Determination of in-situ rock density and strength with SH-Love wave tomography

FDOT BED31-977-12

GRIP Meeting

August 14-15th, 2025

Rock Density and Strength with SH-Love Wave Tomography

Project Manager:
Kelly Shishlova, P.E.

Principal Investigators:
Khiem Tran, Ph.D.
Michael McVay, Ph.D.

Graduate Assistants
Nhat Tran, PhD student
Kunyu Yang, Ph.D.

University of Florida

Presentation outline

- Project benefits
- Implementation
- Introduction and project background
- Project objectives
- Task outline and deliverables
- Summary of research conclusion
- Recommendation
- Future research
- Publication

Project qualitative benefits

- Florida has significant soil/rock uncertainty (layering & properties), weathered conditions (soil & rock interleaved). Core sampling is limited by invasive nature and restricted spatial coverage. This often leads to gaps in the data, compromising the foundation design.

- New SH-Love seismic method allows high-resolution characterization over large volumes. Soil/rock properties (density and moduli) can be characterized at foot pixels to large depths (> 40 ft).

Project quantitative benefits

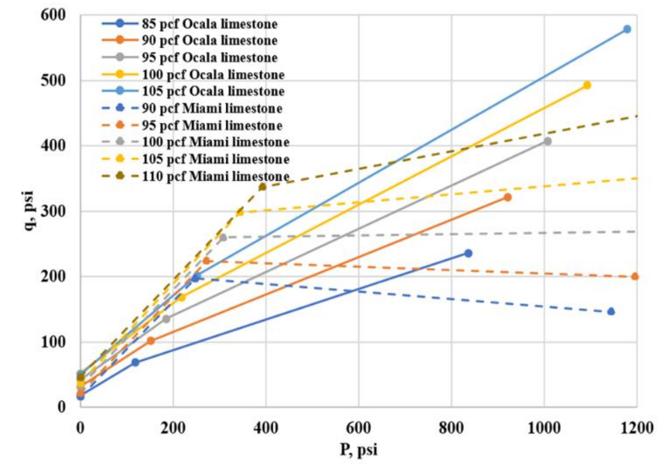
- The new SH-Love wave seismic method significantly reduces soil/rock uncertainty (layering, properties, anomalies), and thus reduces cost in the design, construction of shallow foundations.
- The method reduces the number of borings required for shallow foundation design.

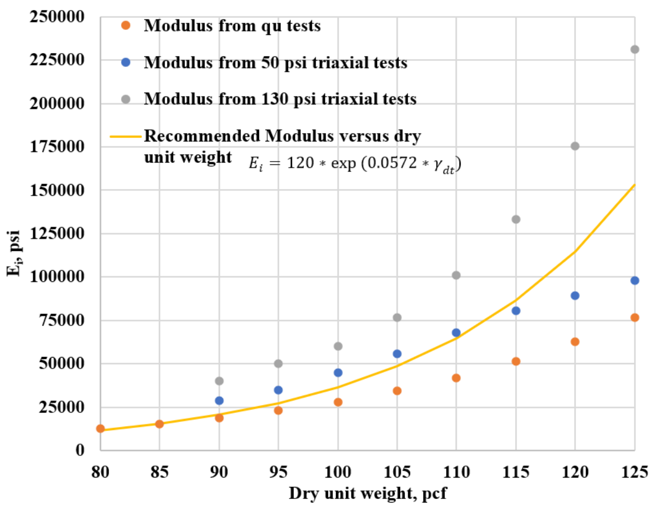
Implementation

- GUI software for SH-Love FWI method has been developed and transferred to FDOT for future use.
- Conduct seismic shear testing (horizontal source and geophones)
- Perform analysis of shear wave seismic data
  - Density, S-wave velocity, moduli in 2D domain
  - Identification of layering, anomalies, etc.
- Use shear and Young moduli for shallow foundation design (bearing and settlement)

Introduction and background

- Design and construction of shallow foundations rely heavily on accurate subsurface information, particularly regarding rock density and variability
- Recent FDOT projects have shown that mass density (or unit weight) controls the rock strength as well as its stress-strain behavior for most Florida limestone formations.
- Current practice for estimating mass properties is to measure intact sample properties (density, modulus) and multiply them by estimated recoveries. However, this often leads to gaps in the data (e.g., borings far from footing, layering), potentially compromising the design (e.g., missed layer), especially in heterogeneous Florida limestone formations.
- This project is to develop a new method for determination of rock density and modulus over a large volume on a foot scale without requirement of borings.





Project objectives

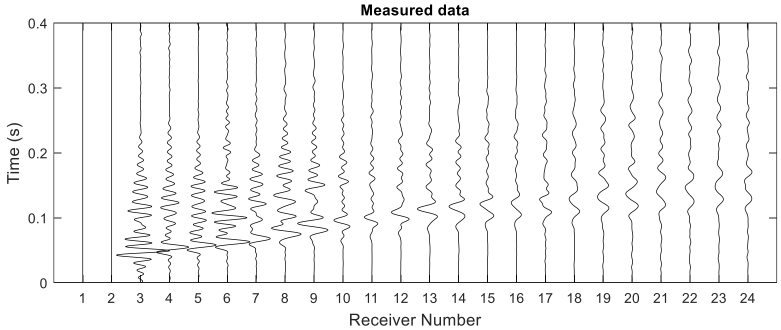
- The project objective is to develop an advanced testing system (hardware and software) for determination of rock density and moduli.
- The system enables to provide both density and moduli at relevant resolutions (foot pixels) for entire rock volume supporting foundations without requirement of borings.
- The GUI module is transferred to FDOT for future uses in site investigations of soil/rock properties and stratigraphy.

Research approach

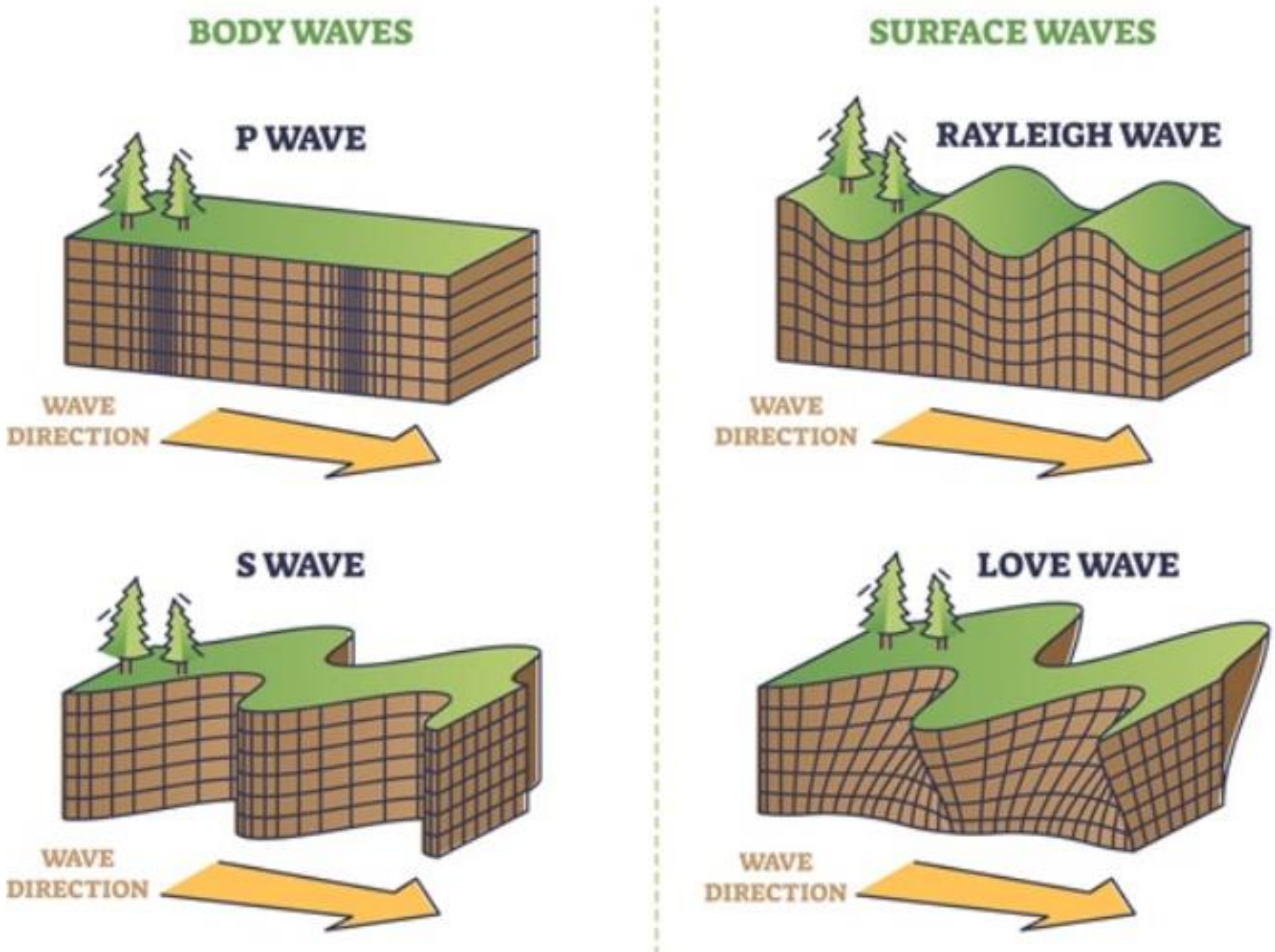
- The research is to develop an SH-Love full waveform inversion (2D SH-Love FWI) method, which can characterize 2D density and moduli of subsurface soil/rock at foot-pixels down to 30-60 ft depth

- Horizontal shear (SH) and Love waves are generated by applying a horizontal source and recorded by an array of horizontal geophones on the ground surface.

- Recorded waveform data are analyzed to extract density, S-wave velocity, moduli of subsurface materials.



Seismic wave types



VectorMine (Dreamstime.com)

Benefits of using SH and Love waves

- SH and Love waves (horizontal source) are more sensitive to material density than vertical S-wave, P-wave, and Rayleigh waves (PSV) (vertical source), and thus the density could be extracted more accurately.

- SH-Love wave simulation requires less computing time (40% that of PSV waves), and the 2D SH-Love FWI can be done quickly (15 minutes) in the field.

- Both density (ρ) and S-wave velocity (Vs) can be characterized. Shear (G) and Young (E) moduli can be computed for determination of shallow foundation's settlement and bearing capacity, and other geotechnical analyses.

$$G = \rho V_s^2$$

$$E = 2G(1 + v)$$

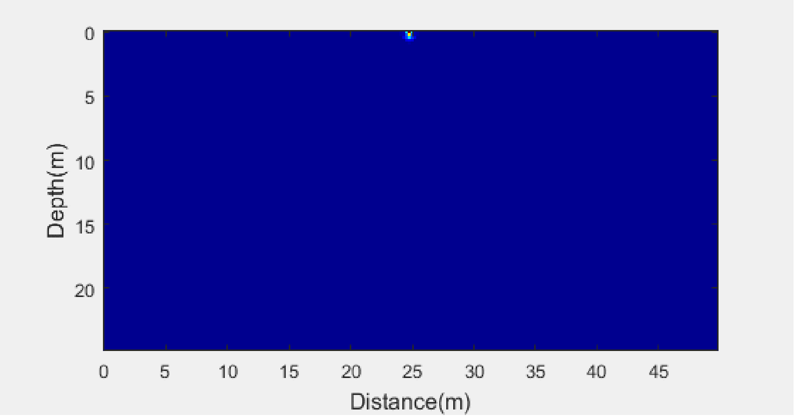
Task 1: Develop 2D SH-Love FWI algorithm

- Forward modelling

$$\rho(x,z)\frac{\partial v_y}{\partial t} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} + f_y \qquad (1)$$

$$\frac{\partial \sigma_{xy}}{\partial t} = \mu(x,z)\frac{\partial v_y}{\partial x} \qquad (2)$$

$$\frac{\partial \sigma_{yz}}{\partial t} = \mu(x,z)\frac{\partial v_y}{\partial z} \qquad (3)$$



ρ: density, $\mu = G = \rho V_s^2$

- Perfectly matched layers (PML) are applied at the bottom and vertical boundaries to absorb outgoing waves.

