Determination of in-situ rock density and strength with SH-Love wave tomography

FDOT BED31-977-12

GRIP Meeting 2024

Project Manager:
Kelly Shishlova, P.E.

Principal Investigators:
Khiem Tran, Ph.D.
Michael McVay, Ph.D.

Graduate Assistants
Nhat Tran, PhD student
Kunyu Yang, Ph.D.

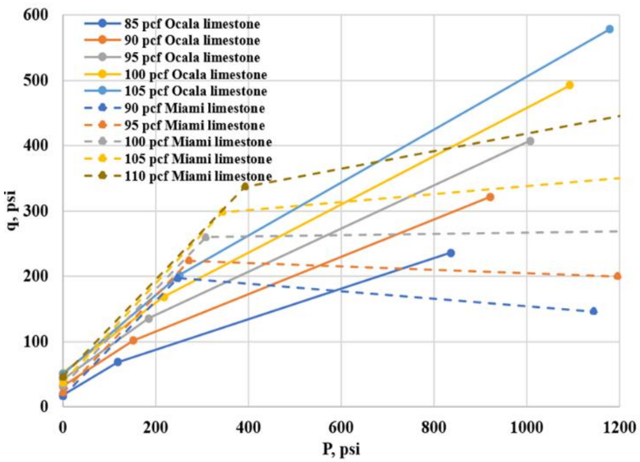
University of Florida

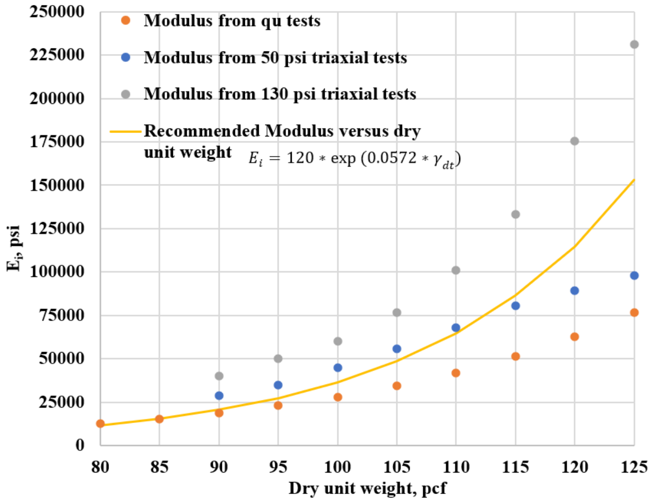
Presentation outline

- Introduction
- Project objectives
- Research approach
- Benefits of using SH and Love waves
- SH-Love FWI algorithm
- Synthetic experiment
- Field experiment at 3 test sites
- GUI development
- Conclusion

Introduction

- Design and construction of shallow foundations rely heavily on accurate subsurface information, particularly regarding rock density and variability
- Recent FDOT projects have shown that mass density (or unit weight) controls the rock strength as well as its stress-strain behavior for most Florida limestone formations.
- Current practice for estimating mass properties is to measure intact sample properties (density, modulus) and multiply them by estimated recoveries. However, this often leads to gaps in the data (e.g., borings far from footing, layering), potentially compromising the design (e.g., missed layer), especially in heterogeneous Florida limestone formations.
- This project is to develop a new method for determination of rock density and modulus over a large volume on a foot scale without requirement of borings.





Project objectives

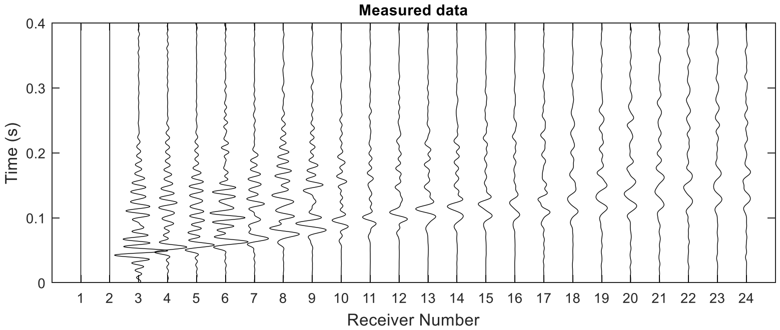
- The project objective is to develop an advanced testing system (hardware and software) for determination of rock density and moduli.
- The system will enable to provide both density and moduli at relevant resolutions (foot pixels) for entire rock volume supporting foundations without requirement of borings.
- The GUI module will be transferred to FDOT for future uses in site investigations of soil/rock properties and stratigraphy.

Research approach

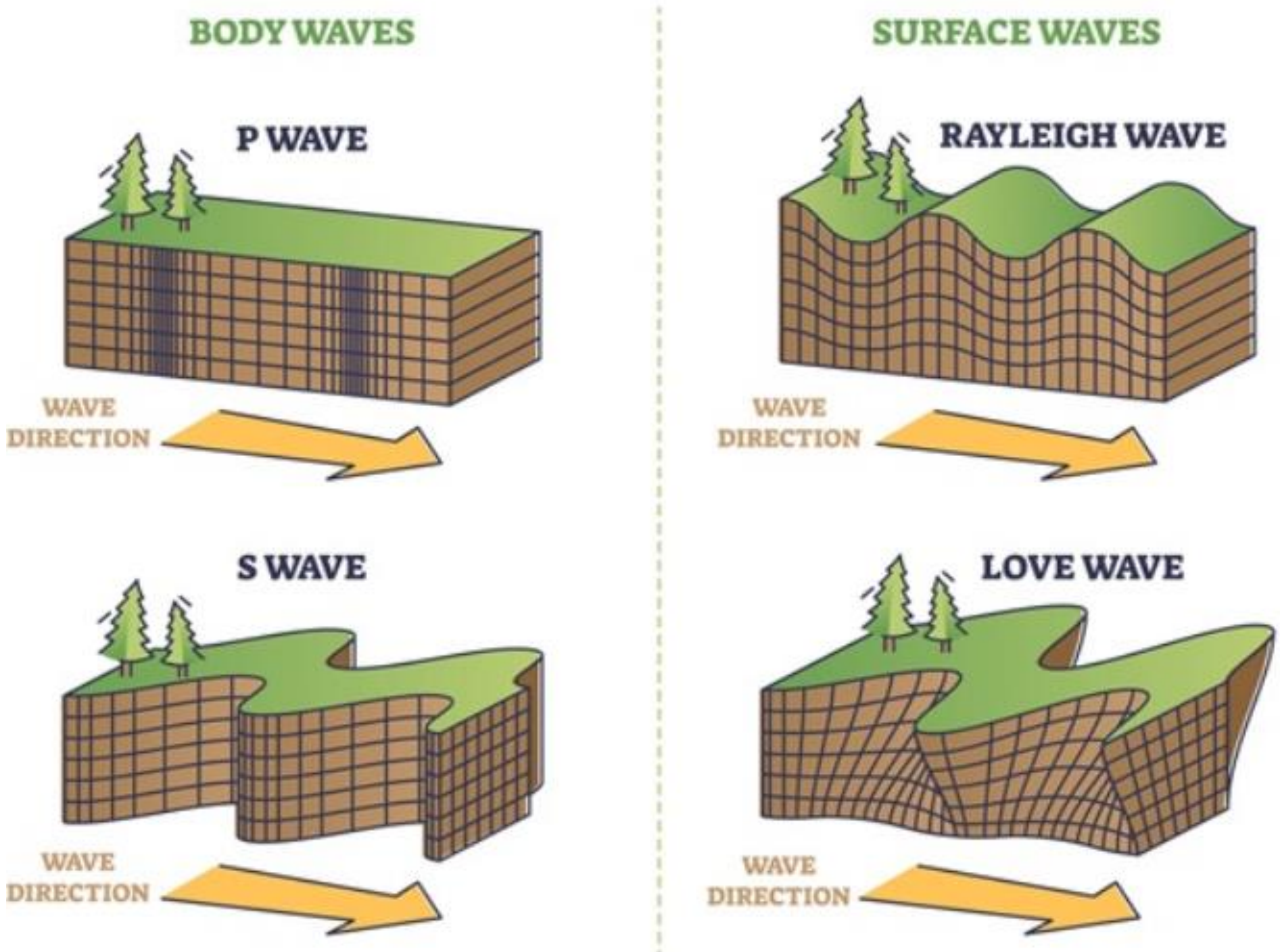
- The research is to develop an SH-Love full waveform inversion (2D SH-Love FWI) method, which can characterize 2D density and moduli of subsurface soil/rock at foot-pixels down to 30-60 ft depth

- Horizontal shear (SH) and Love waves are generated by applying a horizontal source and recorded by an array of horizontal geophones on the ground surface.

- Recorded waveform data are analyzed to extract density and S-wave velocity of subsurface materials.



Seismic wave types



VectorMine (Dreamstime.com)

Benefits of using SH and Love waves

- SH and Love waves (horizontal source) are more sensitive to material density than vertical S-wave, P-wave, and Rayleigh waves (PSV) (vertical source), and thus the density could be extracted more accurately.

