Determination of in-situ rock density and strength with SH-Love wave tomography

FDOT BED31-977-12

GRIP Meeting 2023

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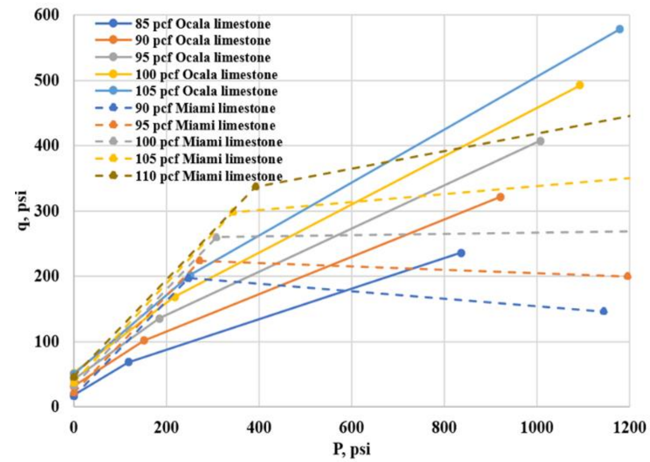
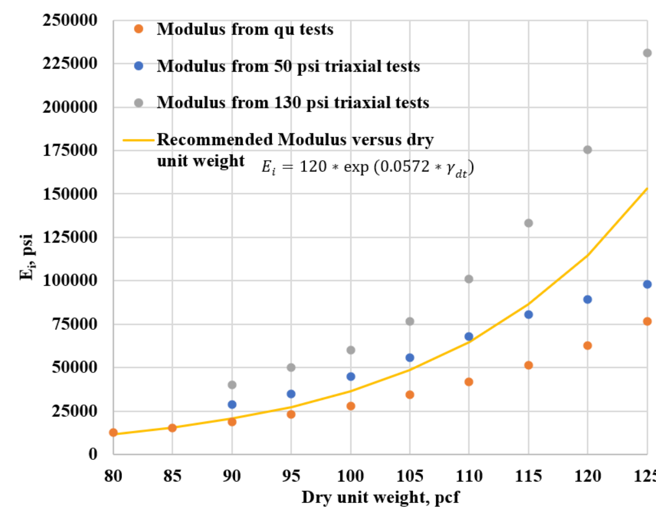
Presentation outline

- Introduction
- Project objectives
- Research approach
- Benefits of using SH and Love waves
- SH-Love FWI algorithm
- Synthetic experiment
- Field experiment at Bell site
- GUI development
- Conclusion

Introduction

- Detailed information on in-situ rock density and variability is important for the design and construction of shallow foundations.
- The recent FDOT projects BDV31-977-51 & BDV31-977-124 have shown that mass density (or unit weight) controls the rock strength as well as its stress-strain behavior for most Florida limestone formations.
- The current practice of measuring density is by weighing rock samples taken from limited borings within the footprint of the foundation.
- This project aims on developing a new method for determination of rock density and strength over a large volume without requirement of borings.





Project objectives

- The project objective is to develop an advanced testing system (hardware and software) for determination of rock density and strength.

- The system will enable to provide both density and strength at relevant resolutions (foot pixels) for entire rock volume supporting foundations without requirement of borings.

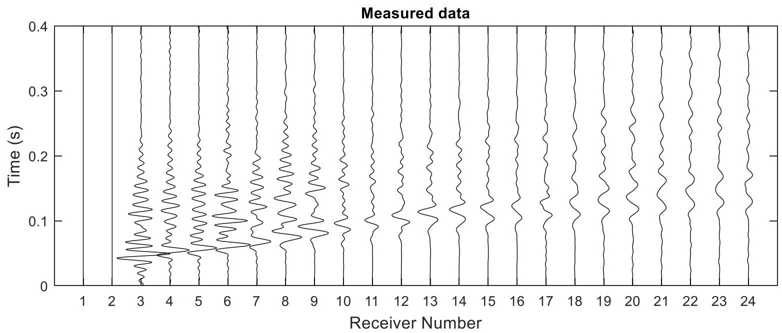
- The GUI module will be transferred to FDOT for future uses in site investigations of soil/rock properties and stratigraphy.