2D SH-Love FWI

- Model update (adjoint-state method)
  - Step 1. Calculate the residual between estimated data and observe data

$$\Delta \boldsymbol{d}_{s,r} = \boldsymbol{D}_{s,r}(\mathbf{m}) - \boldsymbol{d}_{s,r} \qquad (4)$$

  - Step 2. Compute the Least-squares error

$$\mathrm{E}(\mathbf{m}) = \frac{1}{2}\Delta \boldsymbol{d}^T \Delta \boldsymbol{d}, \qquad (5)$$

  - Step 3. Compute the gradients (adjoint-state gradient approach)

$$\frac{\partial E}{\partial V_s} = -\frac{2}{V_s^3 \rho}\sum_{i=1}^{N}\int_0^T dt(\sigma_{xy}^f \sigma_{xy}^b + \sigma_{yz}^f \sigma_{yz}^b), \qquad (6)$$

$$\frac{\partial E}{\partial \rho} = -\frac{1}{V_s^2 \rho^2}\sum_{i=1}^{N}\int_0^T dt(\sigma_{xy}^f \sigma_{xy}^b + \sigma_{yz}^f \sigma_{yz}^b + V_s^2 \rho^2 \frac{\partial v_y}{\partial t} u_y^b), \qquad (7)$$

2D SH-Love FWI

- Model update
  - Step 4. Smooth gradients (Tikhonov regularization) to minimize the ill-posed problem

$$\left(\frac{\partial E}{\partial V_s}\right)_r = \frac{\partial E}{\partial V_s} + \lambda_1 \boldsymbol{D} V_s \qquad (8)$$

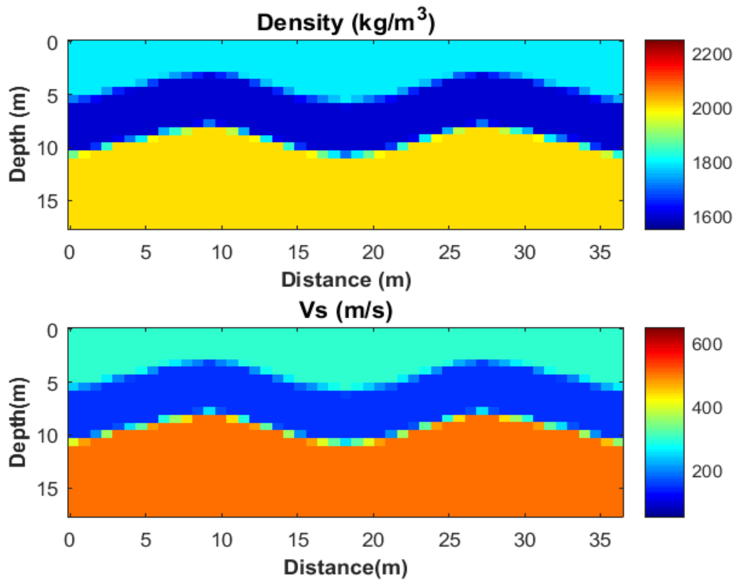
$$\left(\frac{\partial E}{\partial \rho}\right)_r = \frac{\partial E}{\partial \rho} + \lambda_2 \boldsymbol{D} \rho \qquad (9)$$

  - Step 5. update Vs and density models (steepest-descent method)

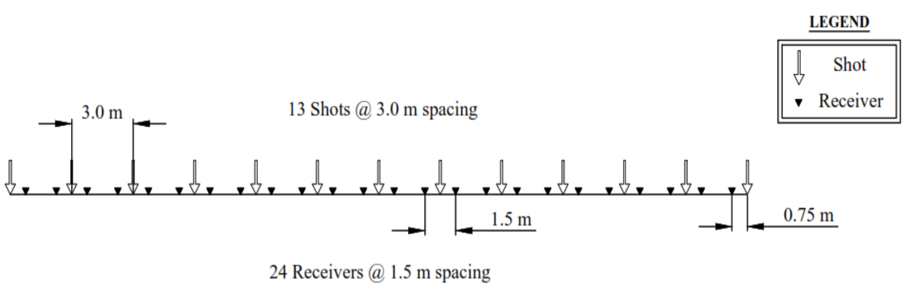
$$V_{s_{n+1}} = V_{s_n} - \alpha_n \boldsymbol{H}_n^{-1} \left(\frac{\partial E}{\partial V_s}\right)_r , \qquad (10)$$

$$\rho_{n+1} = \rho_n - \beta_n \boldsymbol{H}_n^{-1} \left(\frac{\partial E}{\partial \rho}\right)_r , \qquad (11)$$