- SH-Love wave simulation requires less computing time (40% that of PSV waves), and the 2D SH-Love FWI can be done quickly (15 minutes) in the field.

- Both density (ρ) and S-wave velocity (Vs) can be characterized. Shear (G) and Young (E) moduli can be computed for determination of shallow foundation's settlement and bearing capacity, and other geotechnical analyses.

$$G = \rho V_s^2$$

$$E = 2G(1 + v)$$

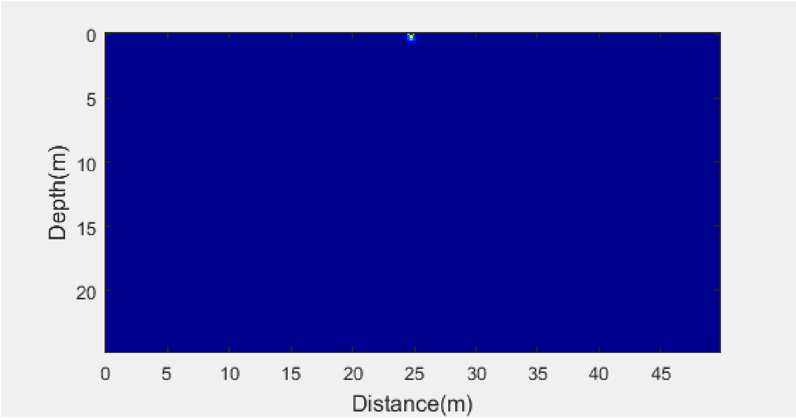
Task 1: Develop 2D SH-Love FWI algorithm (completed)

- Forward modelling

$$\rho(x,z)\frac{\partial v_y}{\partial t} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} + f_y \qquad (1)$$

$$\frac{\partial \sigma_{xy}}{\partial t} = \mu(x,z)\frac{\partial v_y}{\partial x} \qquad (2)$$

$$\frac{\partial \sigma_{yz}}{\partial t} = \mu(x,z)\frac{\partial v_y}{\partial z} \qquad (3)$$



ρ: density, $\mu = G = \rho V_s^2$

- Perfectly matched layers (PML) are applied at the bottom and vertical boundaries to absorb outgoing waves.

2D SH-Love FWI

- Model update (adjoint-state method)
  - Step 1. Calculate the residual between estimated data and observe data

$$\Delta \boldsymbol{d}_{s,r} = \boldsymbol{D}_{s,r}(\mathbf{m}) - \boldsymbol{d}_{s,r} \tag{4}$$

  - Step 2. Compute the Least-squares error

$$\mathrm{E}(\mathbf{m}) = \frac{1}{2} \Delta \boldsymbol{d}^T \Delta \boldsymbol{d}, \tag{5}$$

  - Step 3. Compute the gradients (adjoint-state gradient approach)

$$\frac{\partial E}{\partial V_s} = -\frac{2}{V_s^3 \rho} \sum_{i=1}^{N} \int_0^T dt(\sigma_{xy}^f \sigma_{xy}^b + \sigma_{yz}^f \sigma_{yz}^b), \tag{6}$$

$$\frac{\partial E}{\partial \rho} = -\frac{1}{V_s^2 \rho^2} \sum_{i=1}^{N} \int_0^T dt(\sigma_{xy}^f \sigma_{xy}^b + \sigma_{yz}^f \sigma_{yz}^b + V_s^2 \rho^2 \frac{\partial v_y}{\partial t} u_y^b), \tag{7}$$

2D SH-Love FWI

- Model update
  - Step 4. Smooth gradients (Tikhonov regularization) to minimize the ill-posed problem

$$\left(\frac{\partial E}{\partial V_s}\right)_r = \frac{\partial E}{\partial V_s} + \lambda_1 \boldsymbol{D} V_s \qquad (8)$$

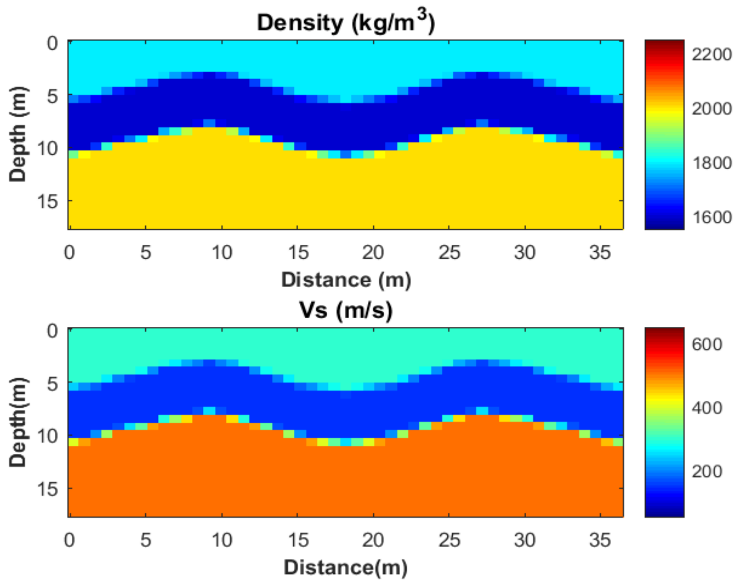
$$\left(\frac{\partial E}{\partial \rho}\right)_r = \frac{\partial E}{\partial \rho} + \lambda_2 \boldsymbol{D} \rho \qquad (9)$$

  - Step 5. update Vs and density models (steepest-descent method)

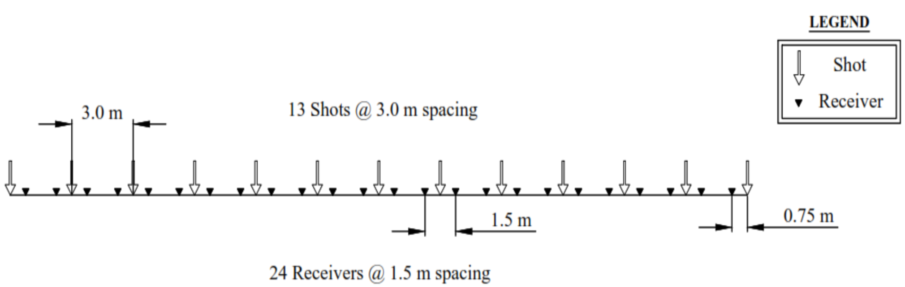
$$V_{s_{n+1}} = V_{s_n} - \alpha_n \boldsymbol{H}_n^{-1} \left(\frac{\partial E}{\partial V_s}\right)_r , \qquad (10)$$

$$\rho_{n+1} = \rho_n - \beta_n \boldsymbol{H}_n^{-1} \left(\frac{\partial E}{\partial \rho}\right)_r , \qquad (11)$$

