as in the **Section 8** on Concrete Structures of the **CHBDC**

## Session 2: Design Assumptions and Materials

### FRP Bar Sizes & Strength:

- FDOT 932-3** and **ASTM D7957 strength (kips)not stress (ksi)**
- CSA-S807** similar but uses stress (MPa)

Bar Designation	Nominal Diameter (in)	Nominal Cross-Sectional Area (in <sup>2</sup> )	Minimum Cross-Sectional Area (in <sup>2</sup> )		Minimum Guaranteed Tensile Load (kips)
			Minimum	Maximum	
2	0.750	0.884	0.851	0.917	18.3
3	0.9375	0.811	0.784	0.837	20.9
4	0.900	0.785	0.763	0.803	21.6
5	0.625	0.311	0.288	0.308	29.1
6	0.750	0.44	0.413	0.439	30.9
7	0.875	0.69	0.667	0.713	34.1
8	1.000	0.79	0.758	0.813	38.8
9	1.125	1.09	0.919	1.117	51.0
10	1.250	1.27	1.074	1.305	59.9

Bar Designation	Diameter (mm)	Cross-Sectional Area (mm <sup>2</sup> )	Minimum Tensile Force (kN)	Maximum Tensile Force (kN)
M6 [2]	6.3 (0.250)	32 (0.549)	33 (0.546)	35 (0.685)
M10 [3]	9.5 (0.375)	71 (0.11)	67 (0.304)	74 (0.161)
M13 [4]	12.7 (0.500)	128 (0.20)	119 (0.180)	130 (0.262)
M16 [5]	15.9 (0.625)	199 (0.31)	186 (0.288)	201 (0.388)
M19 [6]	18.1 (0.712)	264 (0.41)	258 (0.415)	281 (0.539)
M22 [7]	22.2 (0.875)	387 (0.60)	365 (0.566)	400 (0.712)
M25 [8]	25.4 (1.000)	510 (0.79)	478 (0.738)	508 (0.912)
M29 [9]	28.7 (1.125)	645 (1.00)	603 (0.934)	733 (1.37)
M32 [10]	32.3 (1.270)	811 (1.27)	744 (1.156)	894 (1.365)

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior (f, fu, Ef)

#### Tensile Behavior

- The guaranteed (characteristic or specific) tensile strength for FRP reinforcement shall be the mean tensile strength minus three times the standard deviation (ASTM D7957, CSA-S806 & -S6)
- Similarly, the guaranteed (characteristic or specific) rupture tensile strain of FRP reinforcement shall be the mean rupture tensile strain minus three times the standard deviation (ASTM D7957, CSA-S806 & -S6)
- Similarly, the design elastic modulus for FRP reinforcement shall be the mean modulus (ASTM D7957, CSA-S806 & -S6).

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior (f, fu, Ef)

#### Tensile Strength & Modulus of Elasticity of GFRP Bars

- Tensile strength ranges between 77 to 250 ksi (530 to 1700 MPa); **FDOT 932-3 range 77 to 124 minimum.**
- Modulus of elasticity ranges between 5,800 to 9,500 ksi (40 to 65 GPa); **FDOT 932-3 minimum 6,500 ksi.**

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior (Ef)

#### CAN CSA S807-10 – Grades of FRP Bars

© Canadian Standards Association Specification for fibre-reinforced polymers

**Table 2**  
Grades of FRP bars and grids corresponding to their minimum modulus of elasticity, GPa  
(See Clause 8.3 and Table 3)

Designation	Grade I		Grade II		Grade III	
	Individual bars	Bars in a grid	Individual bars	Bars in a grid	Individual bars	Bars in a grid
AFRP	50	40	70	60	90	80
CFRP	80	70	110	100	140	130
GFRP	40	30	50	40	60	50

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior (Ef)

#### CAN CSA S807-19 (Public Review Draft) – Grades of FRP Straight Bars

**Table 2A**  
Grades of FRP straight bars and grids corresponding to their minimum modulus of elasticity, GPa  
(See Clauses 8.1.1, 8.3 and 10.1, and Table 3)