Synthetic experiment

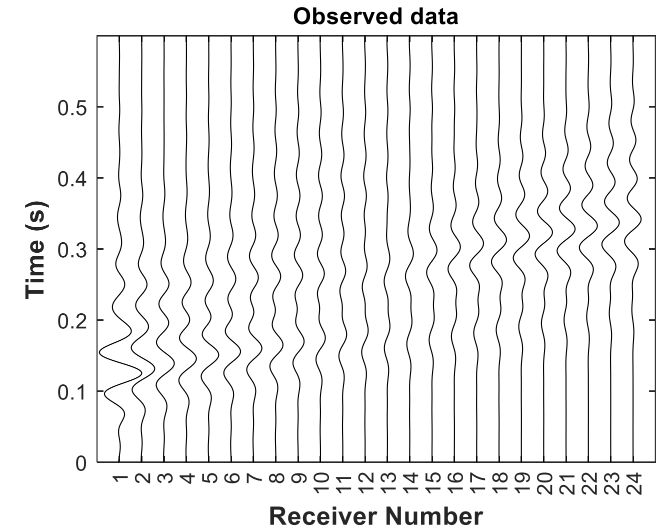


true model

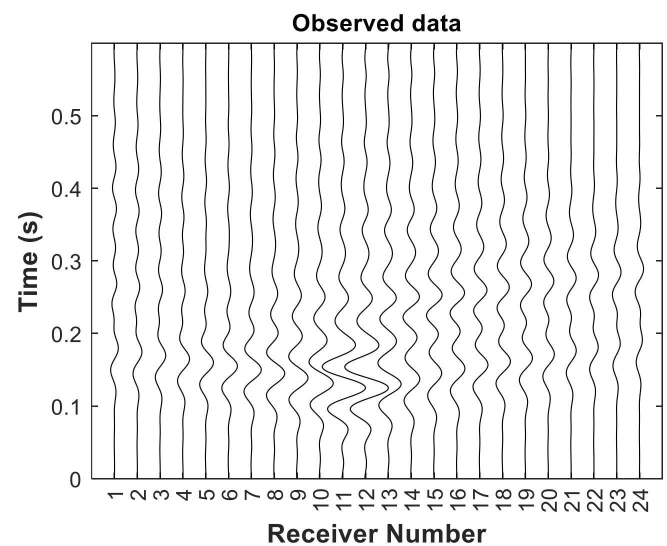


Test configuration

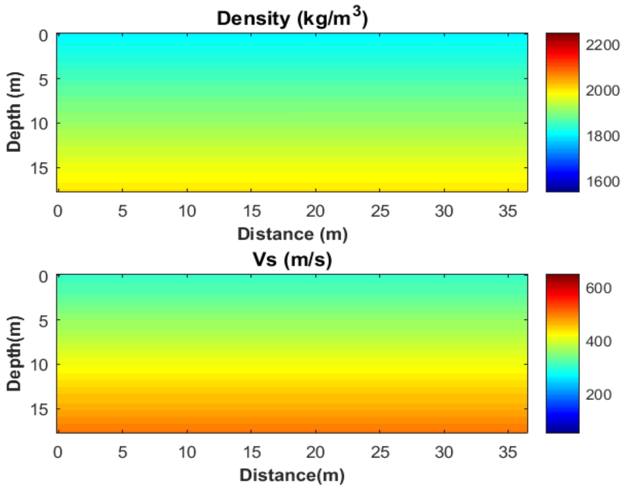
Shot 1



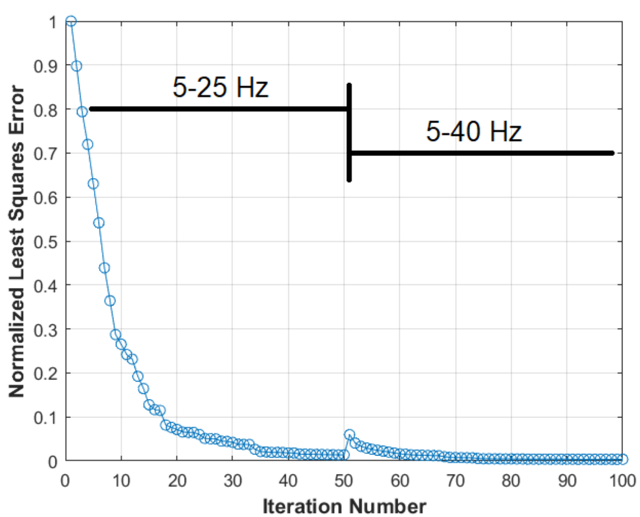
Shot 7



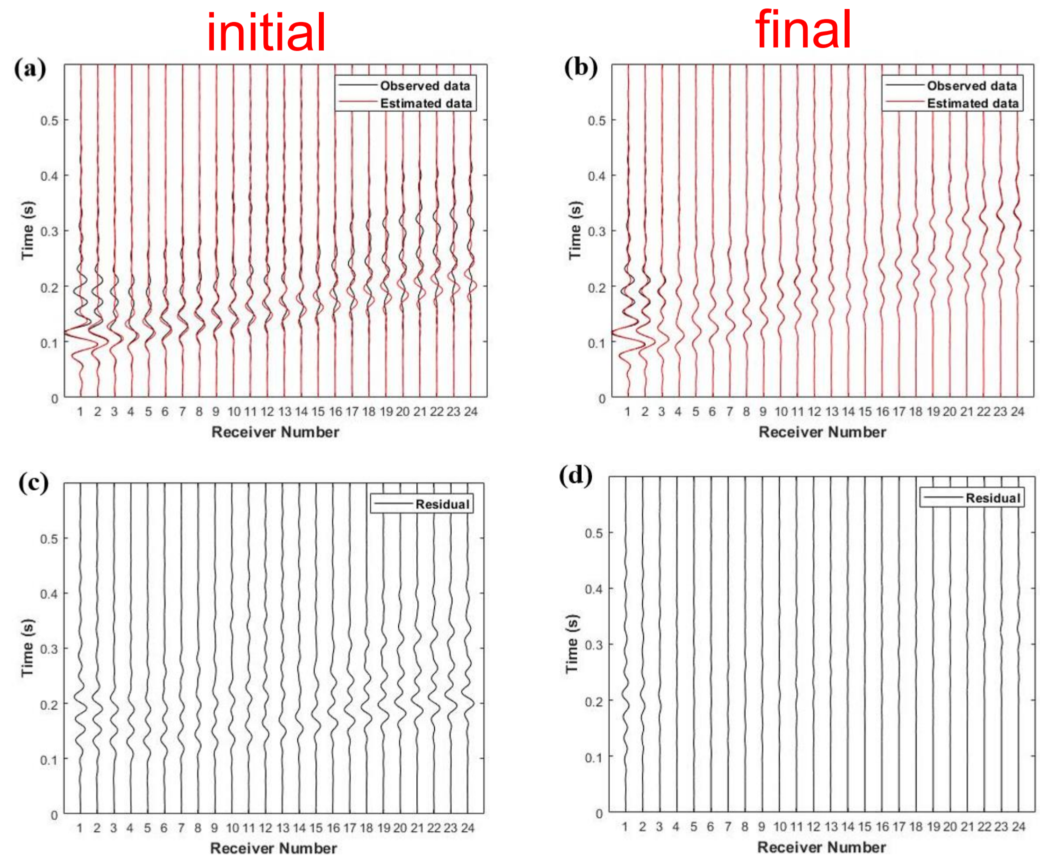
Synthetic experiment



initial model

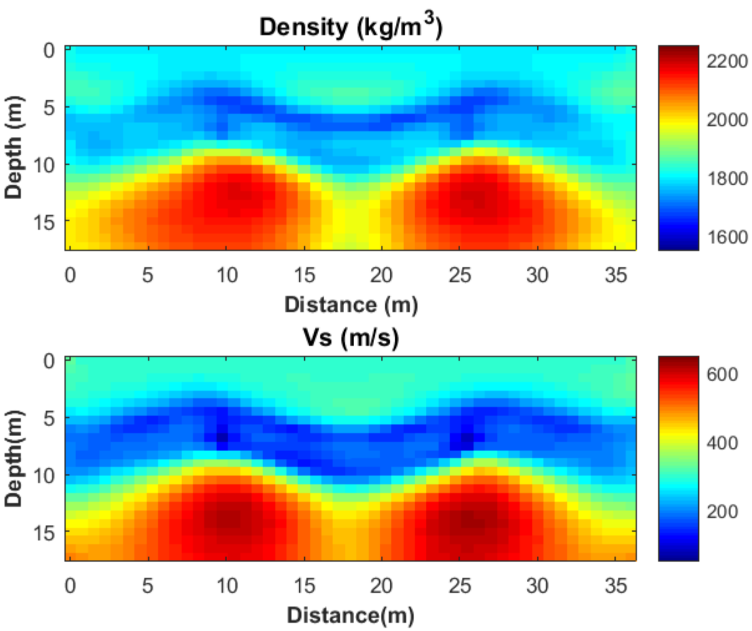


Least-square error

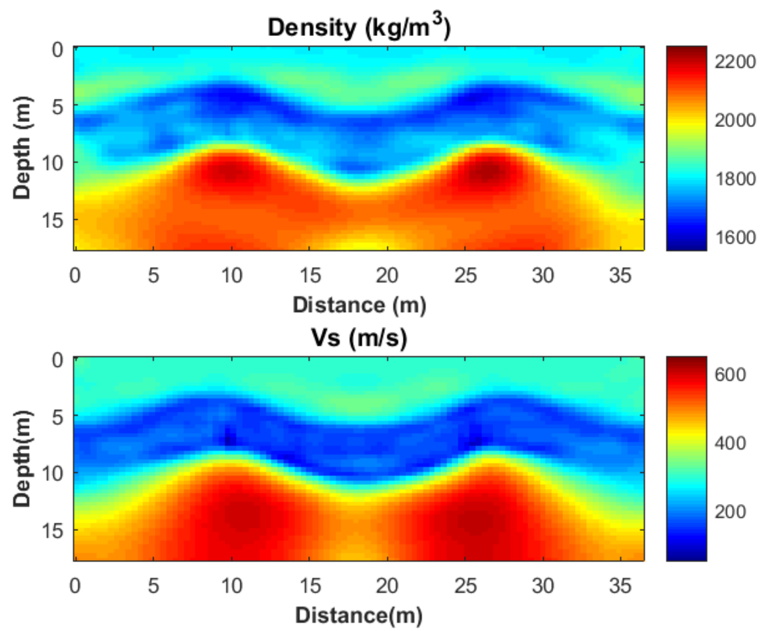


Waveform comparison

Synthetic results



First analysis at 5-25 Hz



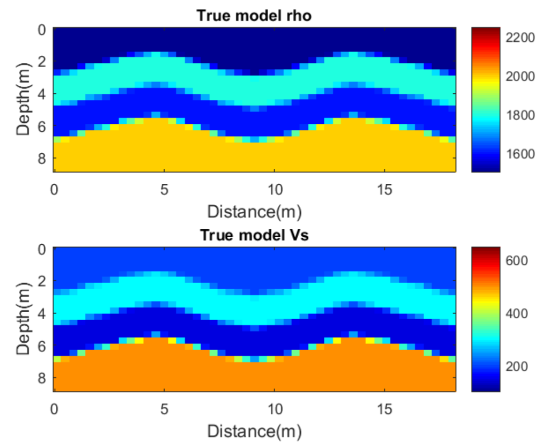
Second analysis at 5-40 Hz

Task 2: Optimize test configurations and wavefield characteristics

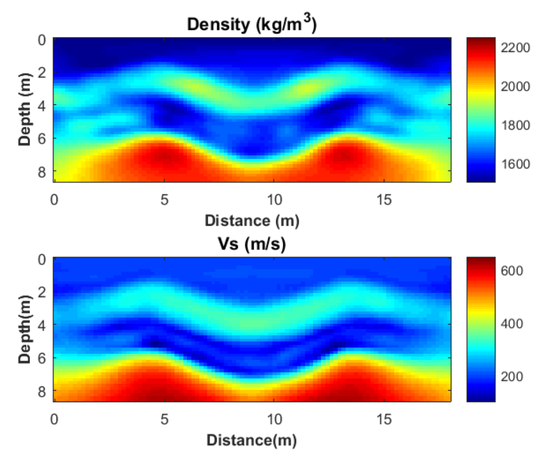
- Characterized depth = ½ test length
- Resolution is max (¼ receiver spacing, ¼ wavelength)

- 60 ft depth: test length 120 ft, geophone spacing of 5 ft and source spacing of 10 ft, data up to 40 Hz (15" pixel)

- 30 ft depth: test length 60 ft, geophone spacing of 2.5 ft and source spacing of 5 ft, data up to 80 Hz (7.5" pixel)



True



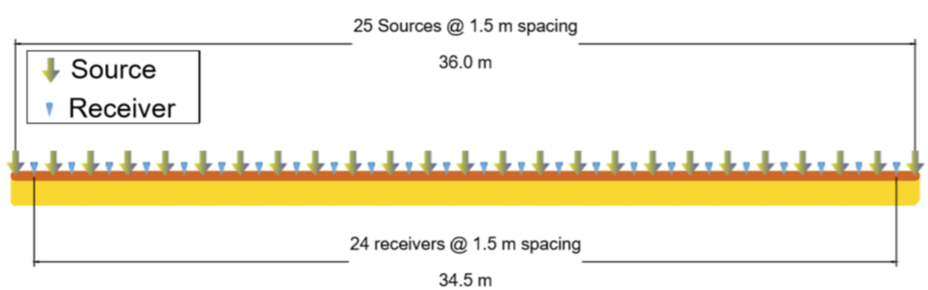
Inverted

Task 3: Verify the 2D SH-Love FWI algorithm with field experiments

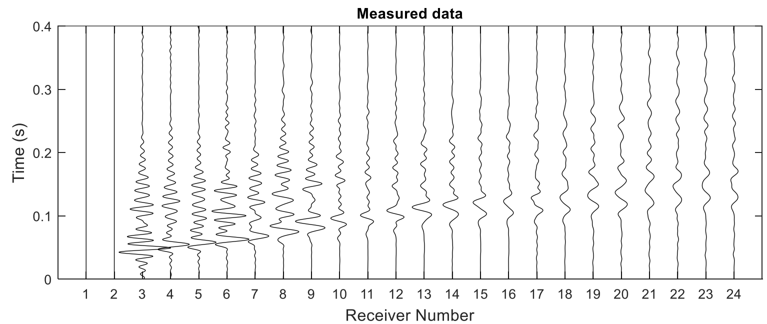
Site 1: Bell site (3rd Shallow Foundation Load Test, BDV31-977-82)

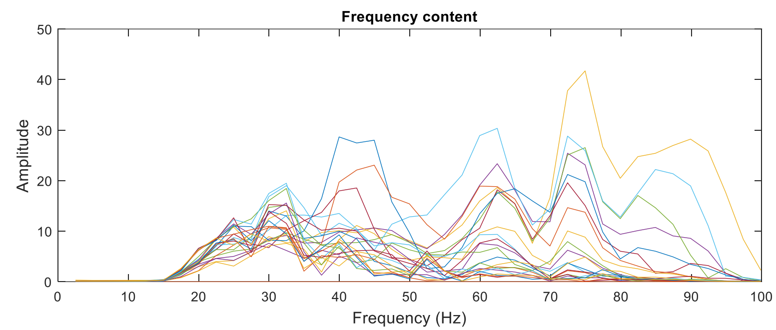
5’ I-beam

Two test lines of 120 ft, 24 geophones at 5 ft, 25 source at 5 ft

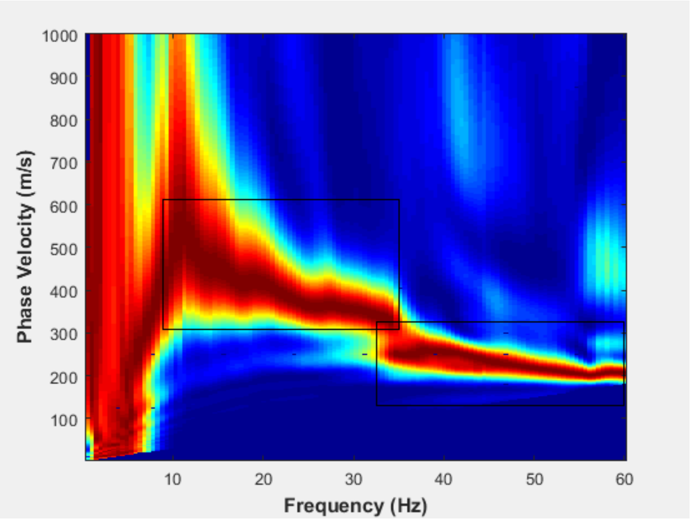


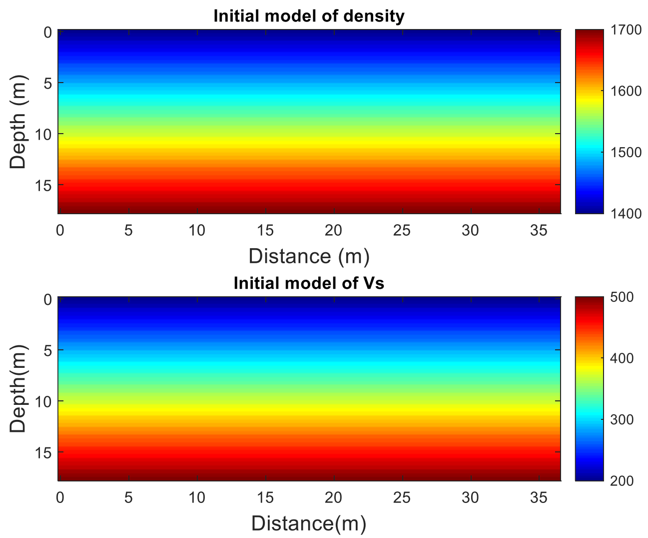
Bell site





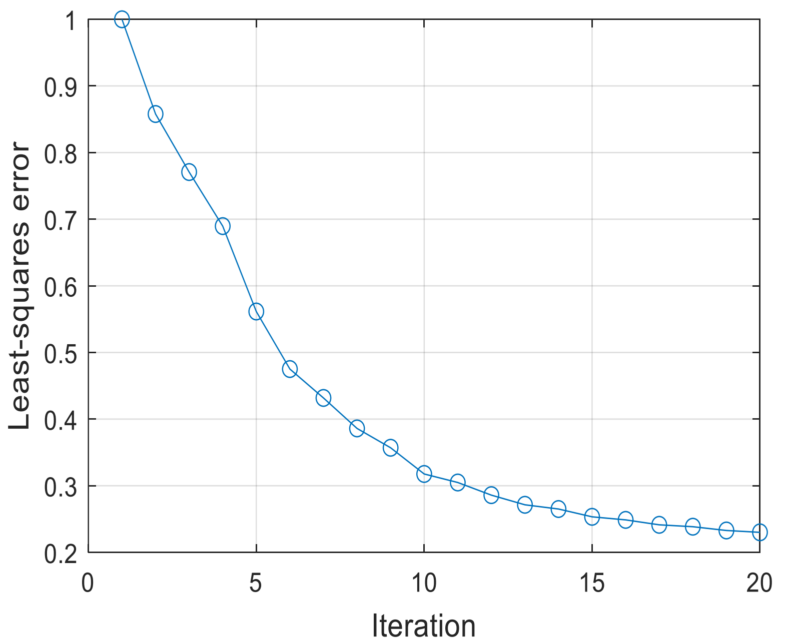
Sample data

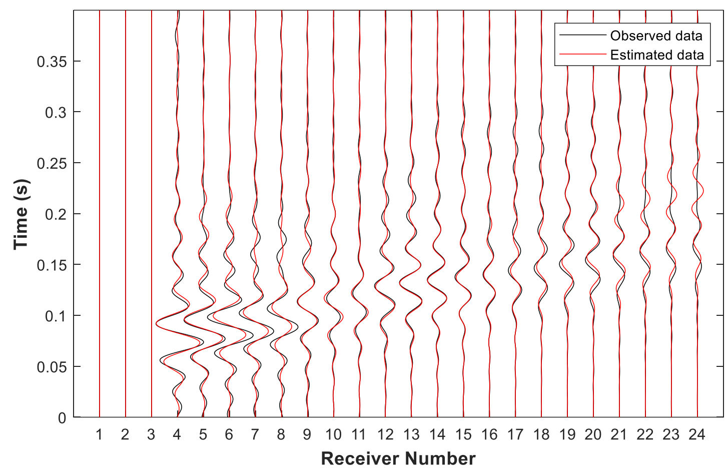


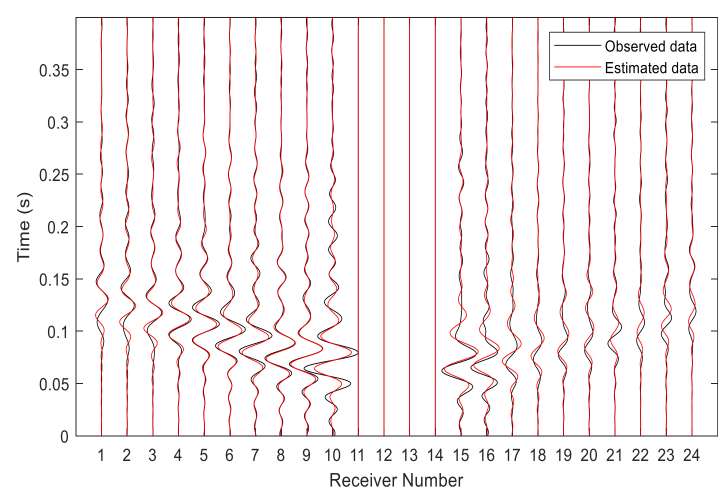


Bell site: data analysis

15 minutes on a standard computer (8-core CPU of 3.70 GHz, 64 GB of RAM)