Synthetic experiment

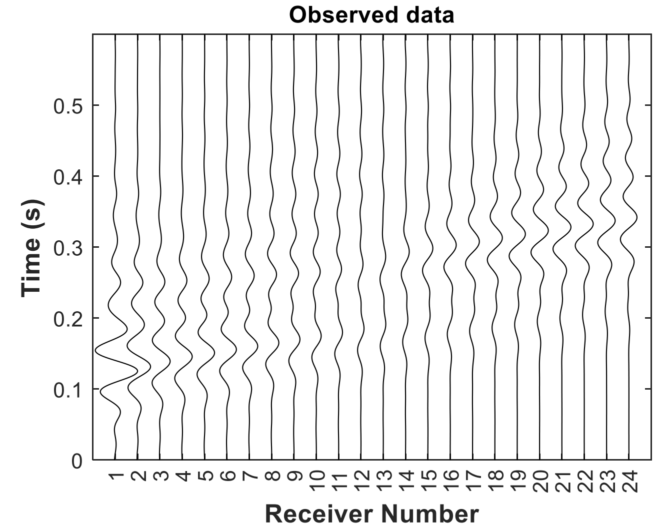


true model

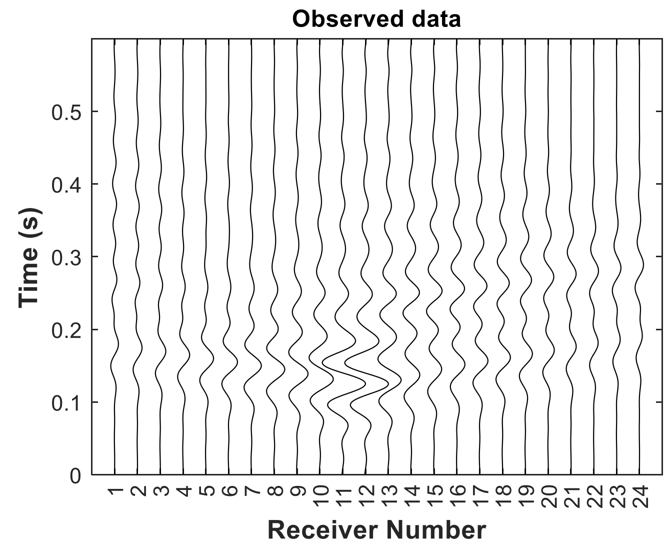


Test configuration

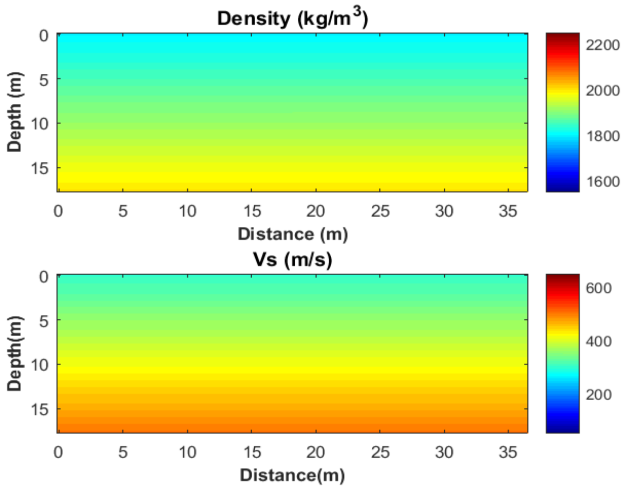
Shot 1



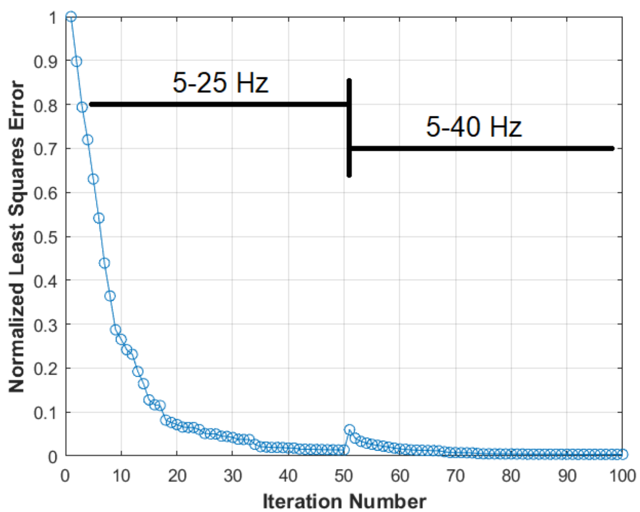
Shot 7



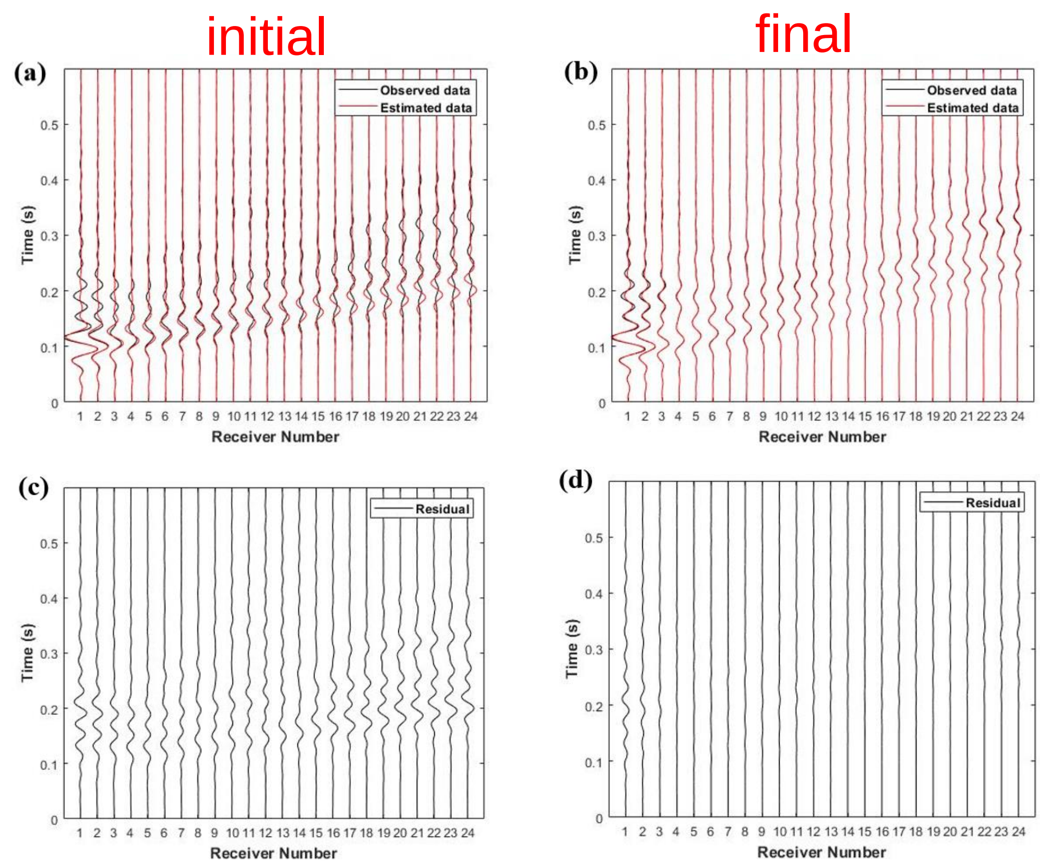
Synthetic experiment



initial model

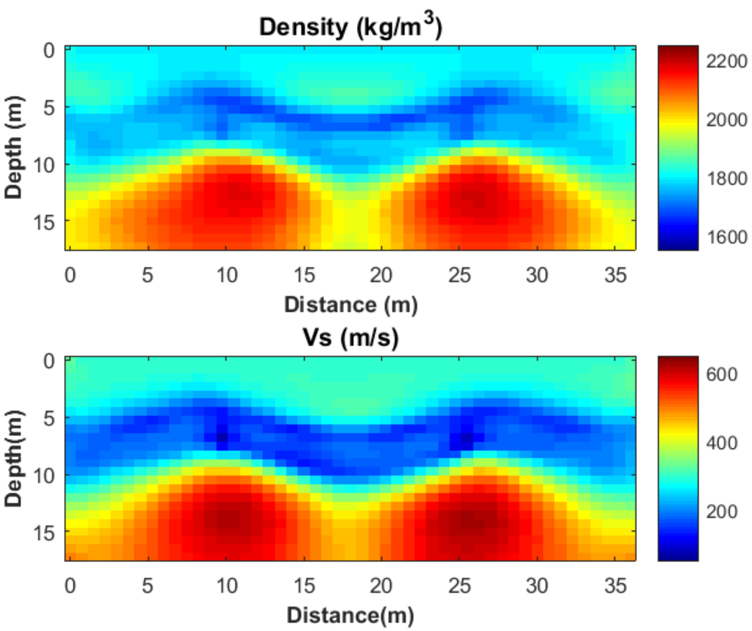


Least-square error

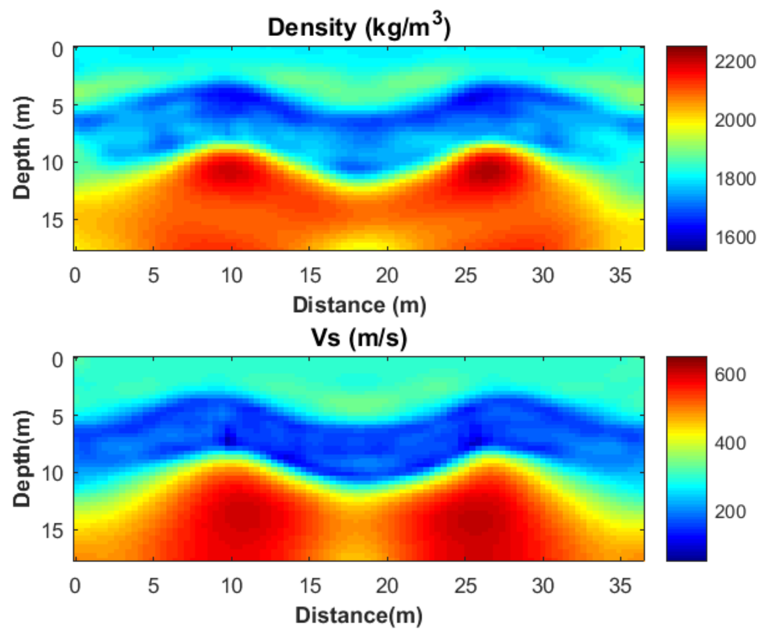


Waveform comparison

Synthetic results

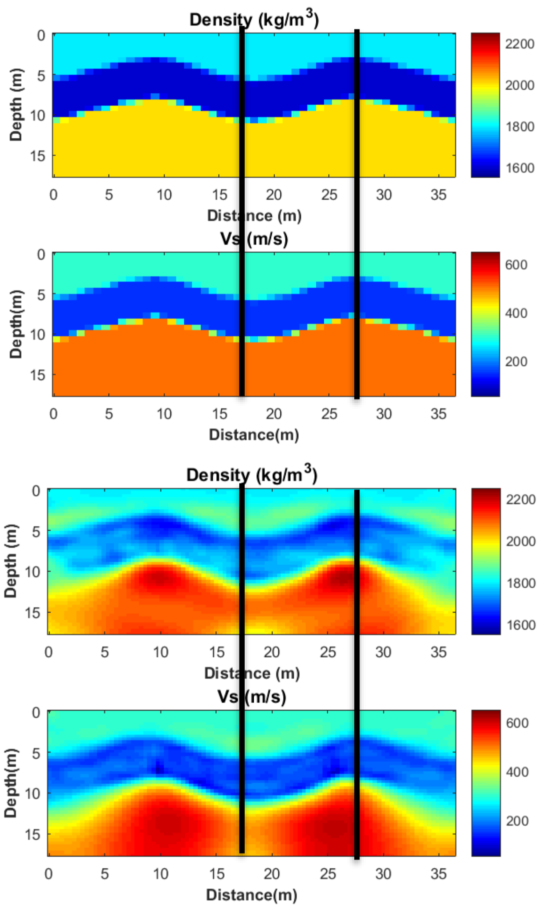


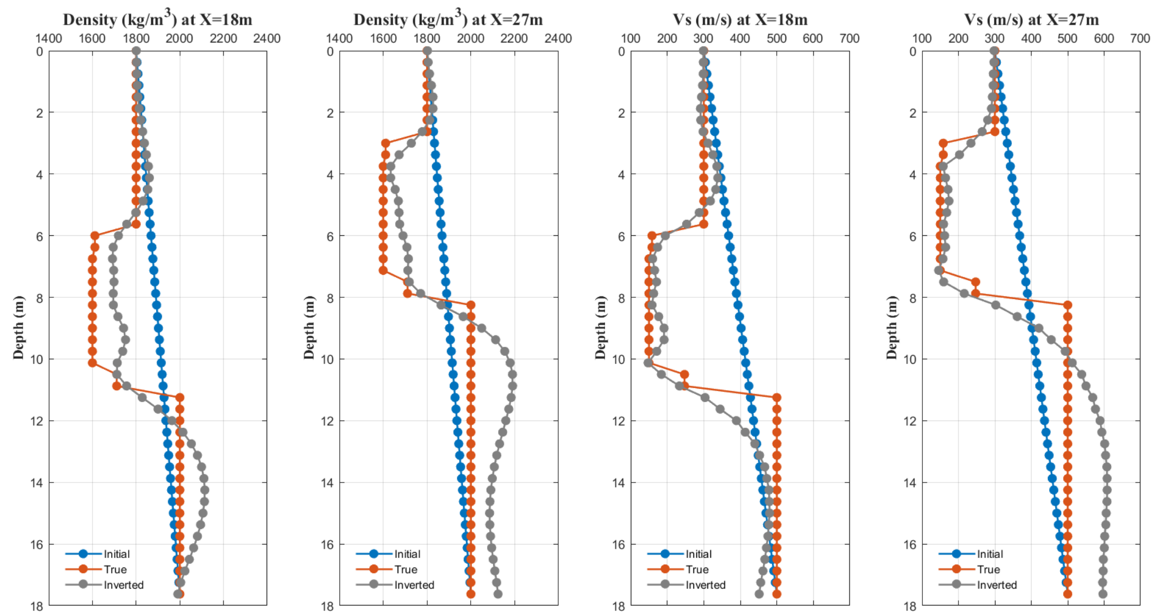
First analysis at 5-25 Hz



Second analysis at 5-40 Hz

Synthetic results





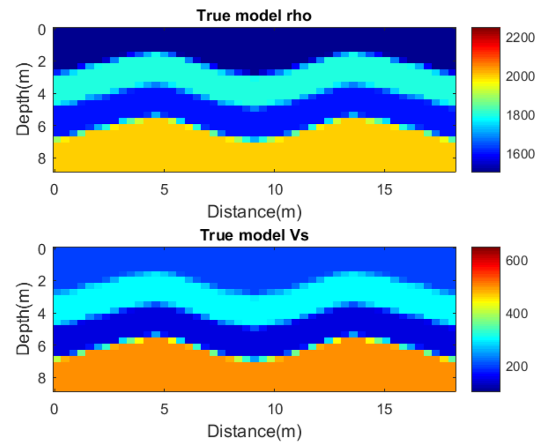
Detailed comparison at two locations