Designation	Grade I		Grade II		Grade III	
	Individual bars	Bars in a grid	Individual bars	Bars in a grid	Individual bars	Bars in a grid
AFRP	50	40	70	60	90	80
BFRP	50	40	60	50	70	60
CFRP	80	70	110	100	140	130
GFRP	40	30	50	40	60	50

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior (Ef)

#### CAN CSA S807-19 (Public Review Draft) – Grades of FRP Bent Bars

**Table 2B**  
Grades of FRP bent bars corresponding to their minimum modulus of elasticity of the straight portion, GPa  
(See Clauses 8.1.1, 8.3 and 10.1, and Table 3)

Designation	Grade IB	Grade IIB	Grade IIIB
	Individual bars	Individual bars	Individual bars
AFRP	50	60	65
BFRP	50	55	60
CFRP	80	100	120
GFRP	40	45	50

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior (f, fu, Ef)

Tensile Properties of V-ROD GFRP bars of Grade I  
(ISIS Canada Manual No. 3)

Lowest

Metric size	Nominal diameter (mm)	Nominal Area (mm <sup>2</sup> )	Tensile modulus of elasticity (MPa)	Guaranteed tensile strength (MPa)
#3	10	71	42500 (6,164 ksi)	899
#4	13	129	44100	825
#5	15	199	42500	800
#6	20	284	44500	733
#8	25	510	43900	654 (95 ksi)

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior ( $f_u, E_f$ )

Tensile properties of V-ROD GFRP bars of **Grade II**  
(ISIS Canada Manual No. 3)

**Medium**

Metric size	Nominal diameter (mm)	Nominal Area (mm <sup>2</sup> )	Tensile modulus of elasticity (MPa)	Guaranteed tensile strength (MPa)
#3	10	71	52500	1200
#4	13	129	53400	1161
#5	15	199	53600	1005
#6	20	284	55400	930
#7	22	387	56600	882
#8	25	510	53500	811
#10	32	819	52900	776

19  
SYMPOSIUM

## Session 2: Design Assumptions and Materials

### Mechanical Properties and Behavior ( $f_u, E_f$ )

Tensile properties of V-ROD GFRP bars of **Grade III**  
(ISIS Canada Manual No. 3)

**Highest**

Metric size	Nominal diameter (mm)	Nominal Area (mm <sup>2</sup> )	Tensile modulus of elasticity (MPa)	Guaranteed tensile strength (MPa)
#3	10	71	65100	1734 (251 ksi)
#4	13	129	65600	1377
#5	15	199	62600	1239
#6	20	284	64700	1196
#7	22	387	62600	1005
#8	25	510	66400 (9,630 ksi)	1064
#10	32	819	65100	1105

20  
SYMPOSIUM

## Session 2: Design Assumptions and Materials

### Tensile Strength of the FRP at Bend ( $f_{fb}$ )

FRP bars can be fabricated with bends, however the tensile strength is reduced:

AASHTO-GS2, CSA S6 and CSA S806 Codes

$$f_{fb} = \left( 0.05 \left( \frac{r_b}{d_b} \right) + 0.3 \right) f_{fu} \leq f_{fu}$$

$r_b$  is the bend radius

$d_b$  is the diameter of reinforcing bar

$f_{fu}$  is the design tensile strength of FRP

FDOT 932-3 Table 3-3  
 $f_{fb} > 60\% f_{fu}$  (straight bars)

21  
SYMPOSIUM

## Session 2: Design Assumptions and Materials

### Compression Behavior of the FRP

FRP compression reinforcement is considered in AASHTO-GS2, CSA-S806 and CSA-S6 (New Edition, 2019)

For the purpose of design, assume zero compression strength and stiffness. (see GS2 - 2.6.4)