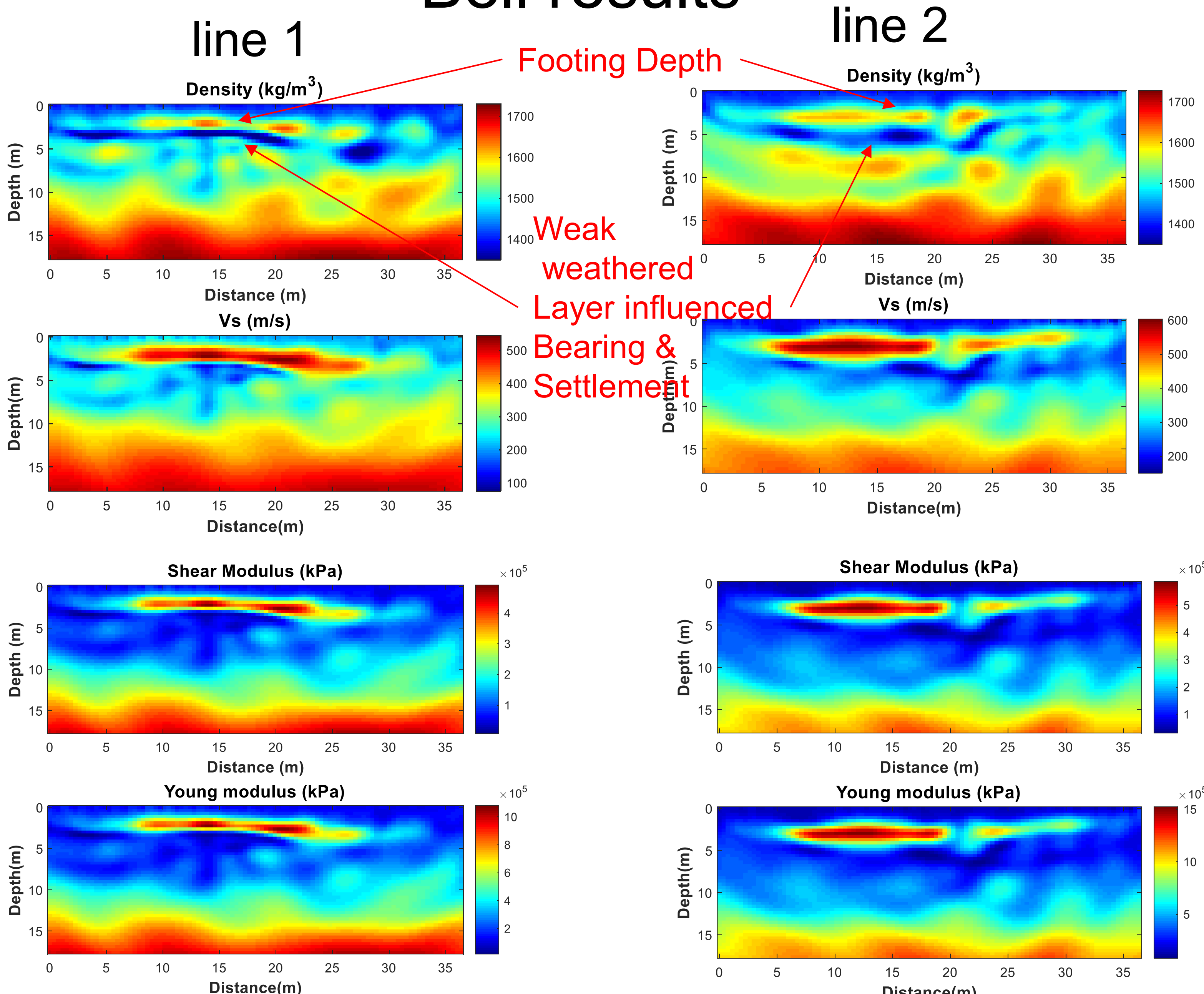




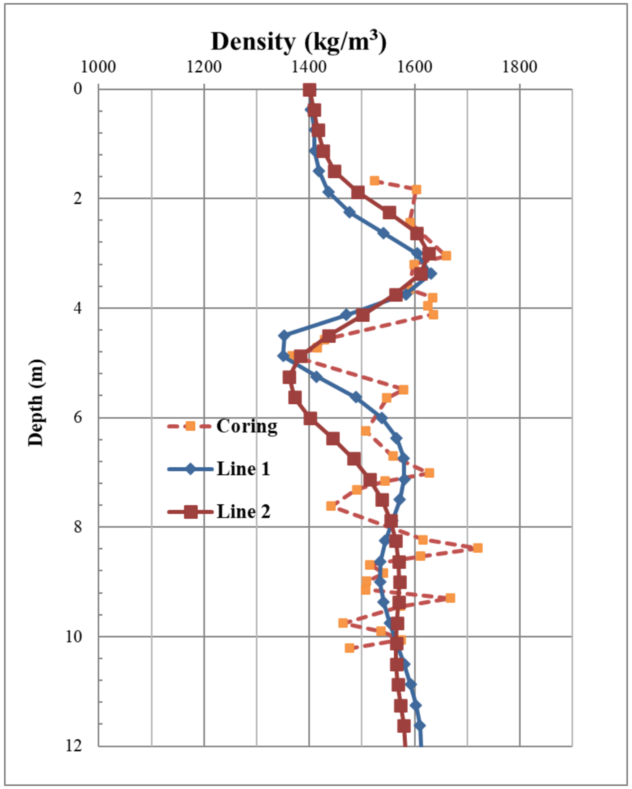


Waveform comparison 10-60 Hz

Bell results



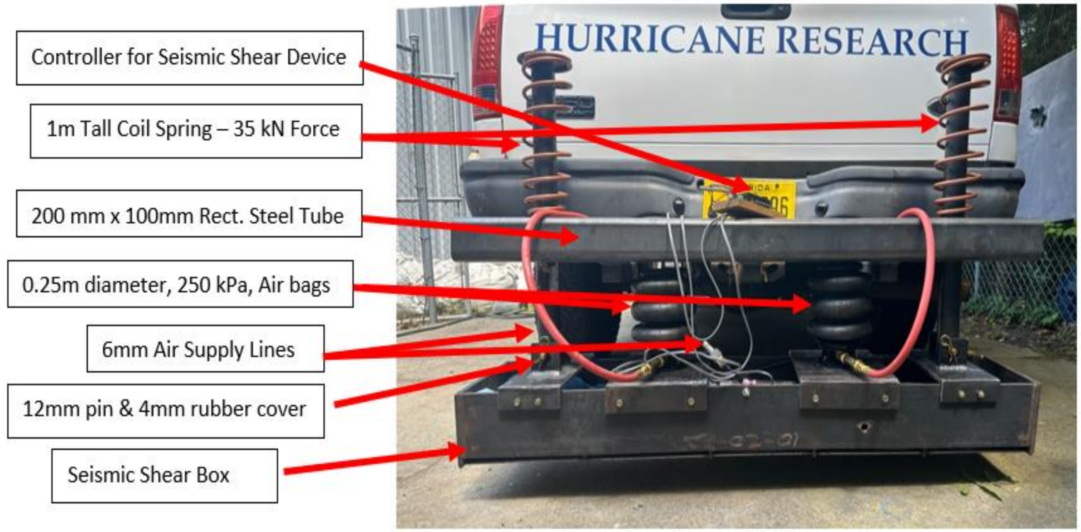
Bell site

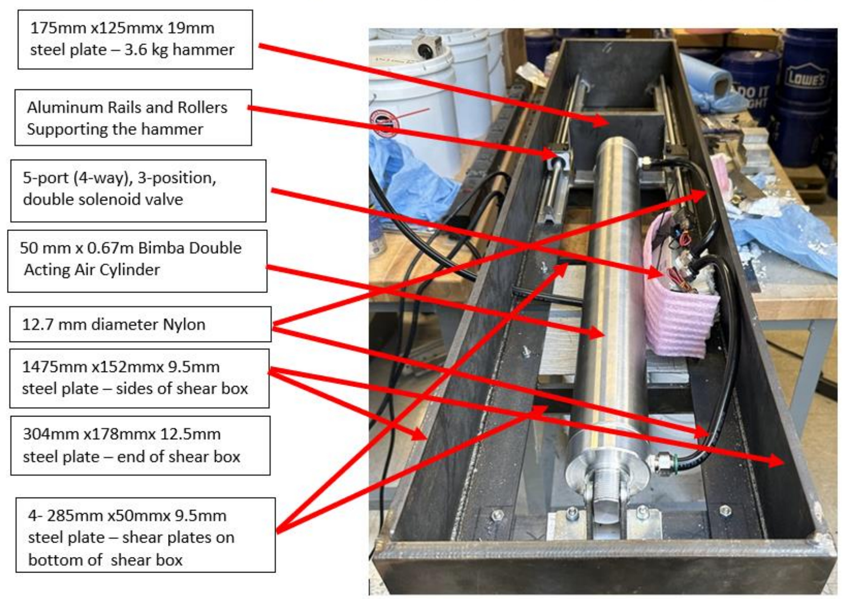


Comparison of inverted densities and coring sample's density

New seismic shear source

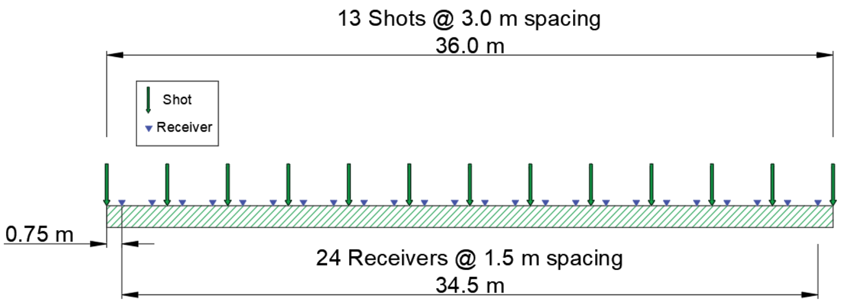
- system includes a seismic shear box , two steel tubes for raising and lowering the box, two air bags that apply downward force for ground coupling, and two coil springs that lift the box once the air bags are deactivated.
- Shear box enables control over both the frequency content and energy of the generated wavefields, which depend on the mass of the hammer and the impact speed. The hammer speed is regulated by adjusting the air (nitrogen) flow rate.
- Same energy is applied at all source locations for consistent wavefields (10-100 Hz).