Task 2: Optimize test configurations and wavefield characteristics (completed)

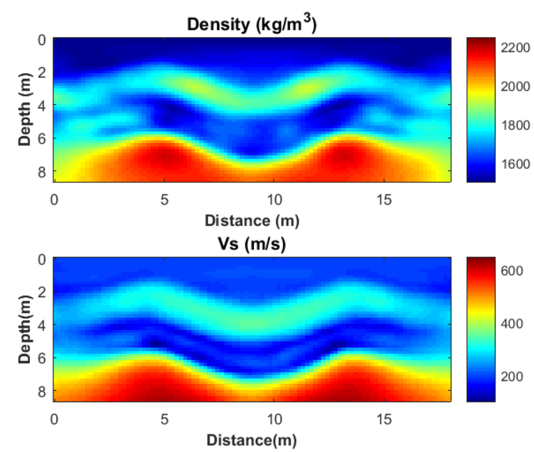
- Characterized depth = ½ test length
- Resolution is max (¼ receiver spacing, ¼ wavelength)

- 60 ft depth: test length 120 ft, geophone spacing of 5 ft and source spacing of 10 ft, data up to 40 Hz (15" pixel)

- 30 ft depth: test length 60 ft, geophone spacing of 2.5 ft and source spacing of 5 ft, data up to 80 Hz (7.5” pixel)



True



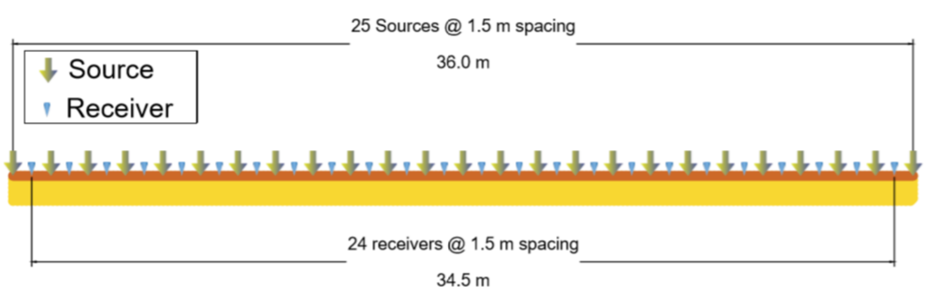
Inverted

Task 3: Verify the 2D SH-Love FWI algorithm with field experiments (ongoing)

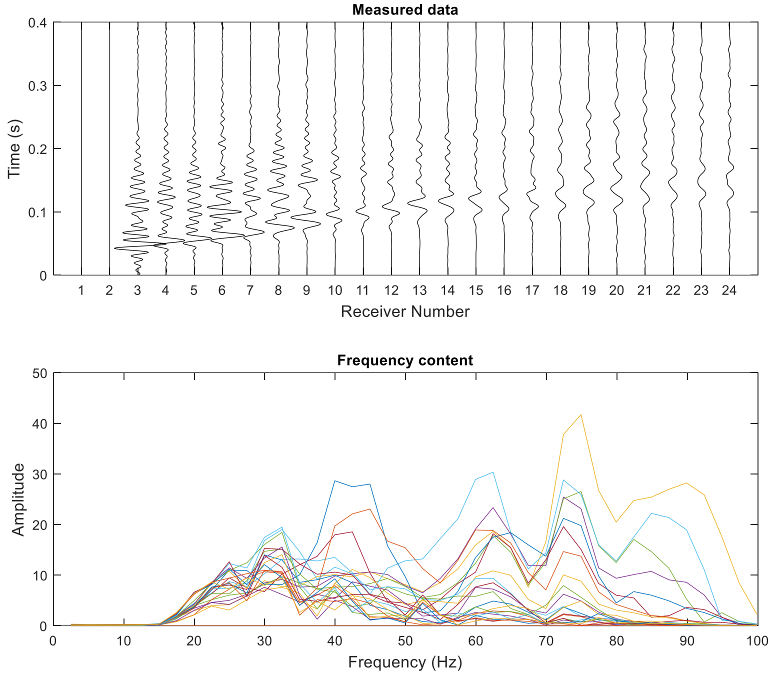
Site 1: Bell site (3rd Shallow Foundation Load Test, FDOTBDV31-977-82)