22  
SYMPOSIUM

## Session 2: Design Assumptions and Materials

### Bond Behavior

- Bond strength is a function of:
  - The bar design and surface roughness
  - Mechanical properties of the bar itself
- Bond Force can be transmitted by:
  - Adhesion resistance at bar interface (chemical bond)
  - Frictional resistance of interface (friction bond)
  - Mechanical interlock due to surface irregularity
- Adequate cover is essential
- $f'_c$  of the concrete affects bond.
- In general the bond of FRP bars to concrete is similar to the bond of steel bars (FDOT 932-3  $\geq 1.1$  ksi)

23  
SYMPOSIUM

## Session 2: Design Assumptions and Materials

### Time Dependent Behavior – Creep Rupture

- FRP reinforcing bars subjected to a sustained load over time can fail after a time period called the **endurance time**.
- This phenomenon is known as **creep rupture** (or **static fatigue**)
- The higher the stress, the shorter the lifetime
- GFRP → BFRP → AFRP → CFRP (least susceptible)

24  
SYMPOSIUM

## Session 2: Design Assumptions and Materials

### Time Dependent Behavior – Creep Rupture (Cc)

The maximum stress in FRP bars (or grids) under loads at serviceability limit state shall not exceed the following fraction of the guaranteed tensile strength **AASHTO-GS2 (CSA-S806 & CSA-S6)**:

- AFRP :  $n/a$  (0.35)
- CFRP :  $C_c \cdot 0.65$  (0.65)
- GFRP :  $C_c \cdot 0.30$  (0.25)

(Also for **CSA** - The maximum strain in GFRP tension reinforcement under sustained service loads shall not exceed 0.002).

## Session 2: Design Assumptions and Materials

### Durability Design

- One of the chief benefits of FRP bars
- FRP bars do not rust, but are susceptible in degrees to high pH (BFRP & GFRP) or moisture (AFRP)
- Depends on type of fiber, resin used, quality of manufacturing, degree of cure, etc.

## Session 2: Design Assumptions and Materials

### Durability Design

#### Example of Durability Related Provisions:

1. Limit on Constituent Material, e.g.
  - Limits on diluents and certain fillers (CSA-S807)
  - Limits on low-profile additives (CSA-S807)
  - No blended resins
2. Lower Limit on Glass Transition Temperature ( $T_g$ ) & Cure Ratio
  - Minimum cure ratio and  $T_g$
3. Material Screening Through Physical & Durability Properties
  - Maximum void content
  - Maximum water absorption
  - Limits on mechanical property loss in different environment conditioning (Alkali)

## Session 2: Design Assumptions and Materials

### Durability Design

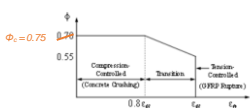
The **AASHTO-GS2, CSA-S806 and -S6** address the durability issue in design of FRP reinforced sections through a common way considering the following:

- The material resistance & environmental reduction factors based on fiber type and exposure conditions
- Limitation of maximum stress under service load
- Limitation of maximum crack-width under service load
- Limitation of maximum stress/strain level under sustained load
- Concrete cover (fire resistance, splitting, & bend development)
- Creep rupture stress limits
- Fatigue stress limits
- Factor for long-term deflection calculation

## Session 2: Design Assumptions and Materials

### Resistance factors (AASHTO-GS2, CSA S806)

- For non-prestressed FRP reinforcement, the resistance factor,  $\Phi_F$  shall be taken as  $\Phi_F = 0.75$  (compression-controlled only in **AASHTO-GS2**; For tension-controlled  $\Phi_F = 0.55$ )
- Concrete and steel resistance factors remain the same.