Research approach

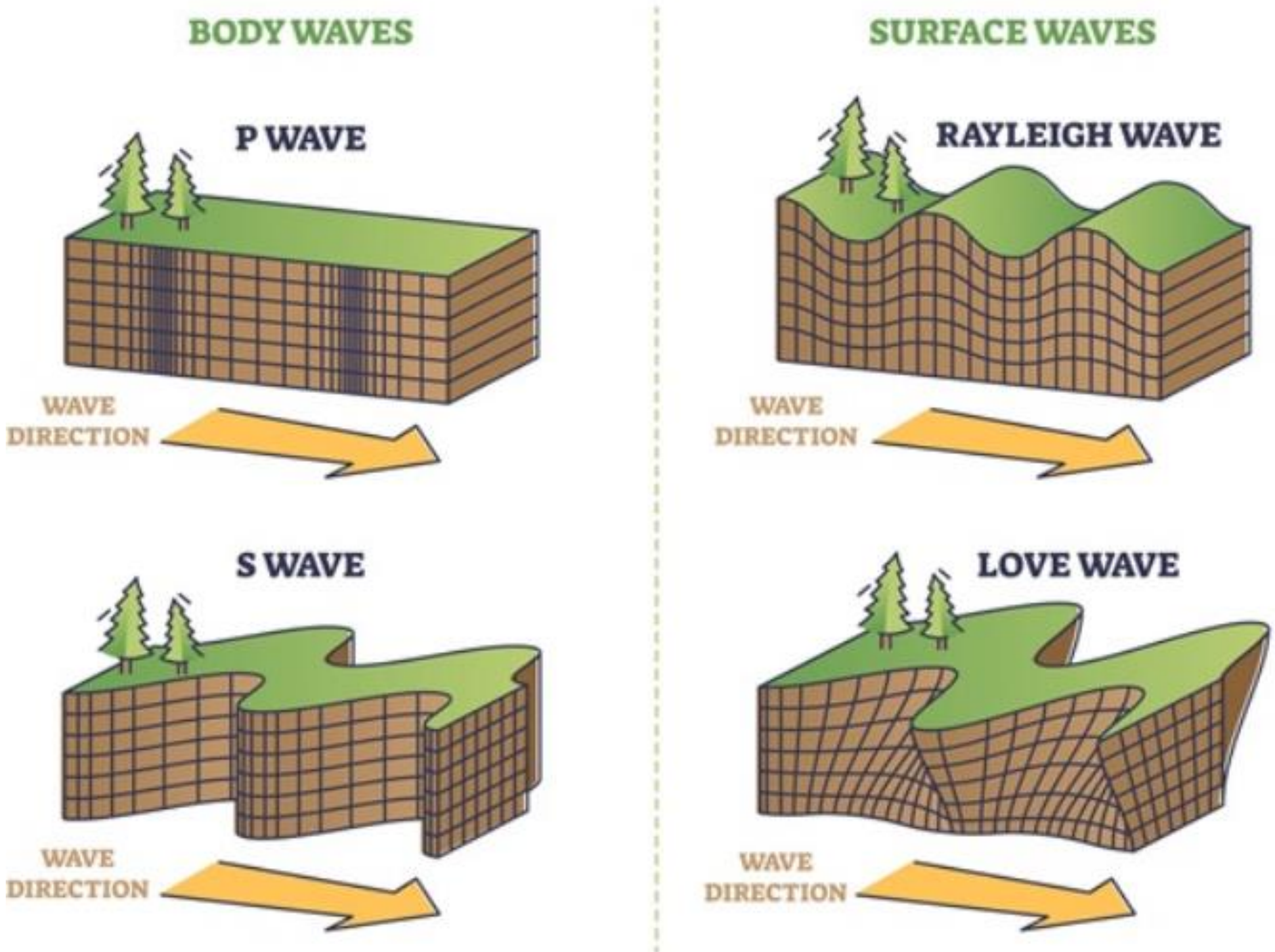
- The research is to develop an SH-Love full waveform inversion (2D SH-Love FWI) method, which can characterize 2D density and S-wave velocity of subsurface soil/rock at foot-scale down to a depth of at least 30 ft.

- The horizontal shear (SH) and Love waves are generated by applying a horizontal source (e.g., horizontally striking sledgehammer on shear beam) and recorded by an array of horizontal geophones on the ground surface.

- The recorded waveform data are analyzed to extract density and S-wave velocity of the subsurface materials.



Seismic wave types



VectorMine (Dreamstime.com)

Benefits of using SH and Love waves

- SH and Love waves (horizontal source) are much more sensitive to material density than vertical S-wave, P-wave, and Rayleigh waves (P-SV) (vertical source), and thus the density could be extracted more accurately.

- SH-Love wave simulation requires less computing time (40% that of P-SV waves), and the 2D SH-Love FWI can be done quickly (15 minutes) in the field.

- Both density (ρ) and S-wave velocity (Vs) can be characterized. Shear (G) and Young (E) moduli can be computed for determination of shallow foundation's settlement and bearing capacity, and other geotechnical analyses.

$$G = \rho V_s^2$$

$$E = 2G(1 + v)$$

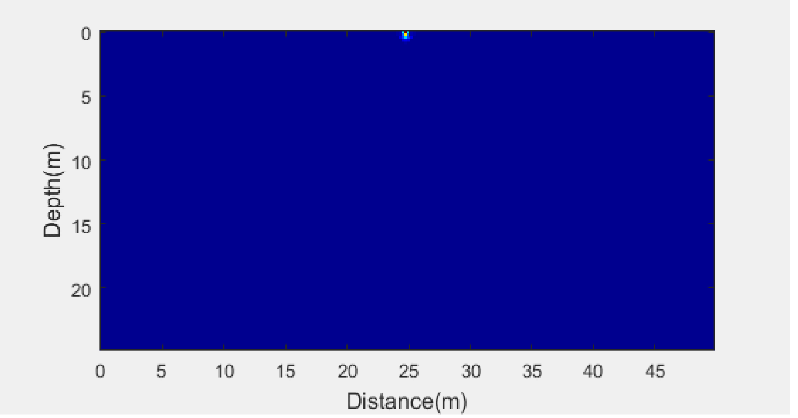
Task 1: Develop 2D SH-Love FWI algorithm

- Forward modelling

$$\rho(x,z)\frac{\partial v_y}{\partial t} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} + f_y \qquad (1)$$

$$\frac{\partial \sigma_{xy}}{\partial t} = \mu(x,z)\frac{\partial v_y}{\partial x} \qquad (2)$$

$$\frac{\partial \sigma_{yz}}{\partial t} = \mu(x,z)\frac{\partial v_y}{\partial z} \qquad (3)$$



ρ: density, $\mu = G = \rho V_s^2$

- Perfectly matched layers (PML) are applied at the bottom and vertical boundaries to absorb outgoing waves.