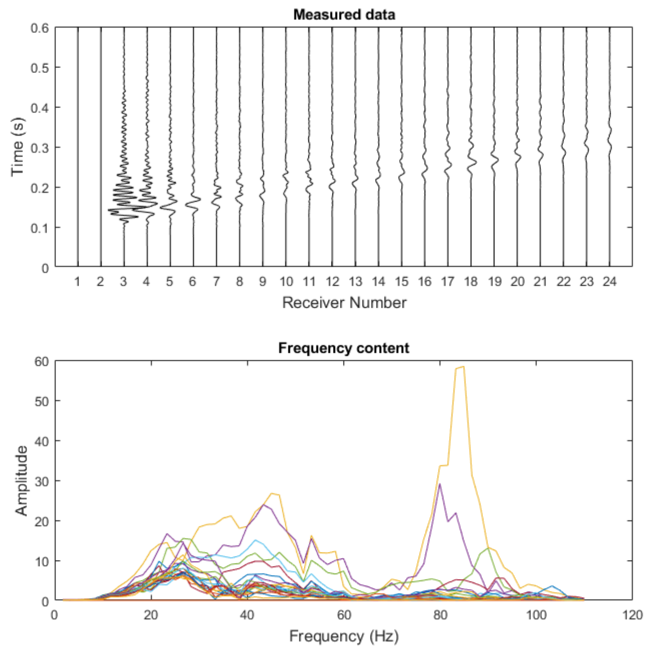


Site 2: Kanapaha site

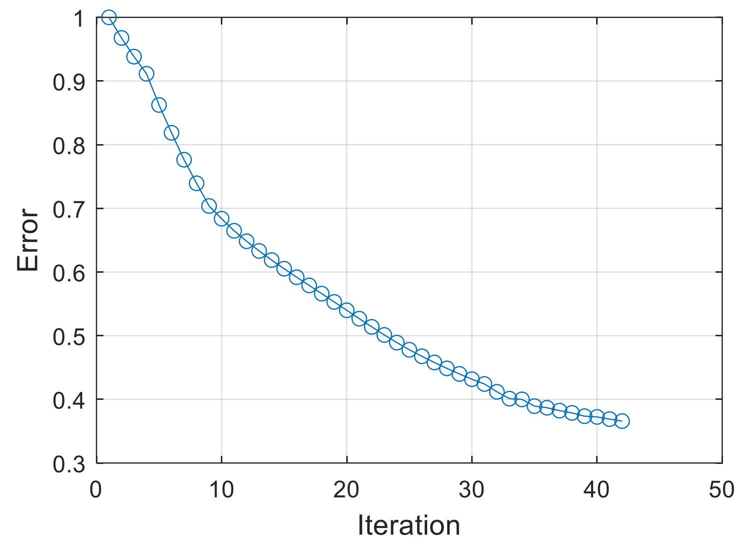
New seismic shear source

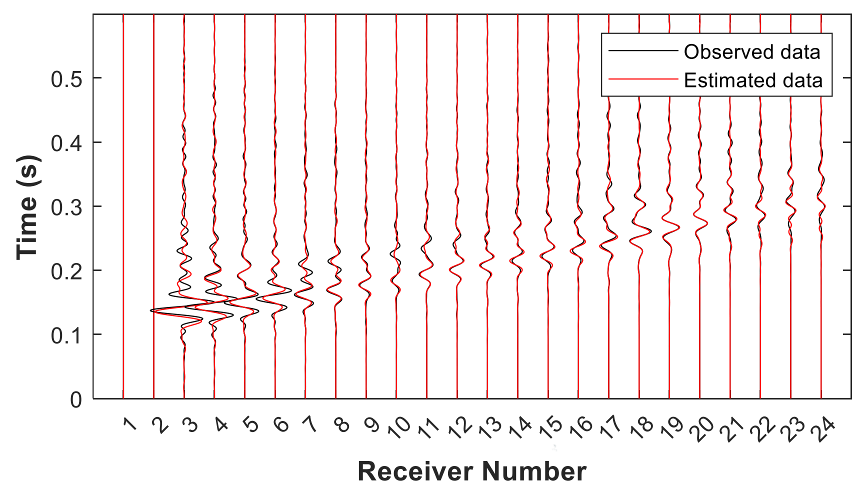


Kanapaha: data analysis



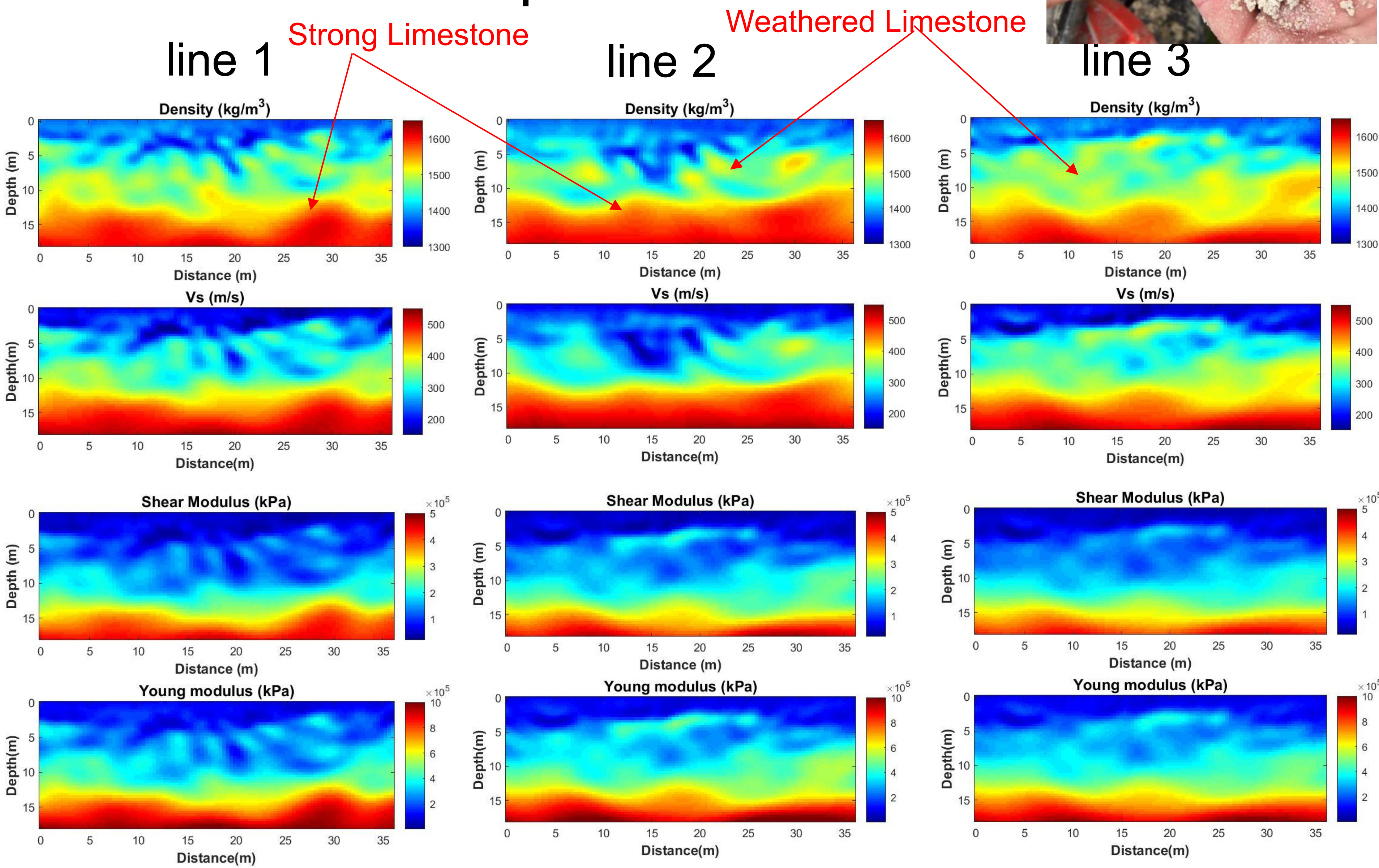
Measured data 10-100 Hz



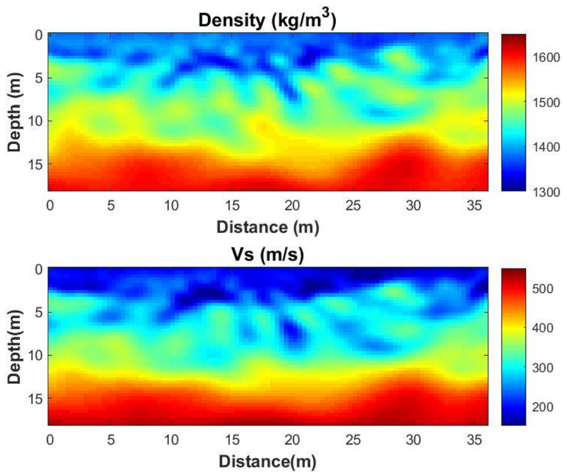


Waveform comparison 10-60 Hz

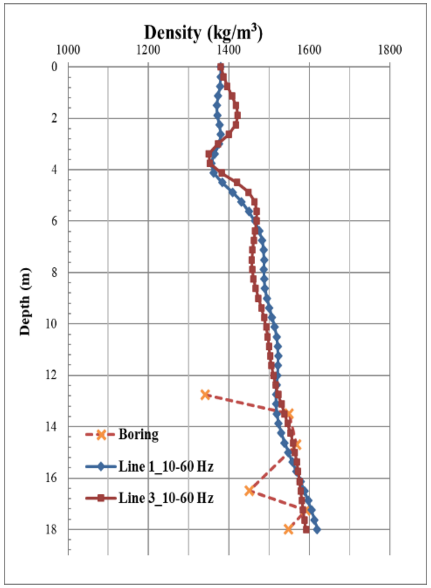
Kanapaha results



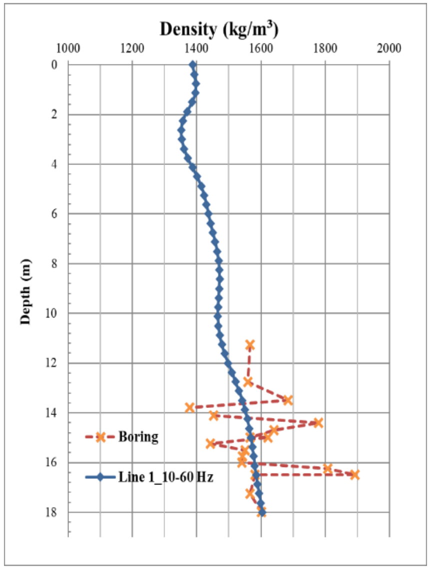
Kanapaha results



line 1

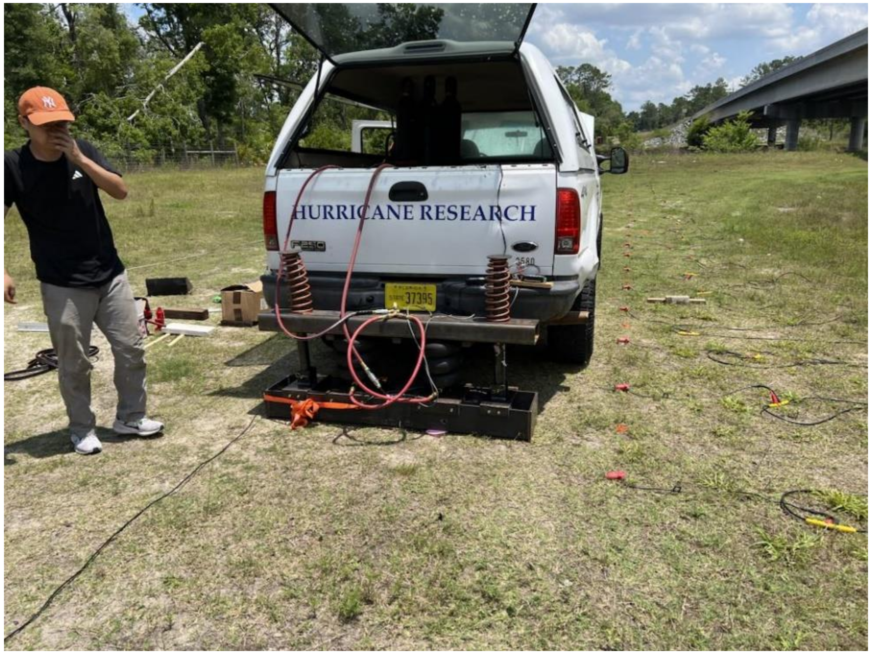