#### 2.5.5.2 – Resistance Factors

The resistance factor,  $\Phi$ , shall be taken as:

- For compression-controlled and tension-controlled reinforced concrete sections as specified in Article 2.6.3:
  - $0.55$  for  $\epsilon_s = \epsilon_{cu}$
  - $1.55 - \frac{\epsilon_s}{\epsilon_{cu}}$  for  $0.80\epsilon_{cu} < \epsilon_s < \epsilon_{cu}$  (2.5.5.2-1)
  - $0.75$  for  $\epsilon_s < 0.80\epsilon_{cu}$

## Session 2: Design Assumptions and Materials

### Material Resistance Factors

Application (CSA-S6)	Resistance Factor $\Phi_{FRP}$	Application (AASHTO)	Resistance Factor $C_e$
AFRP reinforcement in concrete and NSMR	0.65	CFRP tendons embedded (PC) (AASHTO-CFRP-GS)	1.00
AFRP in externally-bonded applications	0.55	CFRP tendons external (PT) (AASHTO-CFRP-GS)	0.90
AFRP and aramid fibre rope tendons for concrete and timber	0.60	GFRP reinforcement in concrete (interior) (AASHTO-GS2)	0.80
CFRP reinforcement in concrete and NSMR	0.80	GFRP [ & BFRP ] reinforcement in concrete (exterior) (AASHTO-GS2) [FDOT]	0.70
CFRP tendons	0.80		
GFRP reinforcement in concrete and NSMR	0.55		
GFRP in externally-bonded applications	0.70		
GFRP tendons for concrete components	0.55		
GFRP tendons for timber decks	0.70		

## Session 2: Design Assumptions and Materials

### Material Resistance Factors

Resistance factors from AASHTO-GS2		
Material	Notation	Factor
Concrete-cast-in-situ	$\phi_c$	0.75
Concrete-precast	$\phi_c$	0.75
Steel reinforcement	$\phi_s$	0.90
CFRP (PC)	$\phi_f$	(0.75)
AFRP	$\phi_f$	n/a
GFRP (& BFRP)	$\phi_f$	0.55

Resistance factors from CSA		
Material	Notation	Factor
Concrete-cast-in-situ	$\phi_c$	0.65
Concrete-precast	$\phi_c$	0.70
Steel reinforcement	$\phi_s$	0.85
CFRP	$\phi_f$	0.75
AFRP	$\phi_f$	0.75
GFRP (& BFRP?)	$\phi_f$	0.75

## Session 2: Design Assumptions and Materials

### Serviceability of FRP Reinforced Concrete Members (Beams & Slabs)

- Deflections under service loads often control design
- Designing FRP-RC beams or slabs for concrete crushing satisfies serviceability criteria for deflections and crack width
- Cracking and deflections are defined as:
  - Cracking — Excessive crack width is undesirable for aesthetic and other reasons (for example, to prevent water leakage) that can damage or deteriorate the structural concrete
  - Deflection — Deflections should be within acceptable limits imposed by the use of the structure (for example, supporting attached nonstructural elements without damage).
- The substitution of FRP for steel on an equal area basis would typically result in larger deflections and wider crack widths

## Session 2: Design Assumptions and Material Properties

### Crack control: (CSA-S806)

The crack control parameter,  $z$ :

$$z = k_b \frac{E_s}{E_f} f_t^3 d_c A$$

$z < 45\,000$  N/mm for interior exposure and  $38\,000$  N/mm for exterior exposure. (Sections satisfied the  $38\,000$  N/mm criterion)

$f_t < 0.25 f_{tu}$  for GFRP bars. (No  $f_t$  limit for AASHTO-GS2, see sustained load crack)

## Session 2: Design Assumptions and Material Properties

### Crack Control Reinforcement (CSA-S6, AASHTO-GS2 similar)

the maximum tensile strain in FRP reinforcement under service loads exceeds 0.0015, cross-sections of the component in maximum positive and negative moment regions shall be so proportioned that the crack-width must not exceed 0.5 mm (0.02-in.) for member subject to aggressive environments otherwise 0.7 mm (0.028-in.), where the crack width is given by:

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

- The value of  $k_b$  shall be determined experimentally, but in the absence of test data it may be taken as:
  - CSA 0.8 for sand-coated and 1.0 for deformed FRP bars.
  - AASHTO-GS2  $k_b = 1/CS$  1.2 for deformed FRP bars.
- For CSA in calculating  $d_c$ , the clear cover shall not be taken greater than (50 mm).