5' I-beam

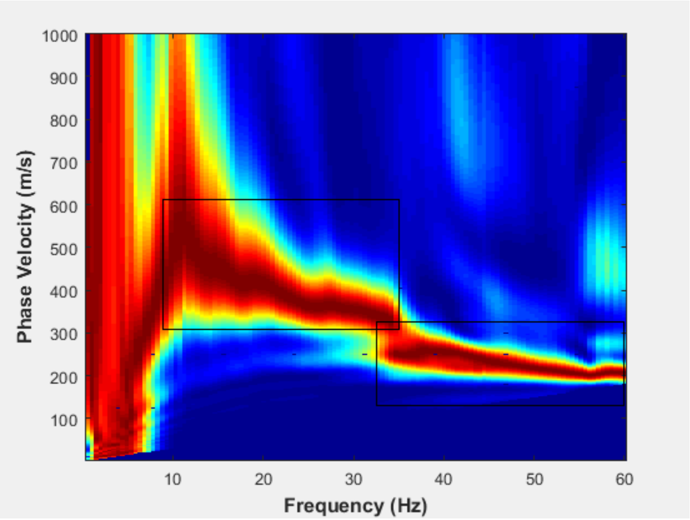
Two test lines of 120 ft, 24 geophones at 5 ft, 25 source at 5 ft

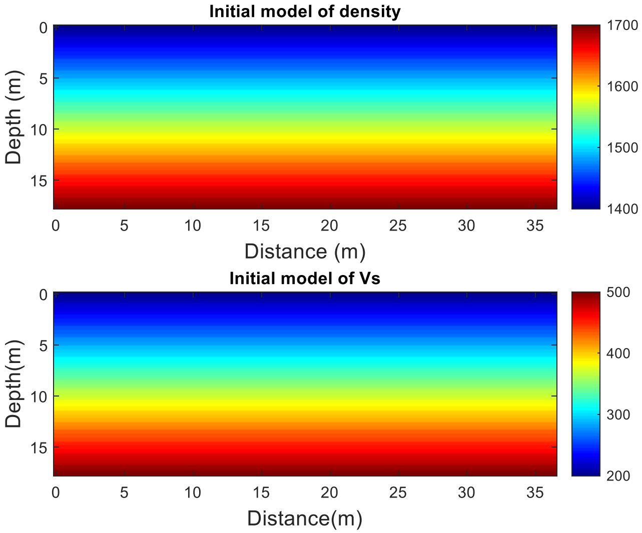


Bell site



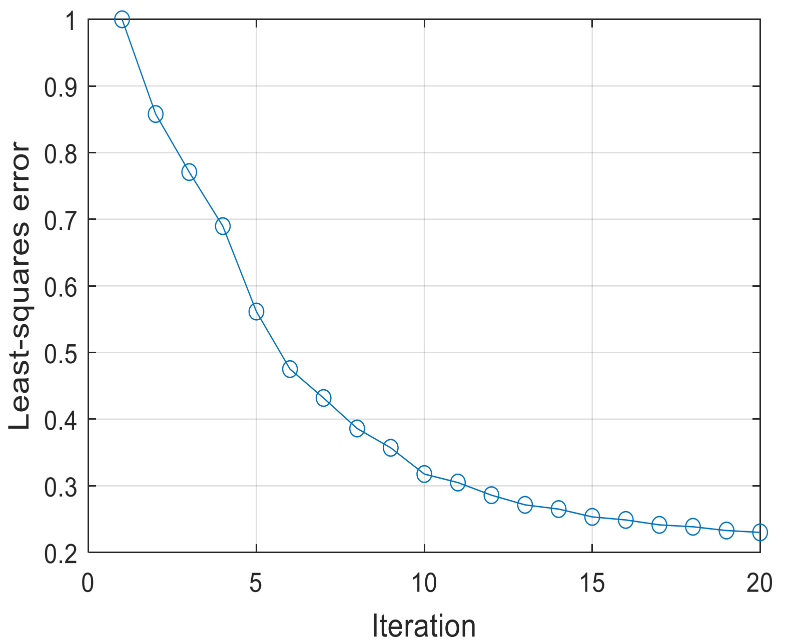
Sample data

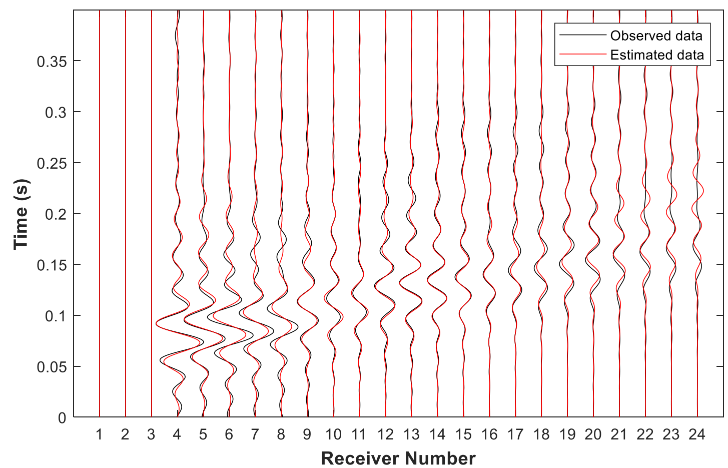


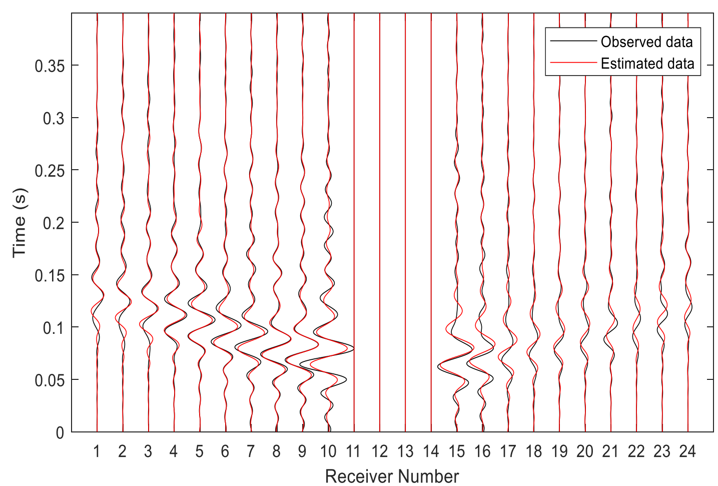


Bell site: data analysis

15 minutes on a standard computer (8-core CPU of 3.70 GHz, 64 GB of RAM)