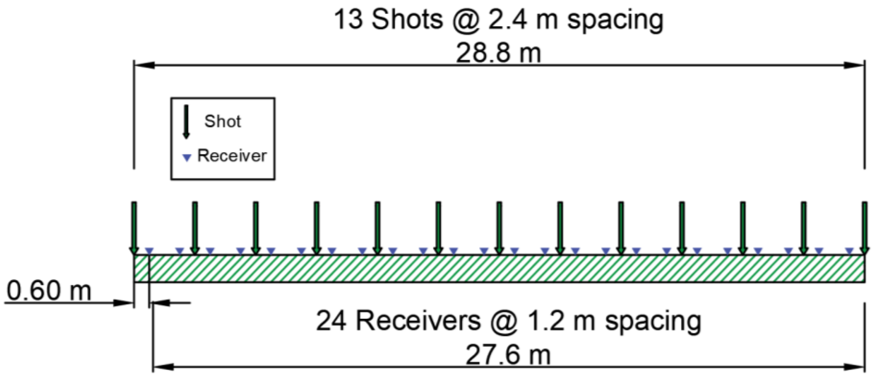
B22



B21

Site 3: CR 250 over Suwannee river

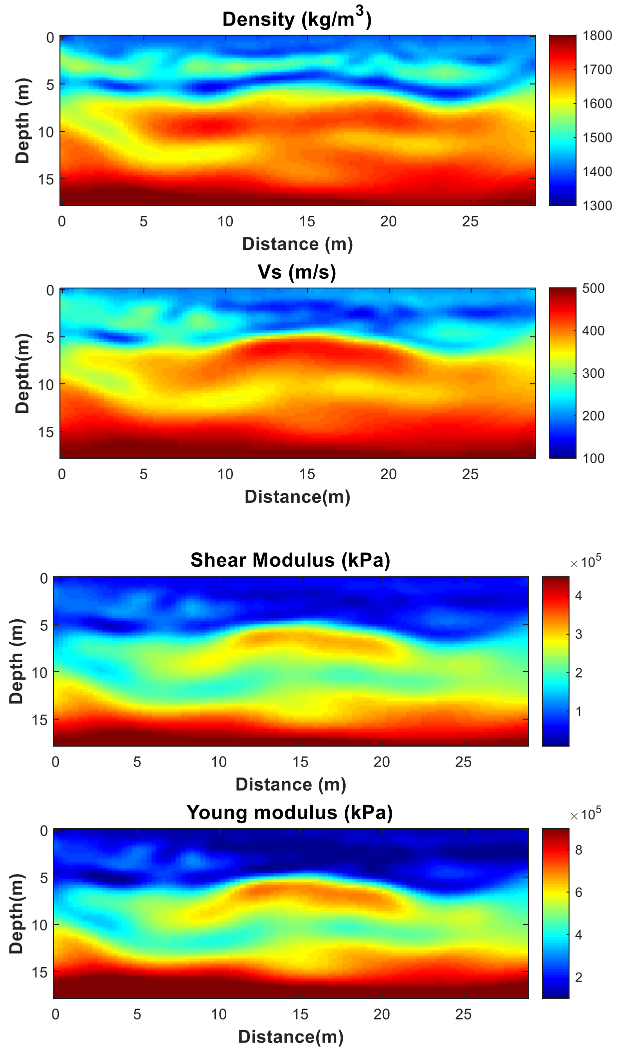




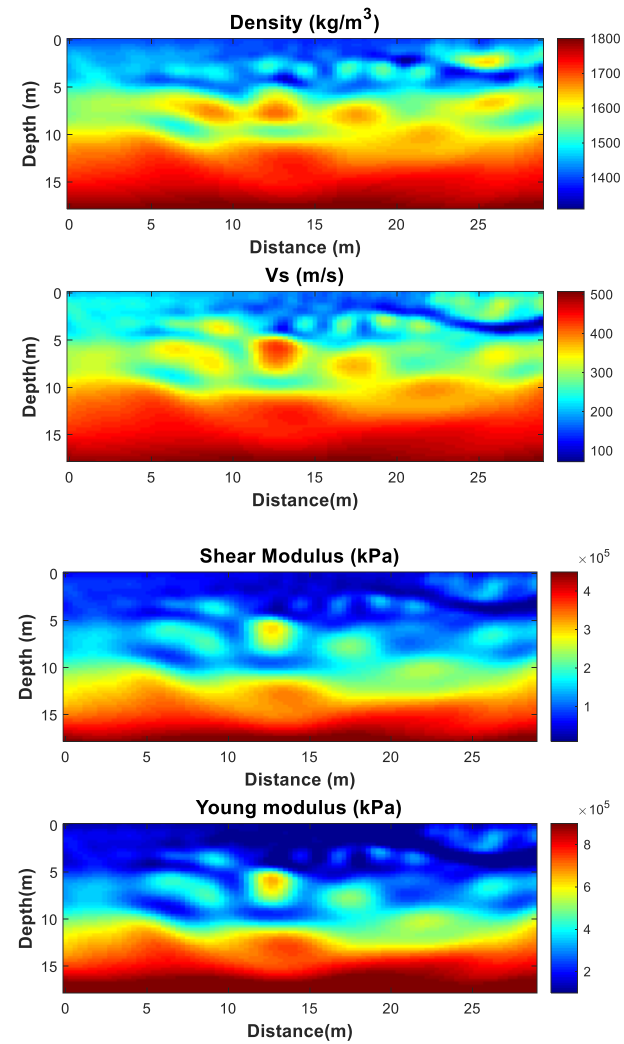
Two parallel test lines of 96 ft, 20 ft apart
24 geophones at 4 ft, 13 source at 8 ft

CR 250 results

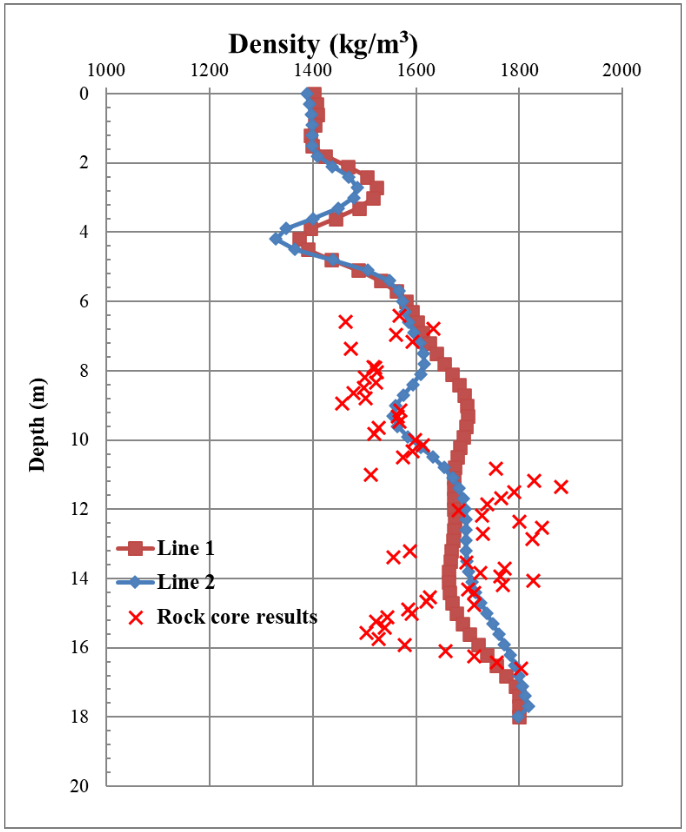
line 1

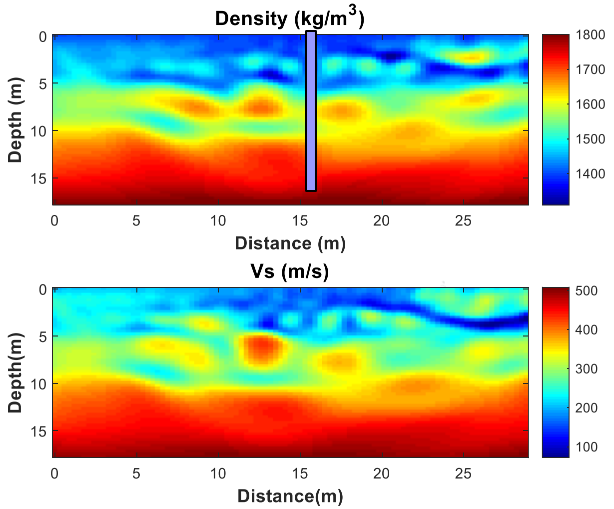


line 2



CR 250 results





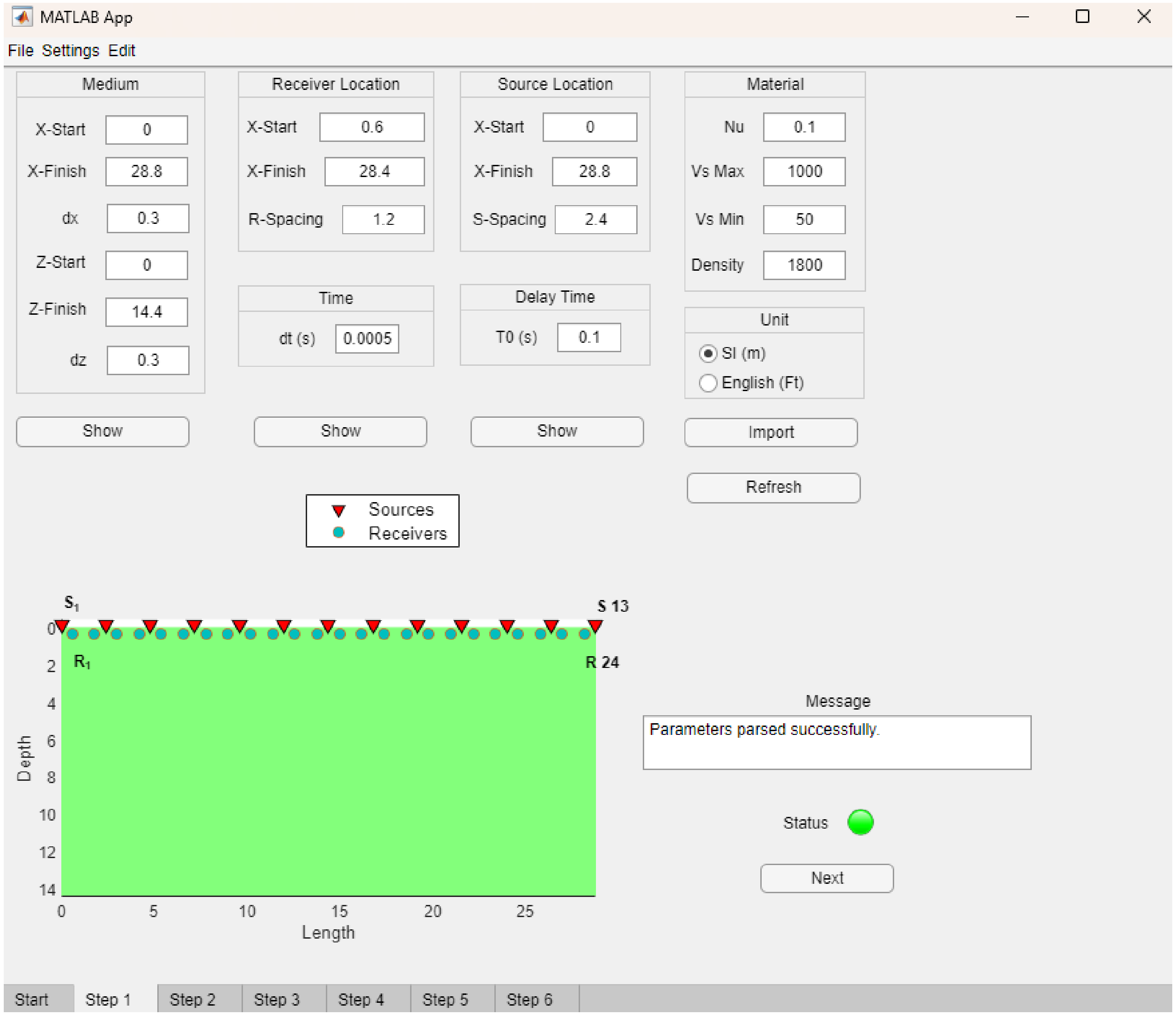
line 2

Coring location is 10 ft away from each of two test lines.