2D SH-Love FWI

- Model update (adjoint-state method)
  - Step 1. Calculate the residual between estimated data and observe data

$$\Delta \boldsymbol{d}_{s,r} = \boldsymbol{D}_{s,r}(\mathbf{m}) - \boldsymbol{d}_{s,r} \quad (4)$$

  - Step 2. Compute the Least-squares error

$$\mathrm{E}(\mathbf{m}) = \frac{1}{2}\Delta \boldsymbol{d}^{T}\Delta \boldsymbol{d}, \quad (5)$$

  - Step 3. Compute the gradients (adjoint-state gradient approach)

$$\frac{\partial E}{\partial V_s} = -\frac{2}{V_s^3 \rho}\sum_{i=1}^{N}\int_0^T dt(\sigma_{xy}^f \sigma_{xy}^b + \sigma_{yz}^f \sigma_{yz}^b), \quad (6)$$

$$\frac{\partial E}{\partial \rho} = -\frac{1}{V_s^2 \rho^2}\sum_{i=1}^{N}\int_0^T dt(\sigma_{xy}^f \sigma_{xy}^b + \sigma_{yz}^f \sigma_{yz}^b + V_s^2 \rho^2 \frac{\partial v_y}{\partial t} u_y^b), \quad (7)$$

2D SH-Love FWI

- Model update
  - Step 4. Smooth gradients (Tikhonov regularization) to minimize the ill-posed inverse problem

$$\left(\frac{\partial E}{\partial V_s}\right)_r = \frac{\partial E}{\partial V_s} + \lambda_1 \boldsymbol{D} V_s \qquad (8)$$

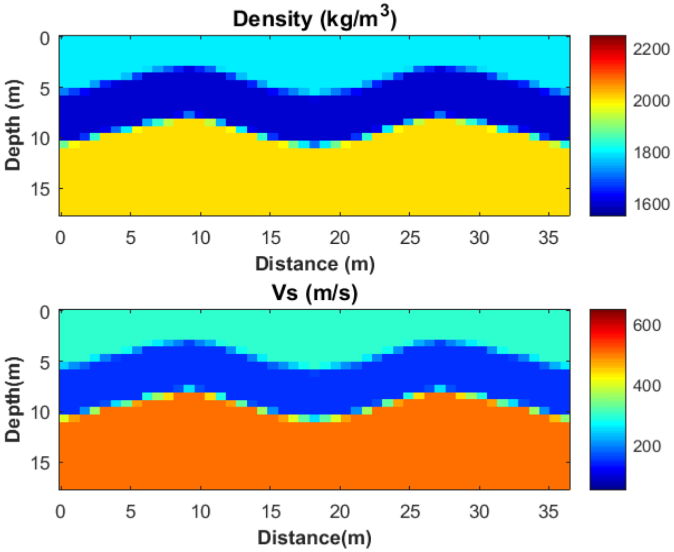
$$\left(\frac{\partial E}{\partial \rho}\right)_r = \frac{\partial E}{\partial \rho} + \lambda_2 \boldsymbol{D} \rho \qquad (9)$$

  - Step 5. update Vs and density models (steepest-descent method)

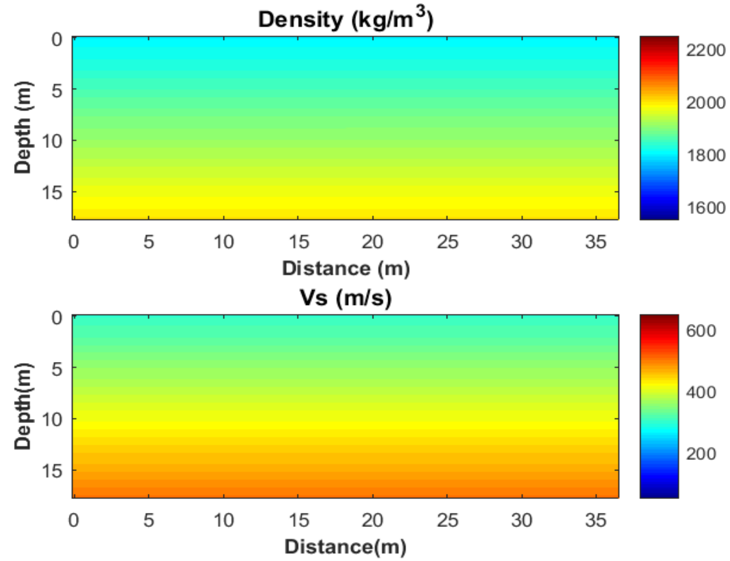
$$V_{s_{n+1}} = V_{s_n} - \alpha_n \boldsymbol{H}_n^{-1} \left(\frac{\partial E}{\partial V_s}\right)_r , \qquad (10)$$

$$\rho_{n+1} = \rho_n - \beta_n \boldsymbol{H}_n^{-1} \left(\frac{\partial E}{\partial \rho}\right)_r , \qquad (11)$$