## Crack Control Reinforcement (CSA S6)

- Check crack widths when tensile strain in FRP at SLS exceeds 0.0015 (stress of 60 MPa in Grade 1) which is almost always the case.
- Maximum permitted crack widths:
  - $w_{cr} \leq 0.50$  mm (0.020-in.) for members subject to aggressive environments
  - $w_{cr} \leq 0.70$  mm (0.028-in.) for members with other exposures
- Maximum permitted crack widths are double what is permitted for reinforcing steel in aggressive environments since GFRP does not corrode.
- Crack width derived from an analytical model:

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

## Crack Control Reinforcement (CSA S6)

Maximum crack width to be checked against the limit

Factor to calculate the maximum crack width (1.5 for mean width, 1 for minimum width)

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

$f_{FRP} / E_{FRP} = \epsilon_{FRP}$  is the average strain in FRP reinforcement

Term  $k_b$  to account for the bond of the bar to concrete:

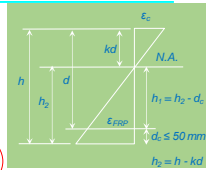
- use  $k_b = 0.8$  for both sand-coated and 1.0 for deformed bars
- AASHTO-GS2 1.2 for all bars

### Crack Control Reinforcement

Geometrical relationship which accounts for the amplification of the average strain from the FRP bar to the exposed surface of the concrete

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} \left( d_c^2 + \left( \frac{s}{2} \right)^2 \right)$$

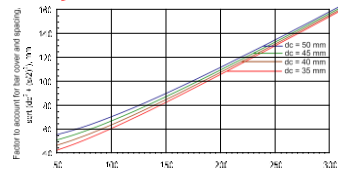
Effect of the bar spacing and bar cover on the basic crack width



s is the bar spacing  
d<sub>c</sub> is the distance from the centroid of tension FRP to the extreme tension surface of the concrete ≤ 50 mm

### Effect of Bar Spacing and Cover on Crack-Width, W<sub>cr</sub>

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \left( d_c^2 + \left( \frac{s}{2} \right)^2 \right)$$



### Session 2: Design Assumptions and Material Properties

#### Deflection Limits of FRP Reinforced Concrete Members (CSA S806)

Table 11

Type of member	Deflection to be considered	Deflection limitation
For full live loading as specified in the design code. Bars to be designed for large deflections.	Immediate deflection due to specified live load L.	L/1600
None for supporting or bracing structural members. Bars to be designed for large deflections.	Immediate deflection due to specified live load L.	L/200
Rest of floor construction supporting or attached to structural members. Bars to be designed for large deflections.	Total deflection due to specified live load L and permanent deflection due to all additional loads.	L/4000
Rest of floor construction supporting or attached to structural members. Bars to be designed for large deflections.	Total deflection due to specified live load L and permanent deflection due to all additional loads.	L/2000

Table 11 Maximum Permissible Computed Deflections (See Clause 6.2.2.1.)

### Session 2: Design Assumptions and Material Properties

#### Deflection Calculation (CSA-S806 - Refined)

- Deflection shall be calculated based on moment-curvature (M/EI) relationship
- Integrate M/EI relationship or use moment-area method

$$\delta_A = \int_0^L \frac{mM}{EI} dx$$

### Session 2: Design Assumptions and Material Properties

#### Deflection Calculation (AASHTO-GS2, CSA S806)

- Effective moment of inertia (used for Direct Method):
  - When a section is uncracked, its moment of inertia is equal to the gross moment of inertia, I<sub>g</sub>
  - When the applied moment, M<sub>a</sub>, exceeds the cracking moment, M<sub>cr</sub>, cracking occurs, which causes a reduction in the stiffness and the moment of inertia is based on the cracked section, I<sub>c</sub>
  - I<sub>e</sub> = bh<sup>3</sup>/12 as before
- Using n<sub>s</sub> as the modular ratio between the FRP reinforcement and the concrete

$$n_s = E_{FRP} / E_c \quad \text{Where: } I_{cr} = \frac{bd^3}{3} - k^3 + n_f A_f d^2 (1-k)^2$$