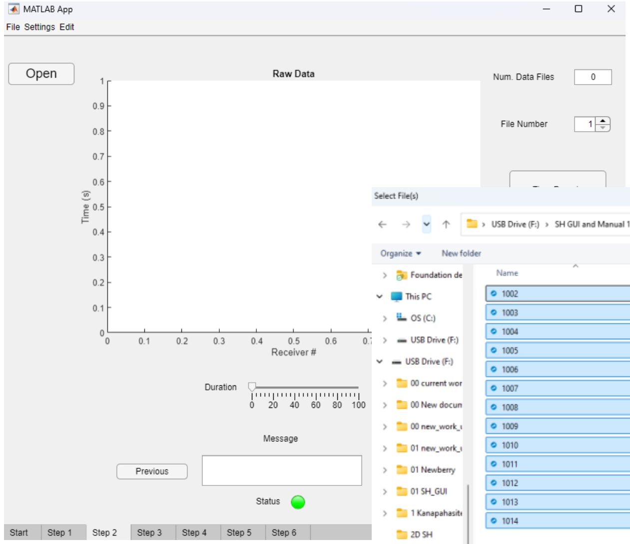
Task 4: Develop GUI software for data processing and analysis

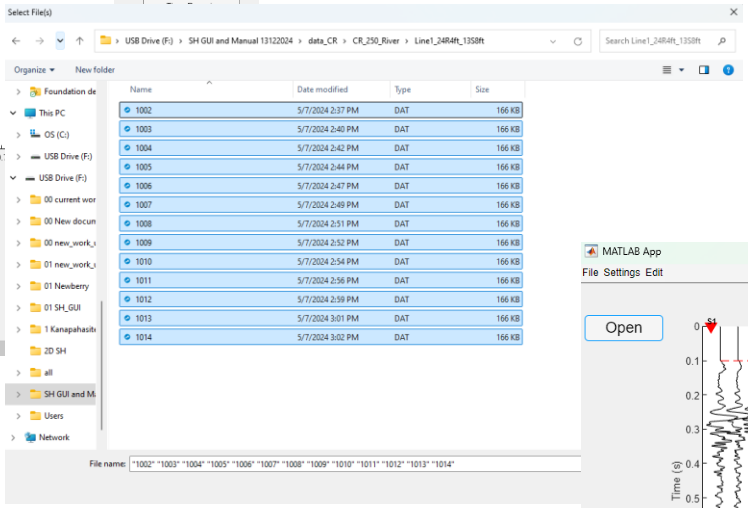
1. Geometry
2. Data condition
3. Analysis