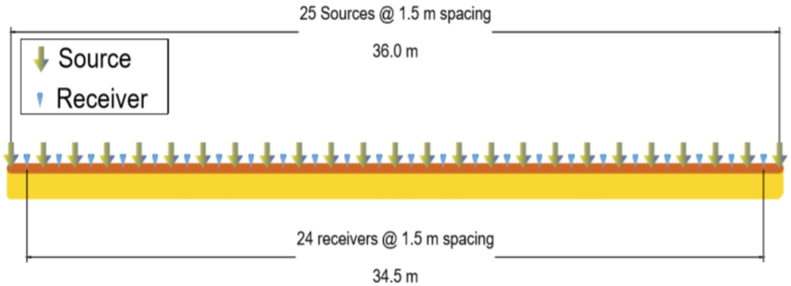
Synthetic experiment



true model

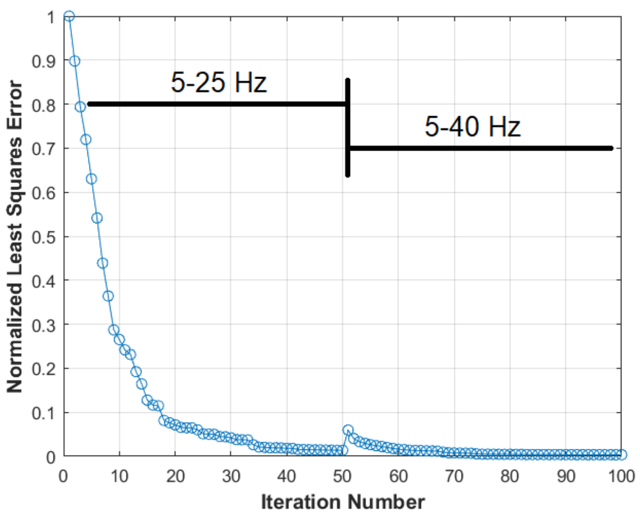


initial model

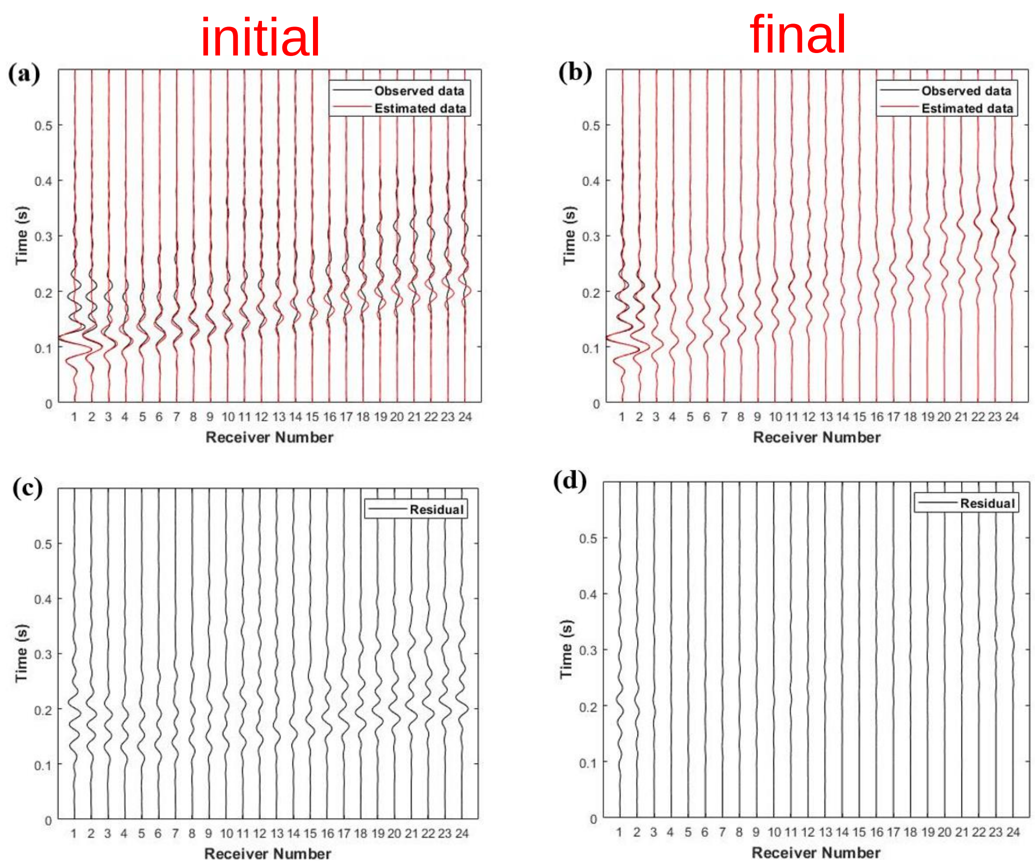


Test configuration

Synthetic experiment

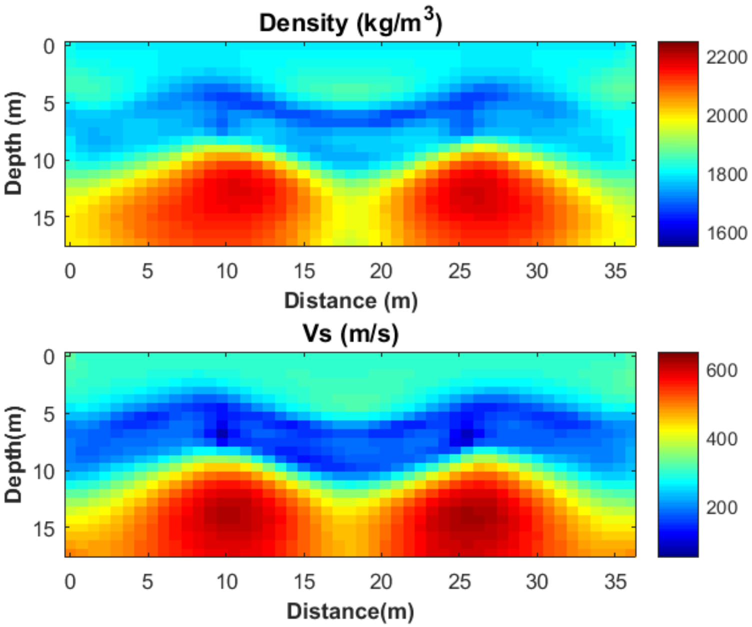


Least-square error

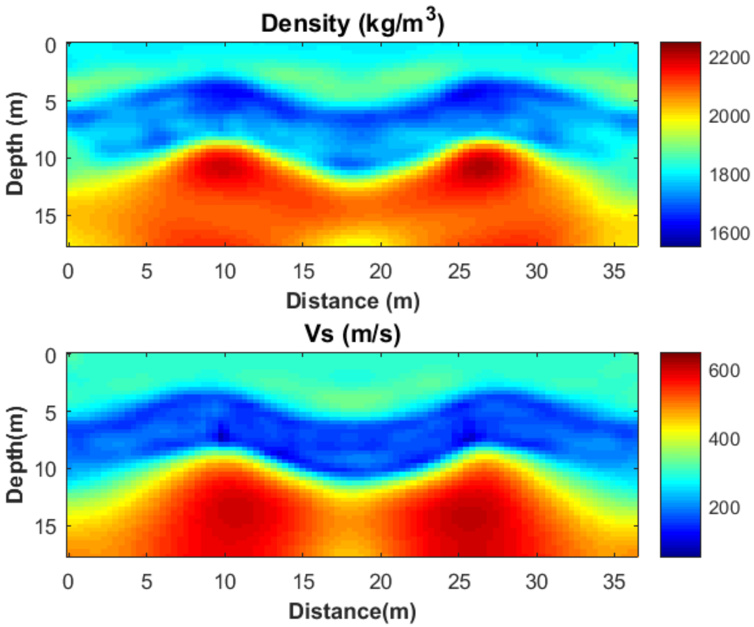


Waveform comparison

Synthetic results

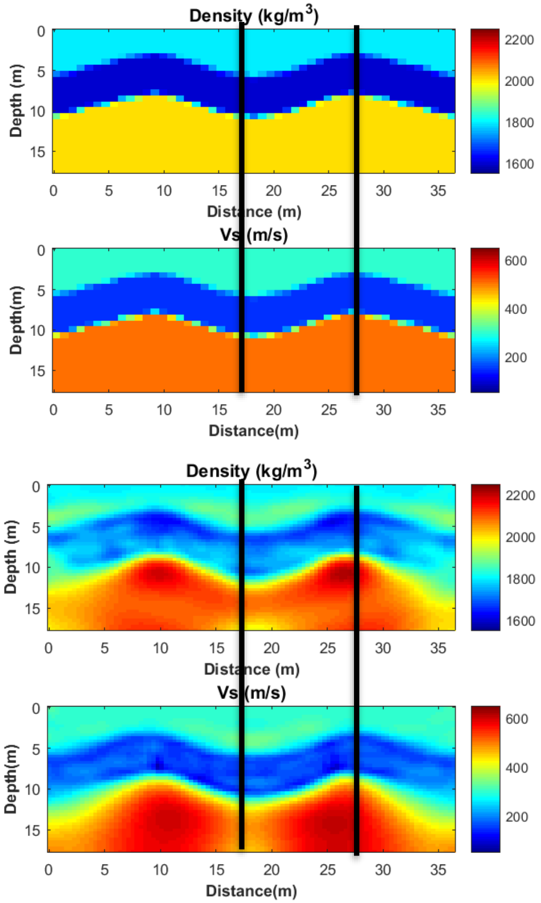