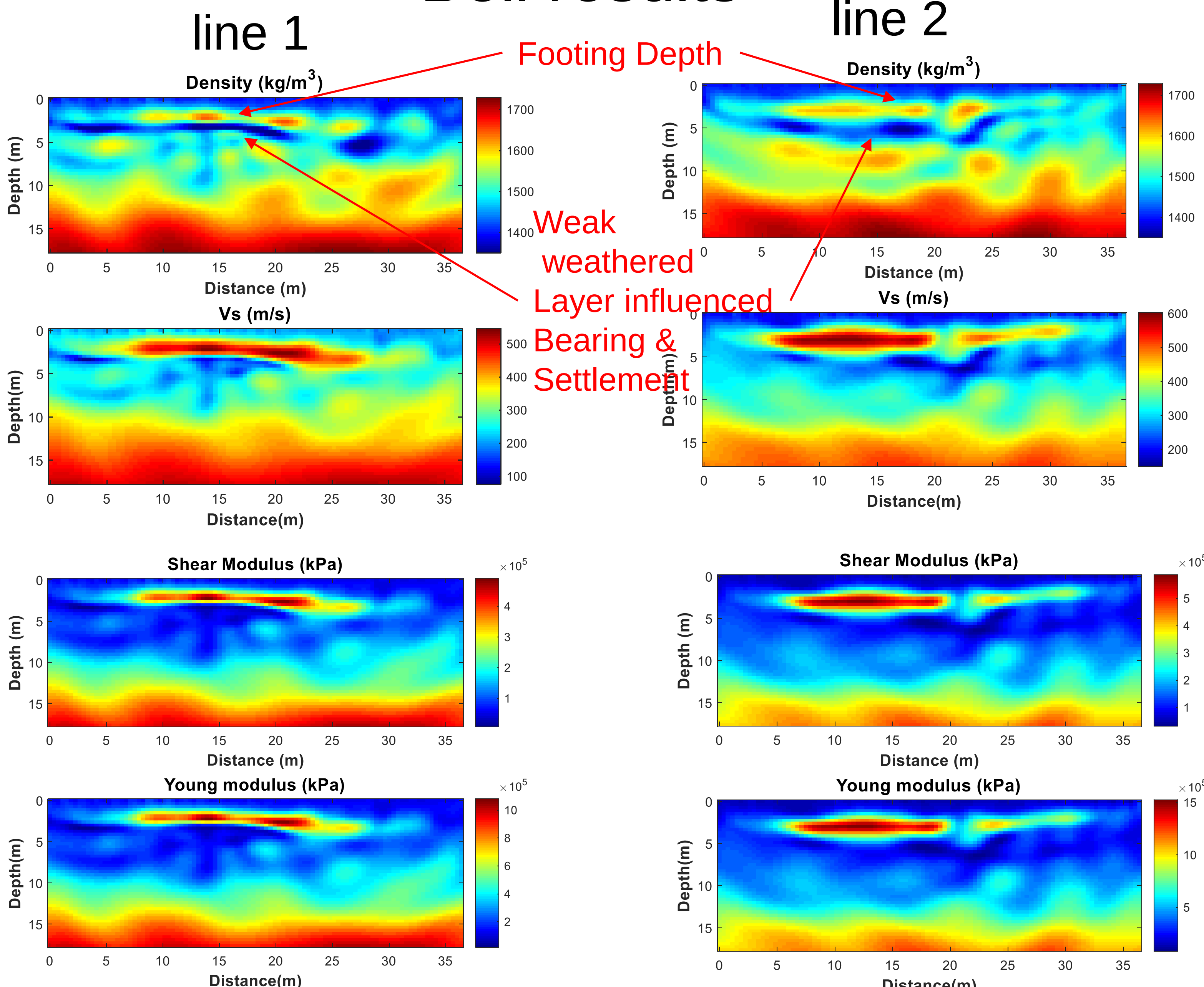




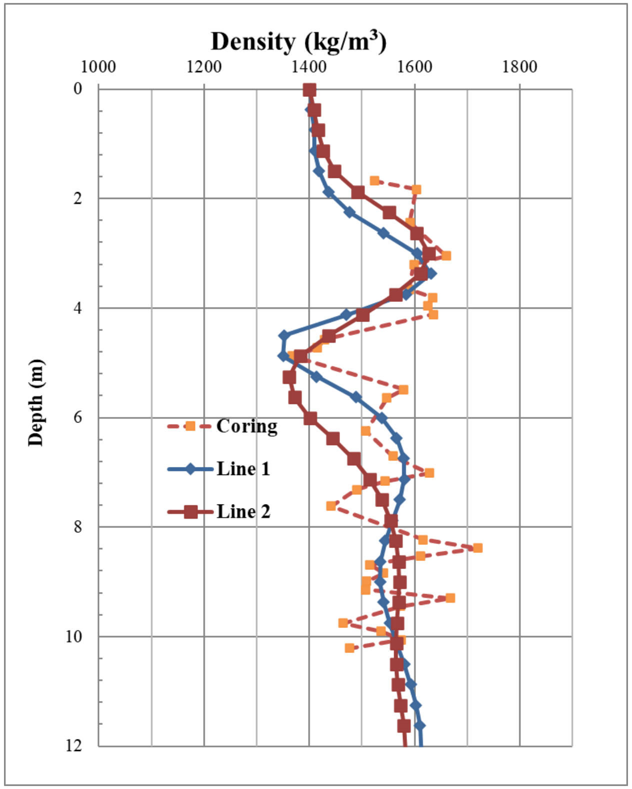


Waveform comparison 10-60 Hz

Bell results



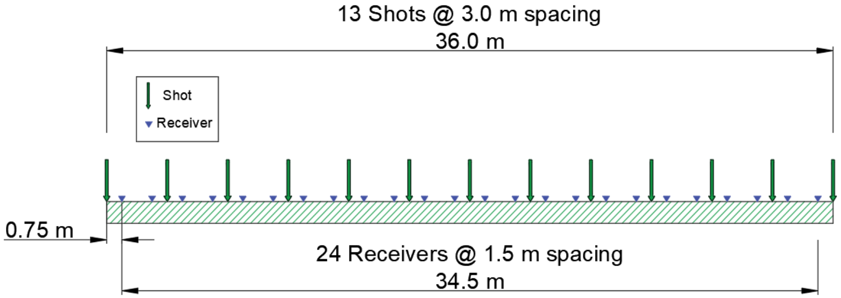
Bell site



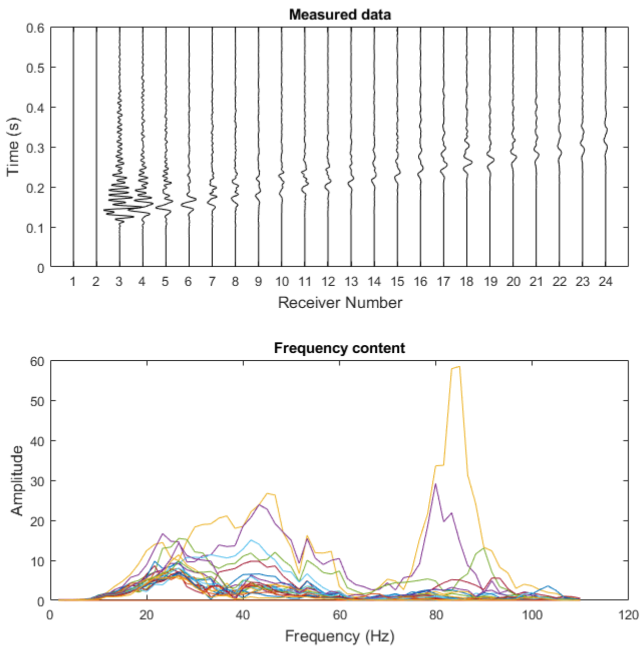
Comparison of inverted densities and coring sample's density