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f$$

### Session 2: Design Assumptions and Material Properties

#### Long-Term Deflection

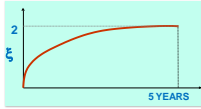
#### Long-term deflection under sustained load

The total immediate plus long-term deflection for flexural members shall be obtained by multiplying the immediate deflections caused by the sustained load considered by a creep factor. For CSA → [1+S], AASHTO-GS2 → 3 for I<sub>cr</sub> or → 4 for I<sub>e</sub>.

- S = 2.0 for 5 years or more
- S = 1.5 for 12 months
- S = 1.3 for 6 months
- S = 1.1 for 3 months

## Session 2: Design Assumptions and Material Properties

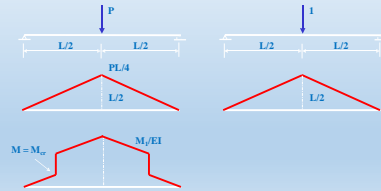
### Long-Term Deflection (CSA)



For same design strength long term deflection is 3 – 4 times greater than members reinforced by steel reinforcement

## Session 2: Design Assumptions and Material Properties

### Deflection Calculation



## Session 2: Design Assumptions and Material Properties

### Deflection Calculation

- Moment-Curvature Relationship for FRP Reinforced Section



## Session 2: Design Assumptions and Material Properties

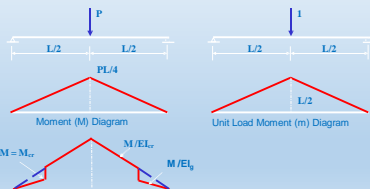
### Deflection Calculation

- Example



## Session 2: Design Assumptions and Material Properties

### Deflection Calculation



## Session 2: Design Assumptions and Material Properties

### Deflection Calculation Maximum Deflection

$$\delta_{max} = \delta_{cr} - \delta$$

Deflection of the fully cracked beam

Correction from the uncracked sections

From Regular Strength of Materials

$$\delta_{cr} = \frac{PL^3}{48E_c I_{cr}}$$



## Session 2: Design Assumptions and Material Properties

Deflection Calculation  
Correction Term

$$\delta_c = 2 \int_0^{L/2} m \eta \frac{M}{EI} dx \quad \text{or} \quad \delta_c = \frac{PL^3}{48EI_{cr}} 8\eta \left(\frac{L}{L}\right)^3$$

## Session 2: Design Assumptions and Material Properties

Deflection Calculation  
Maximum Deflection

$$\delta_{max} = \frac{PL^3}{48EI_c} \left[ 1 - 8\eta \left(\frac{L_c}{L}\right)^3 \right]$$

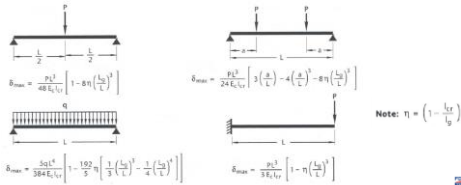
Where:

$$\eta = \left( 1 - \frac{L_c}{L} \right)$$

## Session 2: Design Assumptions and Material Properties

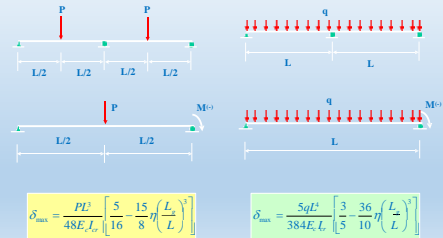
Deflection Calculation

Maximum Deflection of Other Load Cases



## Session 2: Design Assumptions and Material

Deflection of Continuous Beams



## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Development length of FRP bars in tension  $l_d$  shall be taken:

$$l_d = 1.15 \frac{k_1 k_2 k_3 k_4 k_5}{d_{cs}} \frac{f_f}{\sqrt{f'_c}} A_b \geq 300 \text{ mm}$$