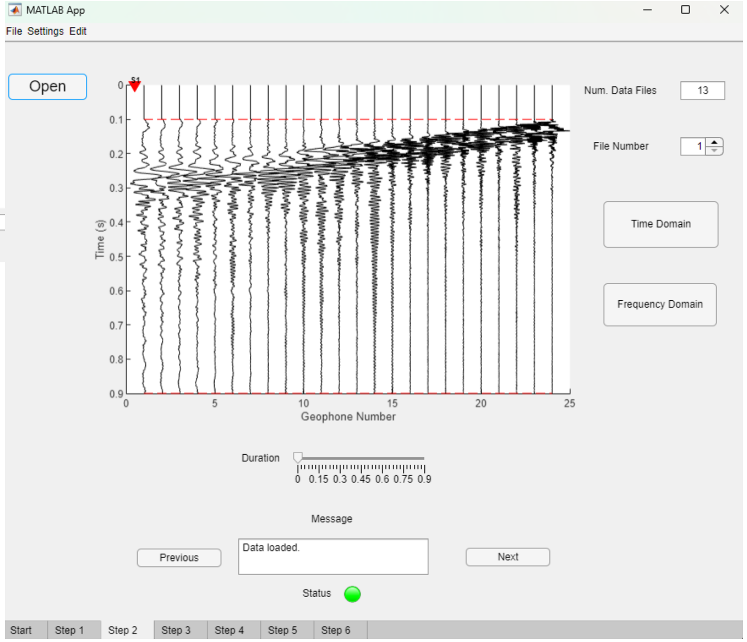
Step 1: Geometry input



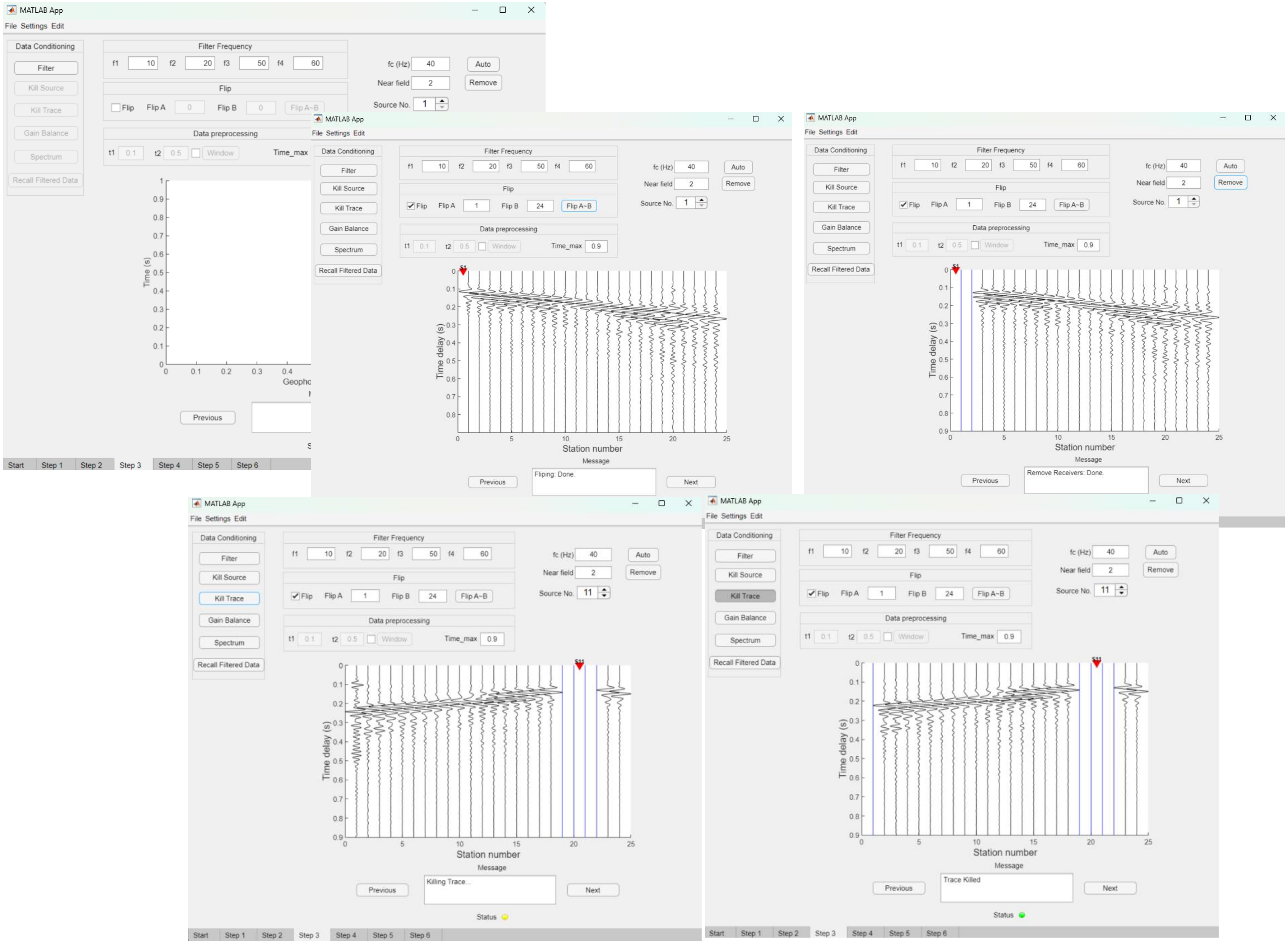
Step 2: Input data



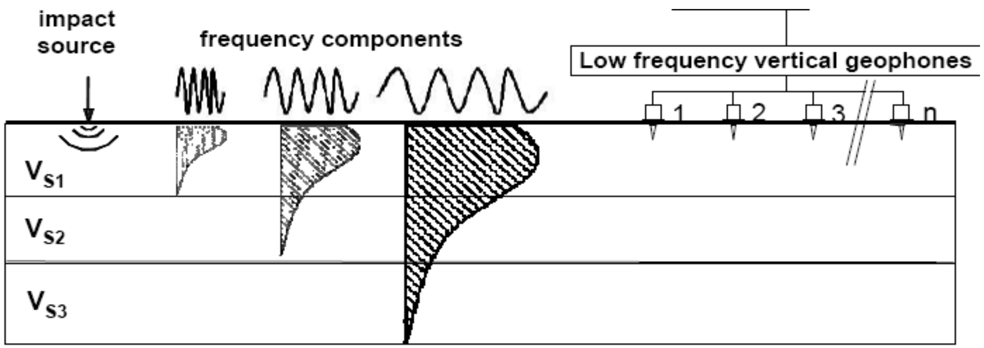


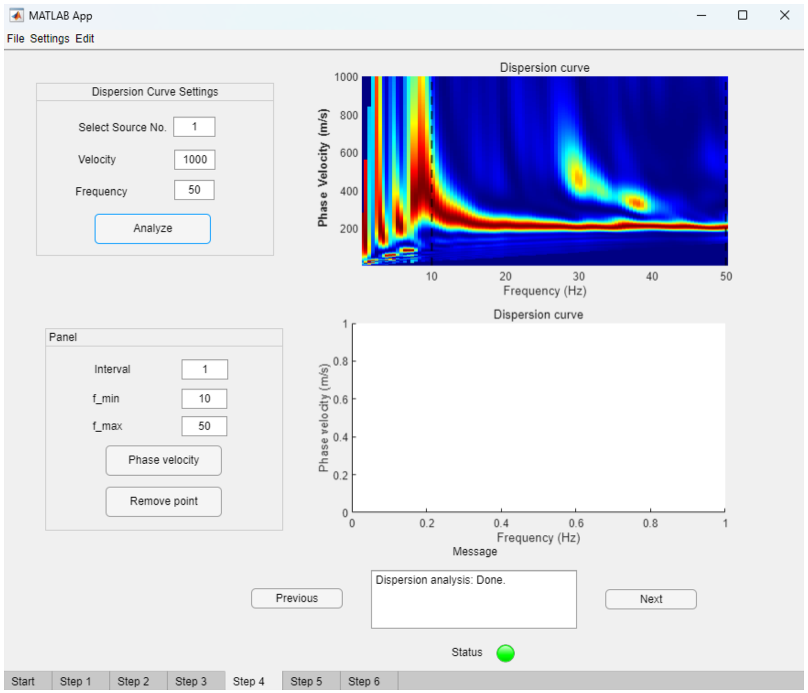


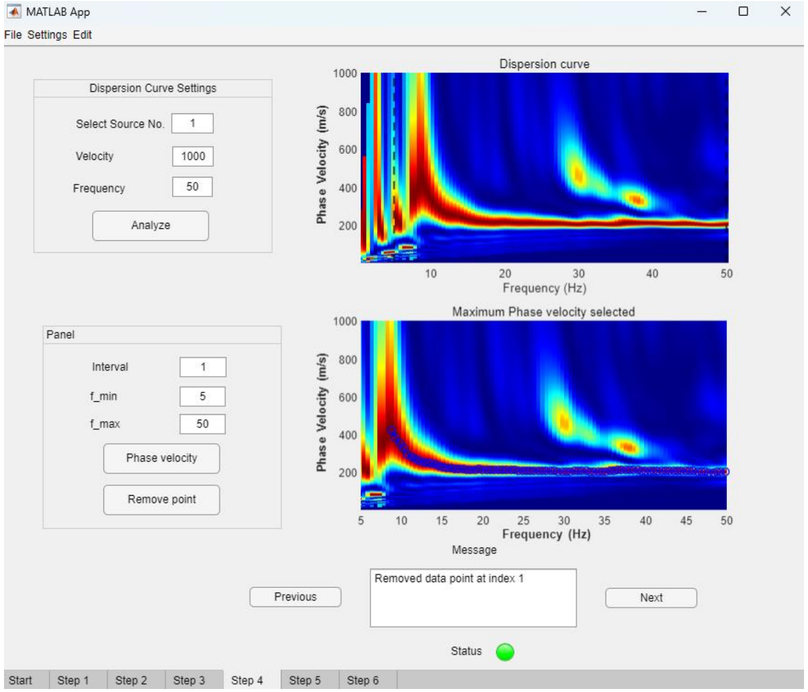
Step 3: Data condition



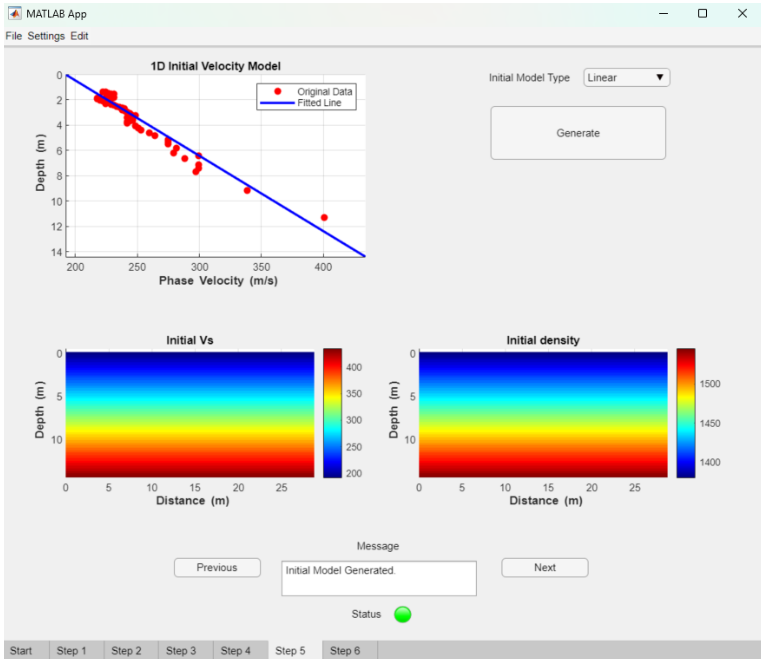
Step 4: Spectral Analysis



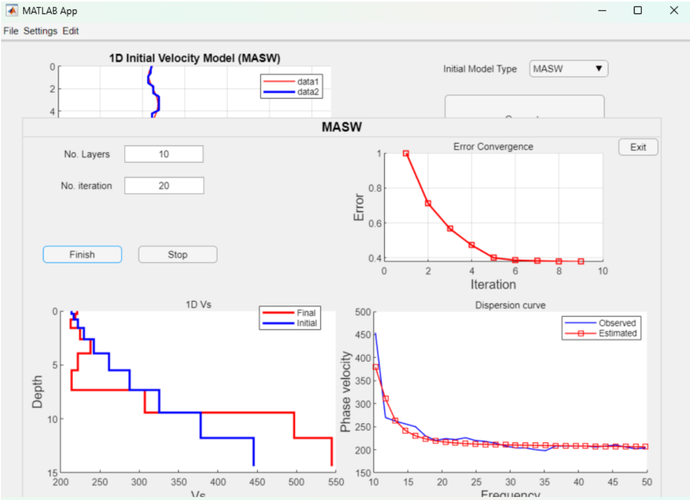


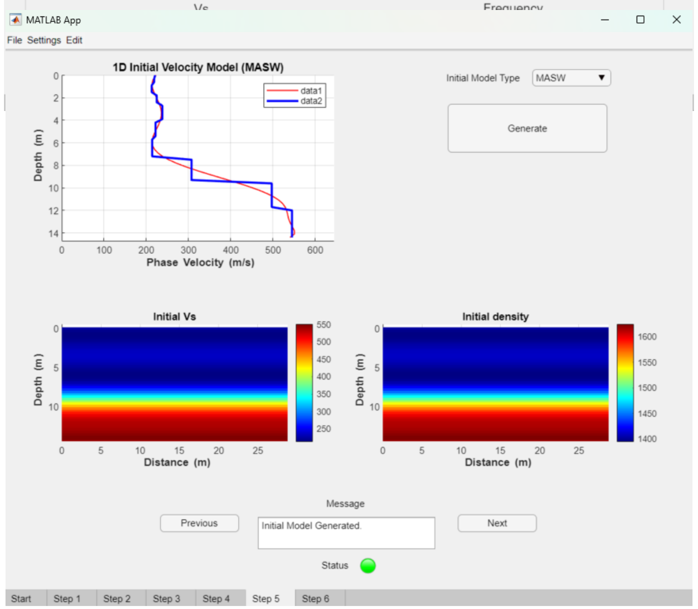


Step 5: Initial model



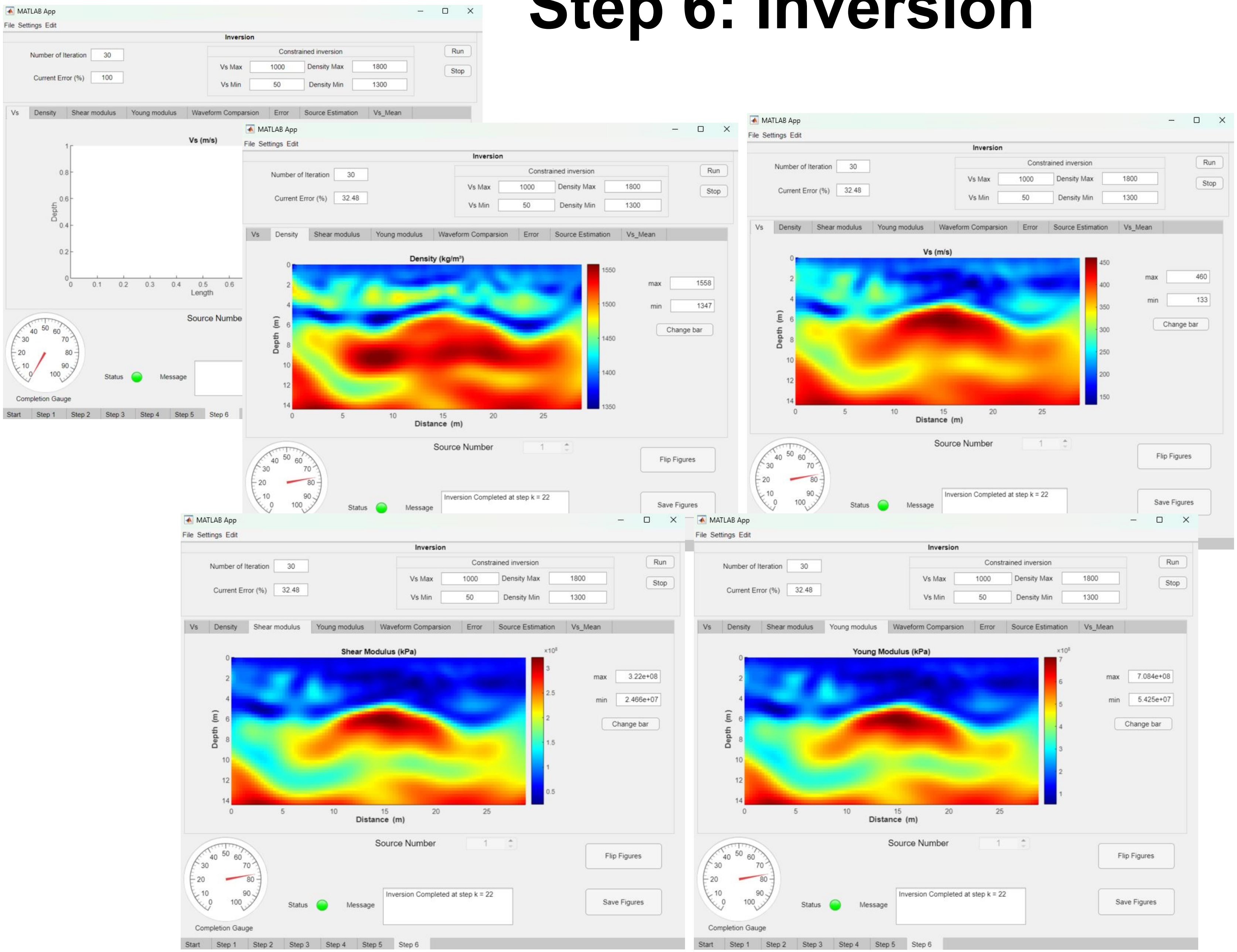
Initial model for Vs and density (Linear)





Initial model for Vs and density (MASW)

Step 6: Inversion



Summary of research conclusions

- A new 2D SH-Love FWI method and its GUI software have been developed for independent characterization of rock density along with moduli.

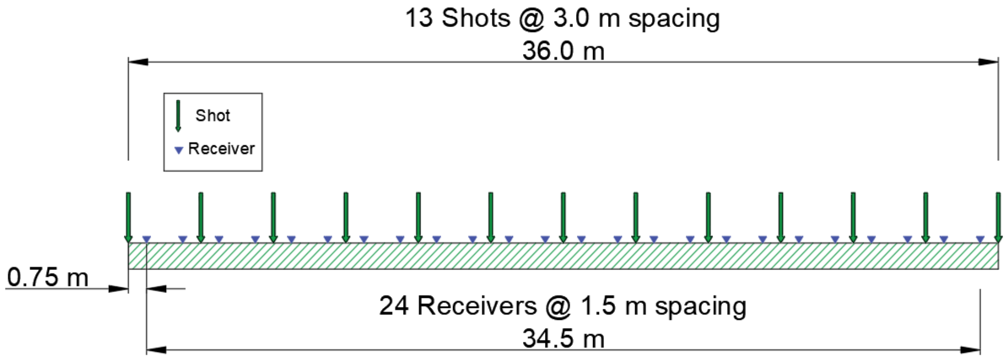
- Field results confirm that the rock density and moduli can be characterized by the SH-Love FWI method. Soil/rock density from seismic testing is consistent with those from coring samples.

- The method provides new capabilities of rock characterization:
  - material volume properties (density and moduli) over a length of 100 ft down to depth of 40 ft at 1 ft resolution.
  - identifies mass properties vs. point properties (sample)
  - clearly shows horizontal layering as well as variability

Recommendation

- ➢ Characterized depth = ½ test length
- ➢ Resolution is max (¼ receiver spacing, ¼ wavelength)

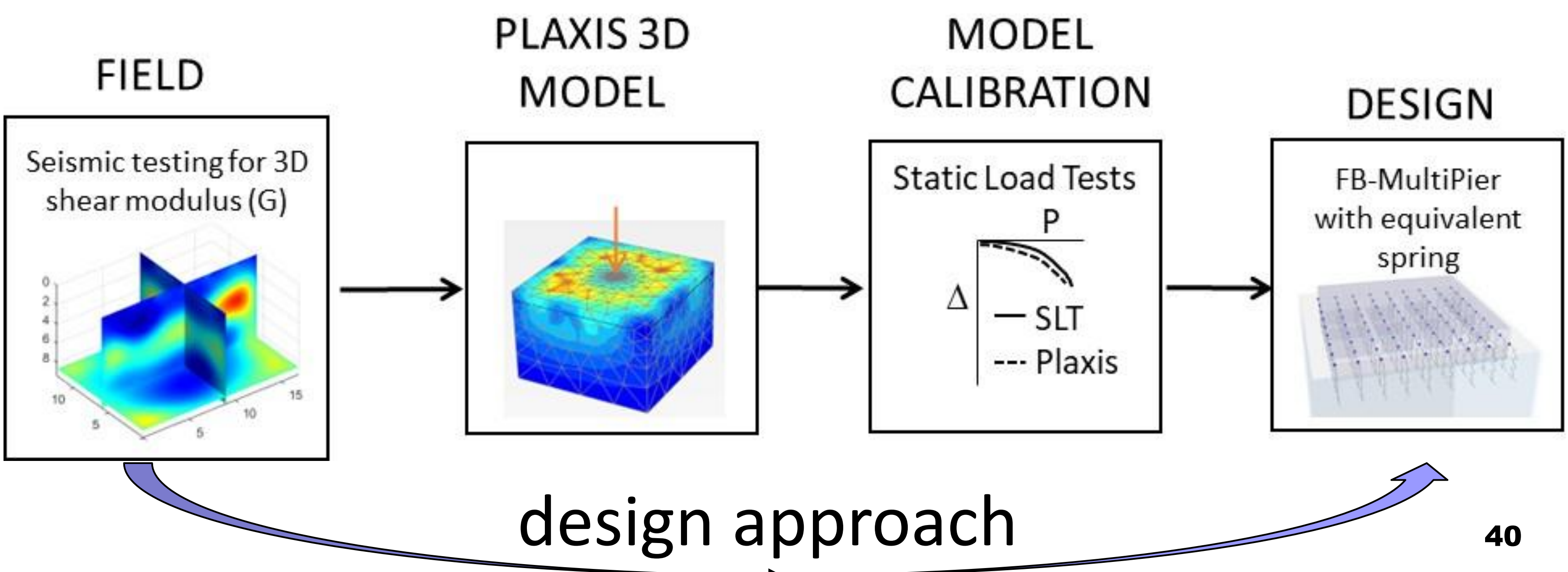
- ➢ 60 ft depth
- test length of 120 ft
- geophone at 5 ft spacing
- source at 10 ft spacing
- Use data up to 60 Hz



Future research

Shallow foundation design with 3D shear modulus to account for site spatial variability

- Seismic shear testing has already conducted at 3 sites with the full-scale load tests of footings (Miami, SR 84, and bell)
- Simulate 3D model load tests of footings using PLAXIS 3D to develop equivalent soil stiffness (spring) from seismic shear modulus.
- Implement the design approach in FB-MultiPier to account for site variability. This will improve prediction of bearing capacity and settlements (uniform, tilting/differential).



Publications

Chen R., Tran K.T., McVay M., Yang K., and Tran M.N. "Characterization of in-situ rock density with SH and Love-wave tomography: field data application", *Journal of Geophysics and Engineering,* https://doi.org/10.1093/jge/gxaf096

Kahbasi A., Tran K.T, Cox B., and Abbas A. (2025) "Deep site characterization with large mobile shaker using 2D time-domain FWI method of SH- and Love-waves", *Journal of Applied Geophysics*, 235, 10565.

Thank You!

Questions & Answers