Site 2: Kanapaha site

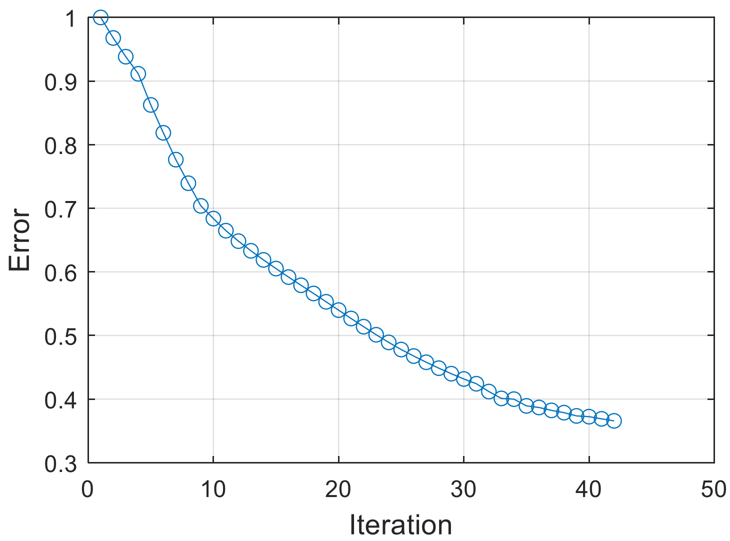
New seismic shear source

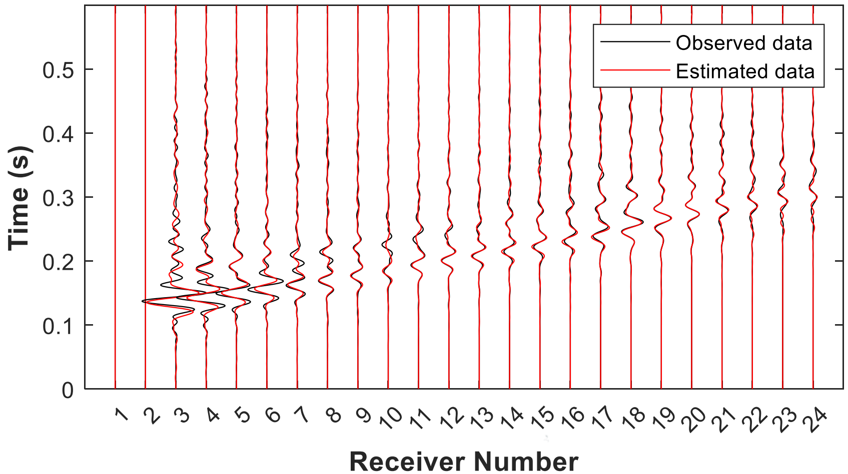


Kanapaha: data analysis



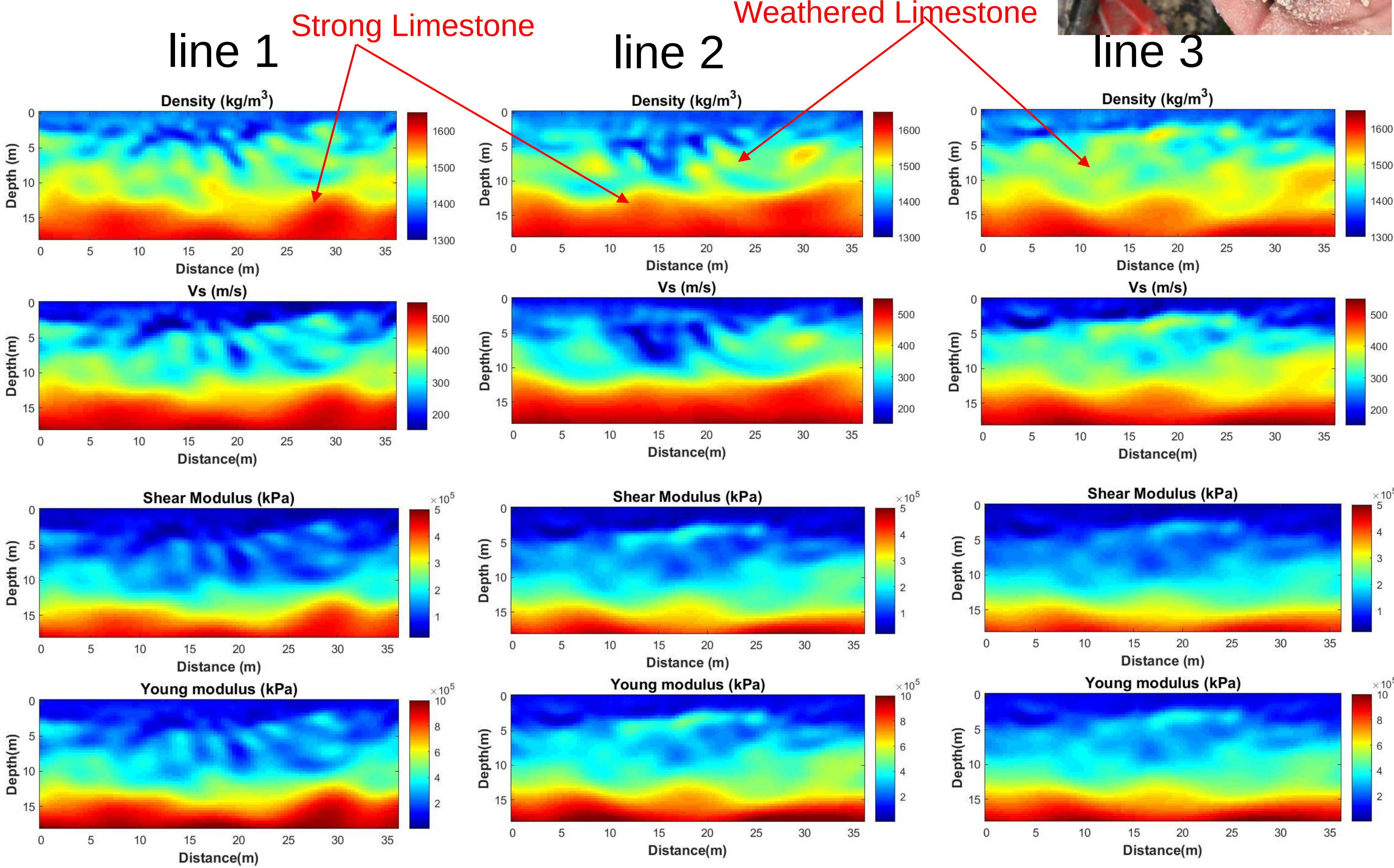
Measured data 10-100 Hz



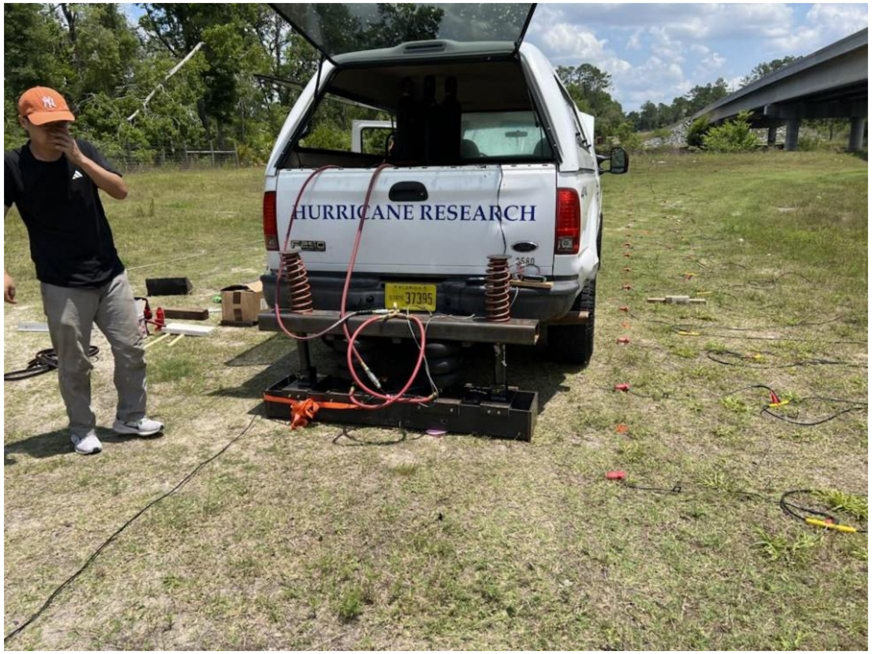


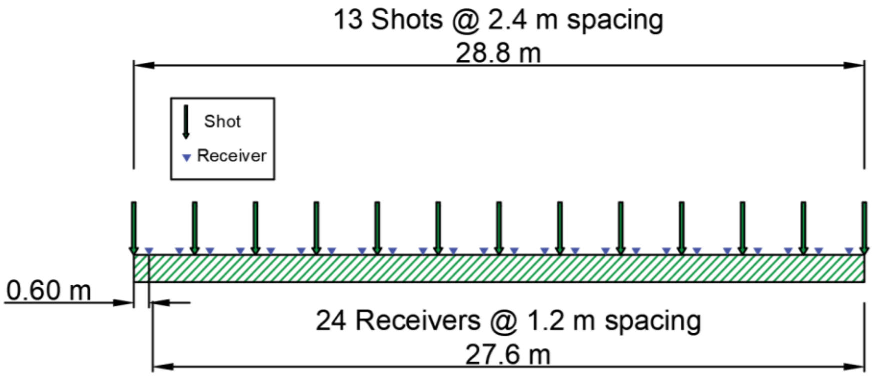
Waveform comparison 10-60 Hz

Kanapaha results



Site 3: CR 250 over Suwannee river

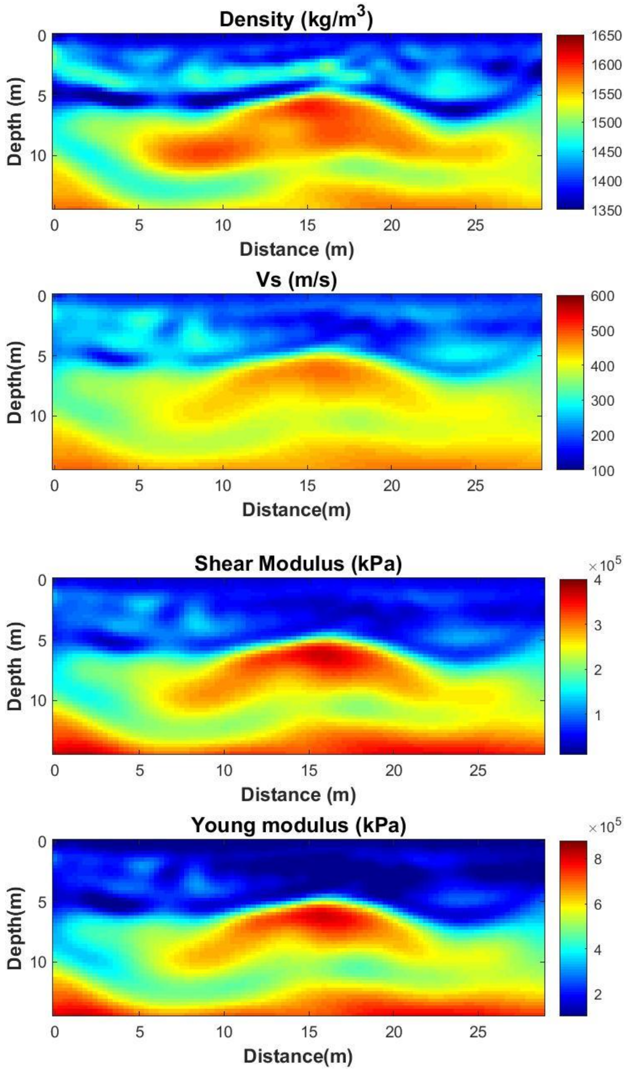




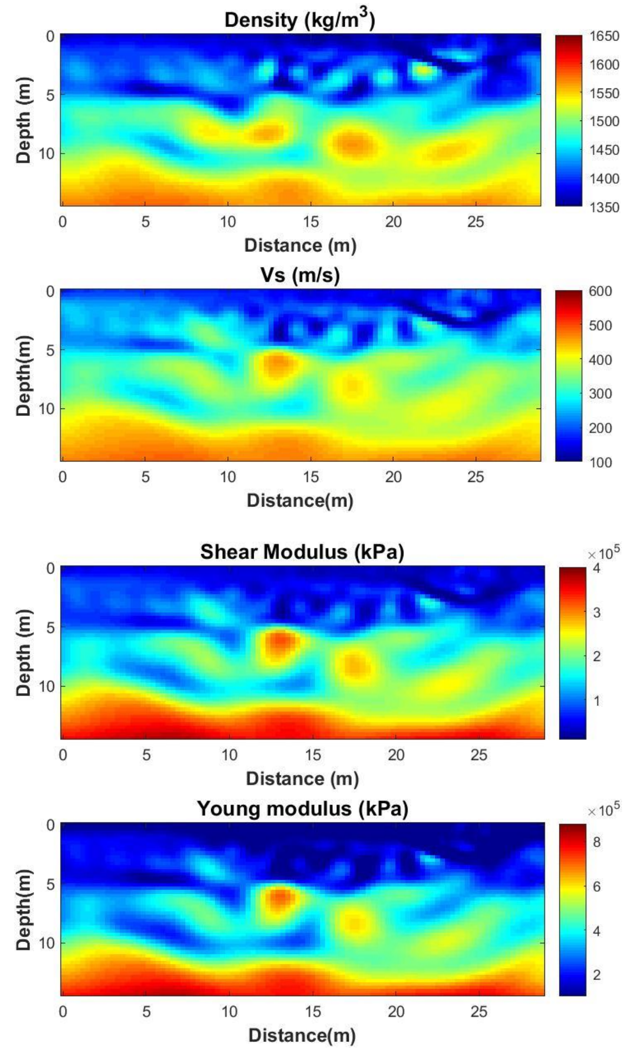
Two parallel test lines of 96 ft, 20 ft apart
24 geophones at 4 ft, 13 source at 8 ft

CR 250 results

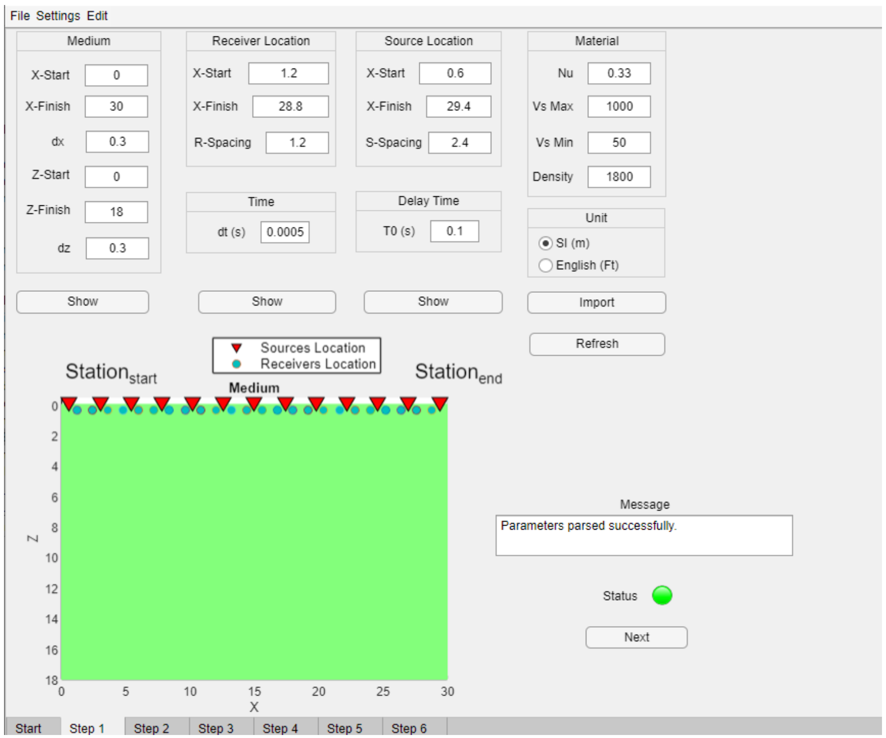
line 1

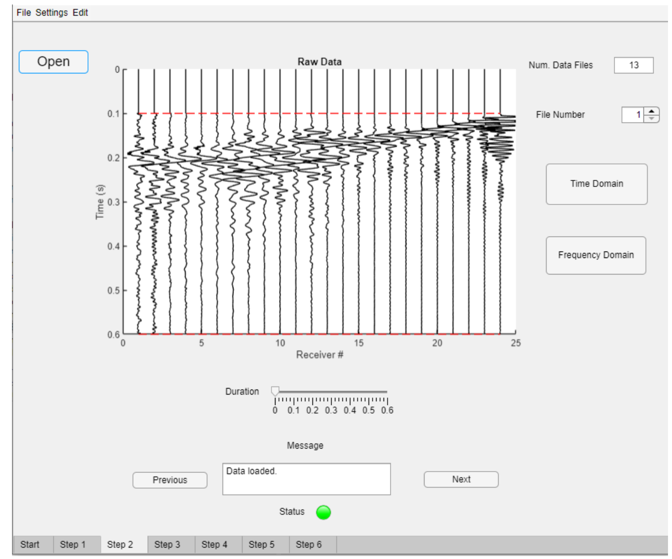


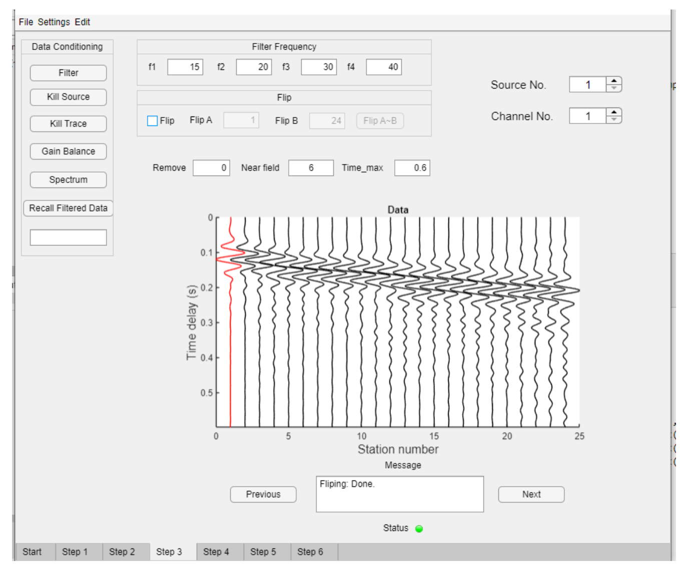
line 2



Task 4: Develop GUI software (ongoing)







- GUI for SH-Love FWI is developed in Matlab and converted to executable file