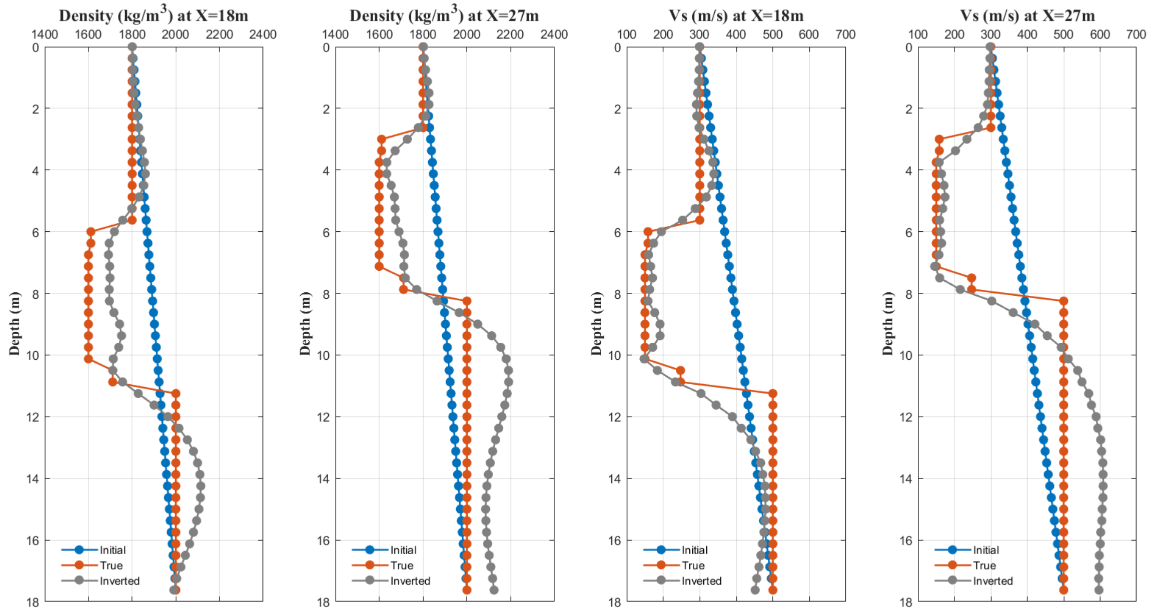
First analysis at 5-25 Hz



Second analysis at 5-40 Hz

Synthetic results



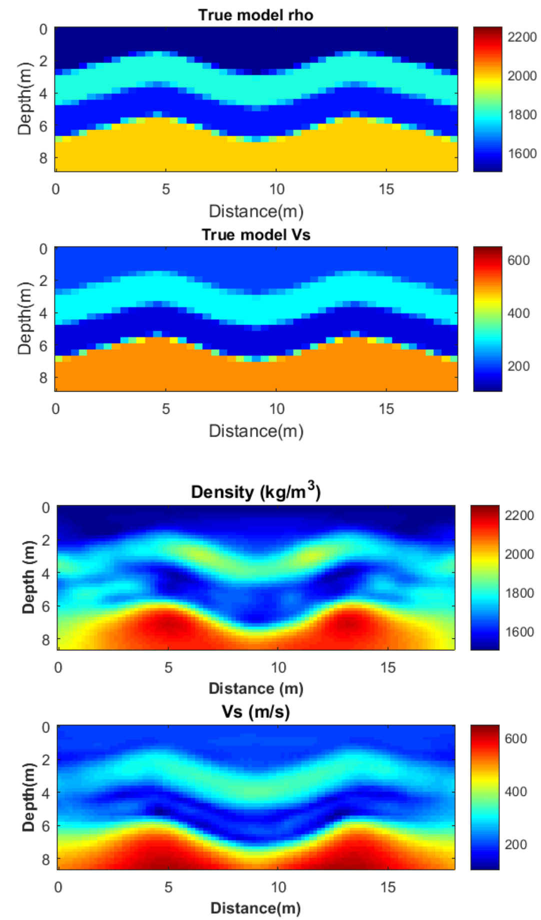


Detailed comparison at two locations

Task 2: Optimize test configurations and wavefield characteristics

- ➢ Relate the characterized resolution with depth to test configurations and frequency content of data.

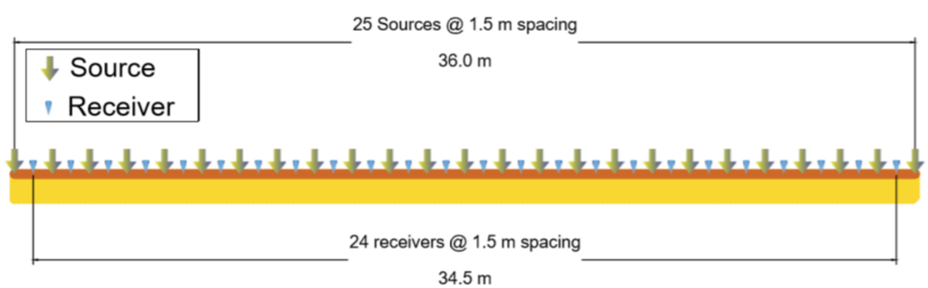