- $k_1$  = bar location factor
- $k_2$  = concrete density factor
- $k_3$  = bar size factor
- $k_4$  = bar fibre factor
- $k_5$  = bar surface profile factor
- $d_{cs}$  = smaller of:
  1. distance from closest concrete surface to the center of the bar
  2. two-thirds of the center-to-center spacing  $d_{cs}$
 shall not be greater than  $2.5 d_b$

## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Development length of FRP bars in tension

The maximum permissible value of  $(f'_c)^{0.5}$  shall be 5 MPa

## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806, AASHTO-GS2)

### Modification factors:

- $k_1$  (Bar location factor) :1.3; 1.0 (1.5; 1.0)
- $k_2$  (Concrete density factor) :1.3; 1.2; 1.0
- $k_3$  (Bar size factor) :0.8; 1.0
- $k_4$  (Bar fibre factor) :1.0; 1.25
- $k_s$  (Bar surface profile) :1.0; 1.05; 1.80

55  
SYMPOSIUM

## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

### Development of bent bar:

$$165k_3 \frac{d_{bs}}{\sqrt{f_c}} \text{ for } f_y \leq 520 \text{ MPa}$$

$$\frac{f_y}{3.1} k_3 \frac{d_{bs}}{\sqrt{f_c}} \text{ for } 520 \leq f_y \leq 1040 \text{ MPa}$$

$$330k_3 \frac{d_{bs}}{\sqrt{f_c}} \text{ for } f_y > 1040 \text{ MPa}$$

- $L_d$  not less than  $12d_b$  or 230 mm
- The tail length of a bent bar,  $l_t$ , should not be less than  $12d_b$
- The bend radius,  $r_b$ , should not be less than  $3d_b$

56  
SYMPOSIUM

## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA-S806)

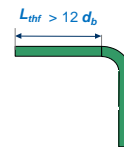
### Anchorage of shear reinforcement:

- Web reinforcement shall be carried as close to the compression and tension surfaces of a member as practically feasible. (Clause 9.9.1)
- Unless it is determined that the shear reinforcement can develop its design strength at mid-height of the beam or column cross-section, FRP web reinforcement shall consist of closed loops or spiral reinforcement.
- The web reinforcement shall have sufficient development length to develop its design stress at mid-height of the member.

57  
SYMPOSIUM

## Session 2: Design Assumptions and Material Properties

### Detailing of shear stirrups



$L_{tnf}$  = length of tail beyond a hook

58  
SYMPOSIUM

## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

### Splices of reinforcement:

- The lap splice length shall be  $1.3l_d$ , where  $l_d$  is the basic development length of the bar (Clause 9.10.3; GS-2.9.7.6);
- Lap splices of bundled bars shall be based on the lap splice length required for individual bars within a bundle, increased by 20% for a two-bar bundle and 30% for a three-bar bundle. Individual bar splices within a bundle shall not overlap (Clause 9.10.4; Not recommended);
- Spliced bars in flexural members shall have a transverse spacing not exceeding the lesser of one-fifth of the required lap splice length or 130mm (Clause 9.10.5; GS-2.9.7.6).

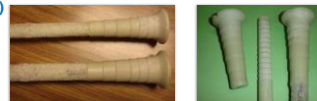
59  
SYMPOSIUM

## Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

### Mechanical anchorage— Clause:

- Mechanical anchorage including headed bars or headed studs may be used, provided their effectiveness has been demonstrated by tests that closely simulate the condition in the field and that they can develop at least 1.67 times the required design strength. AASHTO-GS2 2.9.7.5 – 1.25  $f_u$



60  
SYMPOSIUM

## Session 2: Design Assumptions and Material Properties

### Development Length and Splice of Reinforcement (CSA-S6)

The development length,  $l_d$ , of FRP bars in tension shall be calculated from:

$$l_d = 0.45 \frac{k_1 k_2 \left( \frac{f_{FRP}}{f_r} \right) A}{\left( l_a + K_{tr} \frac{E_{FRP}}{E_s} \right) \left( \frac{f_{FRP}}{f_r} \right) A}$$

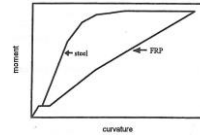
The splice length for FRP bars in tension shall be  $1.3l_d$ .