- Users can graphically input receivers/sources, condition and analyze data

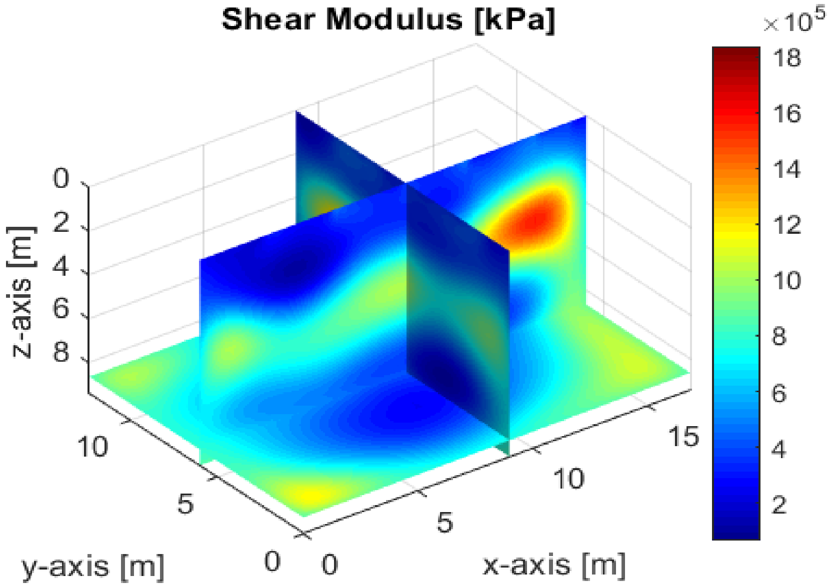
Conclusion

- A new 2D SH-Love FWI method has been developed for independent characterization of rock density along with moduli.

- Field results confirm that the rock density and strength can be characterized by the SH-Love FWI method. Soil/rock density from seismic testing is consistent with those from coring samples.

- The method provides new capabilities of rock characterization:
  - material volume properties (density and moduli) over a length of 100 ft down to depth of 40 ft at 1 ft resolution.
  - identifies mass properties vs. point properties (sample)
  - clearly shows horizontal layering as well as variability

Thank You!



Shear Modulus [kPa]
×10^5
18
16
14
12
10
8
6
4
2
0
2
4
6
8
z-axis [m]
10
5
0
y-axis [m]
0
5
10
15
x-axis [m]