- ➢ Optimal test configurations
- ▪ Resolution is ¼ of wavelength
- ▪ 60 ft depth: test length 120 ft, geophone spacing of 5 ft and source spacing of 10 ft, data up to 40 Hz (15” pixel)
- ▪ 30 ft depth: test length 60 ft, geophone spacing of 2.5 ft and source spacing of 5 ft, data up to 80 Hz (7.5” pixel)



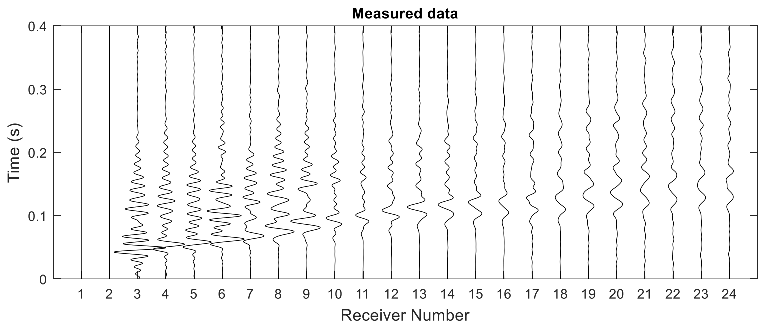
Task 3: Verify the 2D SH-Love FWI algorithm with field experiments