Spliced FRP bars shall not be separated by more than 150 mm.

61  
SYMPOSIUM

## Deformability (CSA-S6)

- Deformability takes into account absorbed energy based on deformability, to ensure adequate deformation of members reinforced with FRP.
- The purpose of calculating deformability in an FRP reinforced section is to provide a comparable deformability as expected of a comparably reinforced steel reinforced section.



(Jaeger et al. 1997)

62  
SYMPOSIUM

## Deformability (CSA-S6, Not in AASHTO-GS2)

- Overall performance factor,  $J$ , must be at least 4.0 for rectangular sections and 6.0 for T-sections.

$$J = \frac{M_{ult} \Psi_{ult}}{M_c \Psi_c}$$

- $M_{ult}$  and  $\Psi_{ult}$  are moment and curvature at ultimate limit state.
  - $M_{ult}$  = moment at ultimate limit state
  - $\Psi_{ult} = \epsilon_{ult} / kd$
- $M_c$  and  $\Psi_c$  are moment and curvature corresponding to a concrete strain of 0.001.
  - $M_c = f_c k (1-k/3) bd^2$
  - $\Psi_c = \epsilon_c / kd$ , where  $\epsilon_c = 0.001$
- Use  $f_c = \epsilon_c E_c$  or  $f_c = 1.8 f_c' (\epsilon_c / \epsilon_c') / (1 + (\epsilon_c / \epsilon_c')^2)$

63  
SYMPOSIUM

## Deformability (CSA-S6)

- For calculating  $M_{ult}$  and  $\Psi_{ult}$ , repeat the same steps as required to calculate  $M_r$ , but with higher resistance factors.
- For calculating  $M_{ult}$  and  $\Psi_{ult}$  use the following:
  - $\Phi_c = 1.00$
  - $\Phi_{FRP} = 1.00$
- Based on the definition of  $J$  as a function of  $M_c$ , tension-controlled members may not have adequate deformability.
- Deformability may govern the design of deep members or T-beam members (i.e. pier caps or diaphragms).

64  
SYMPOSIUM

## How Does GFRP Compare with Steel?

- For typical reinforcement ratios found in a bridge deck slab, factored moment resistance at ULS with GFRP and reinforcing steel is similar (within 30%).
- If the member is subjected to SLS moment:
  - SLS will govern the design of a member with GFRP reinforcement. ULS usually governs the design of a member reinforced with steel.
  - On average, a GFRP design will require 50% to 100% more reinforcement as a design with reinforcing steel.
  - Use the smallest practical bar diameter and bar spacing for an efficient design with GFRP (less of a consideration for design with reinforcing steel).
  - Avoid bar spacing of less than 100 mm to avoid congestion of bars.

65  
SYMPOSIUM

## Questions

### Co-presenters:

**Raphael Kampmann PhD**  
FAMU-FSU College of Engineering  
Tallahassee, FL  
[kampmann@eng.famu.fsu.edu](mailto:kampmann@eng.famu.fsu.edu)

**Marco Rossini, PhD student**  
University of Miami.  
Coral Gables, FL.  
[mrx1465@miami.edu](mailto:mrx1465@miami.edu)

### FDOT Design Contacts:

**Steven Nolan, P.E.**  
FDOT State Structures Design Office,  
Tallahassee, FL.  
[Steven.Nolan@dot.state.fl.us](mailto:Steven.Nolan@dot.state.fl.us)

### FDOT Materials and manufacturing:

**Chase Knight, Ph.D, P.E.**  
State Materials Office,  
Gainesville, FL.  
[Chase.Knight@dot.state.fl.us](mailto:Chase.Knight@dot.state.fl.us)

66  
SYMPOSIUM