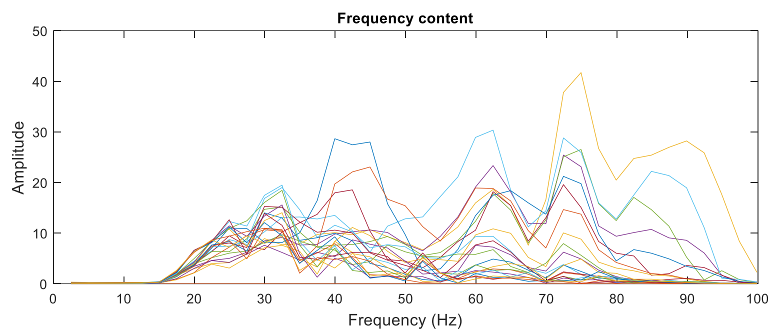
Bell site experiment

5’ I-beam

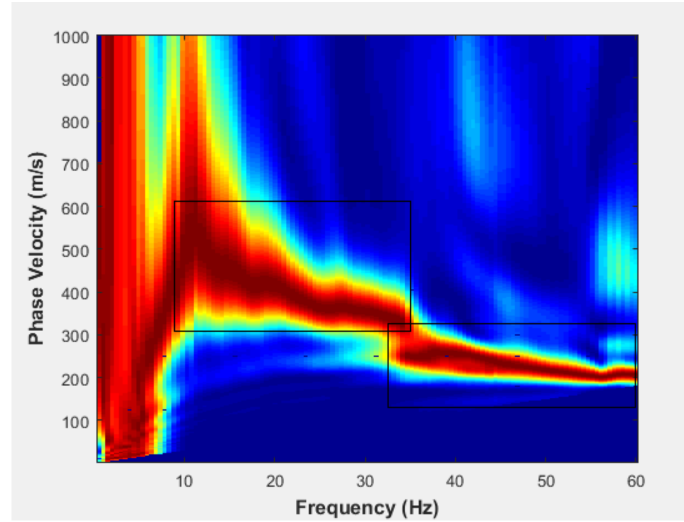


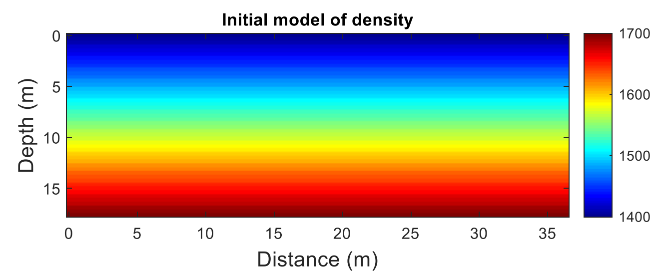
Bell site experiment

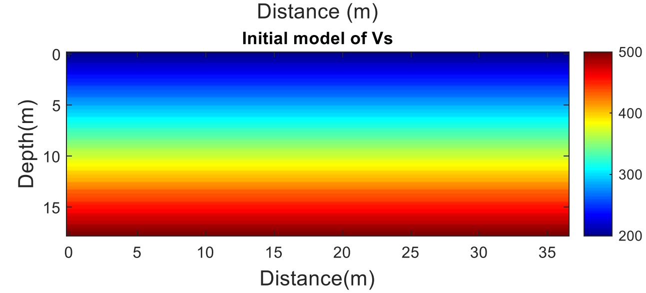




Sample data



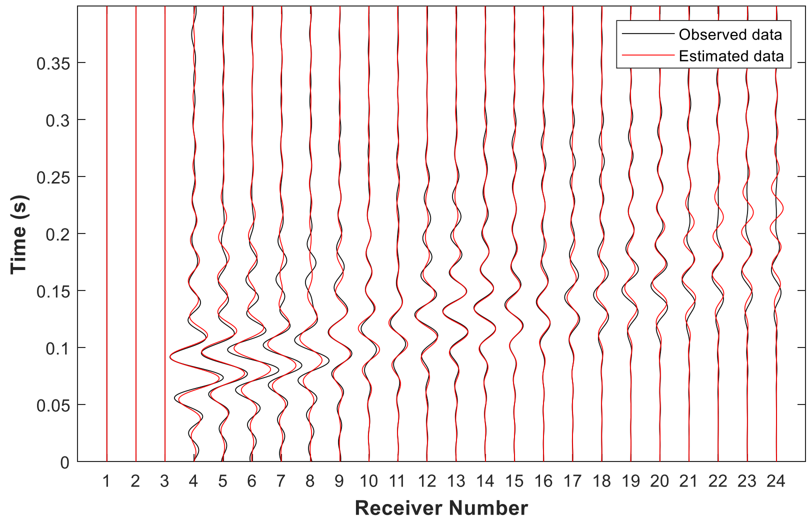




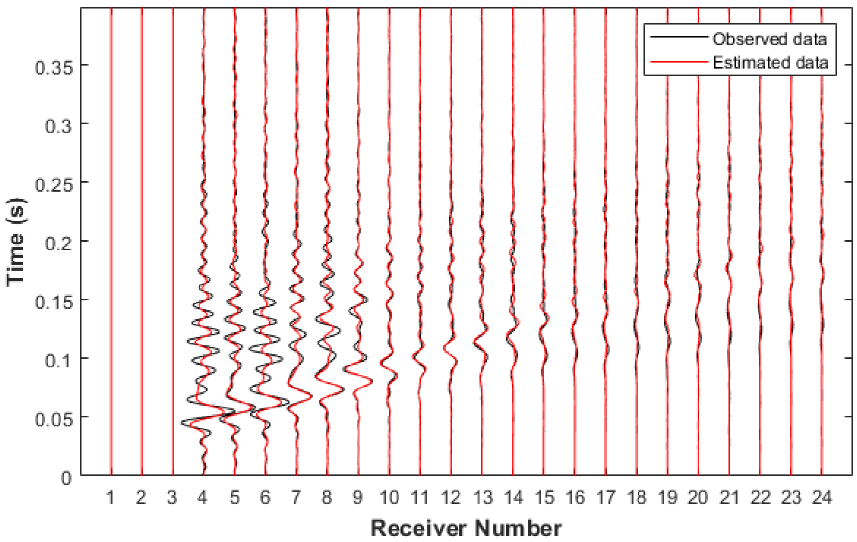
Bell site: data analysis

- First run at 10-50 Hz
- Second run at 10-80 Hz

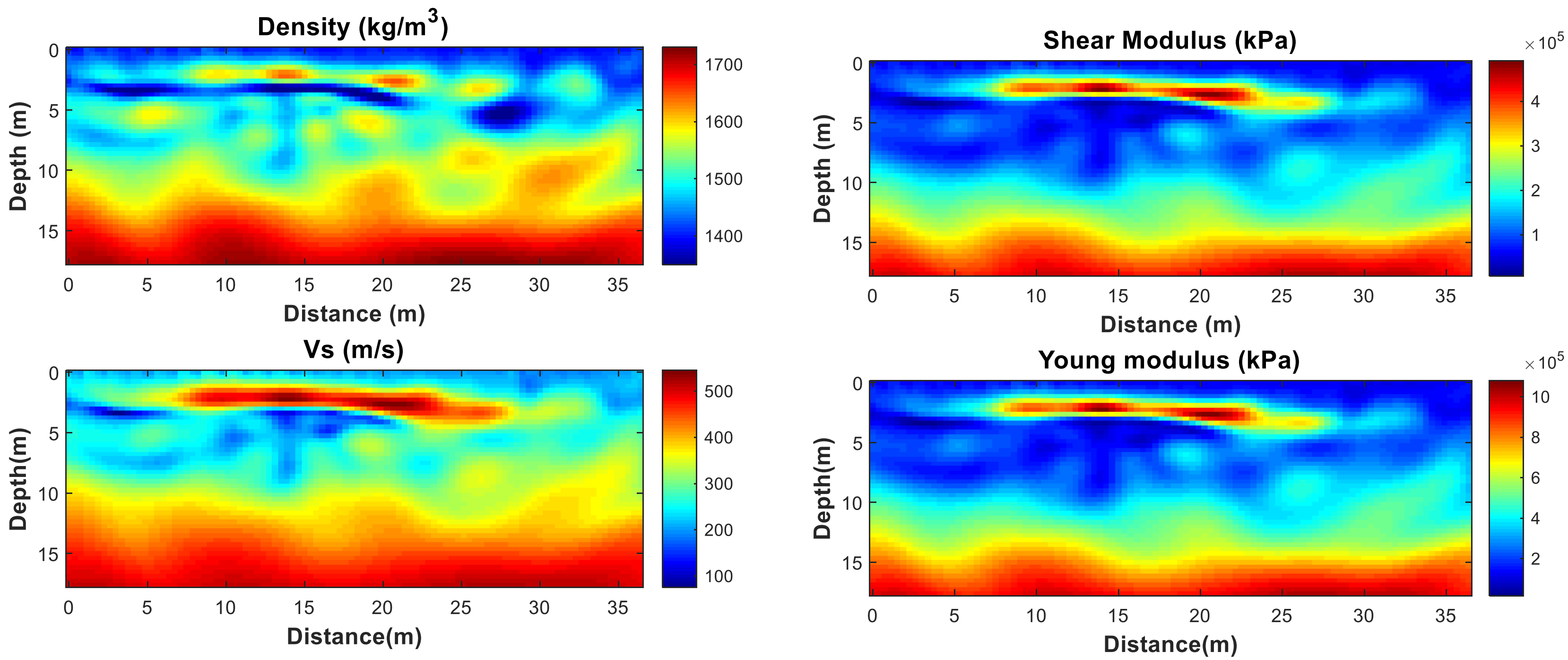
10-50 Hz



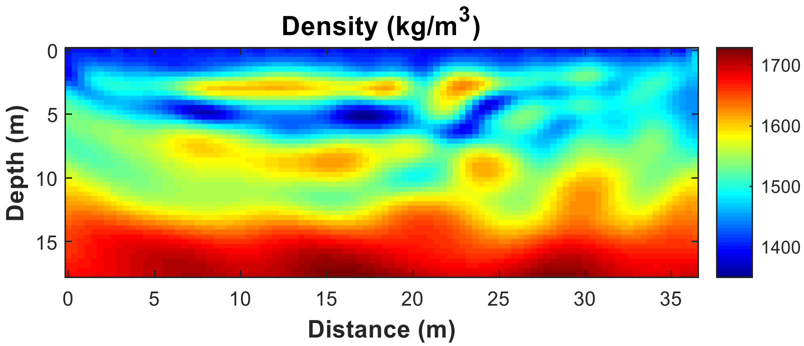
10-80 Hz

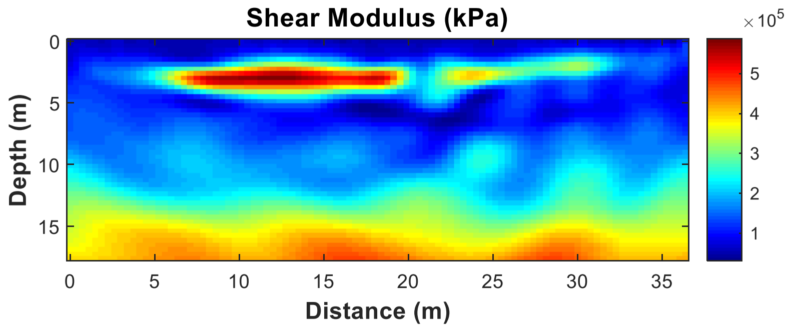


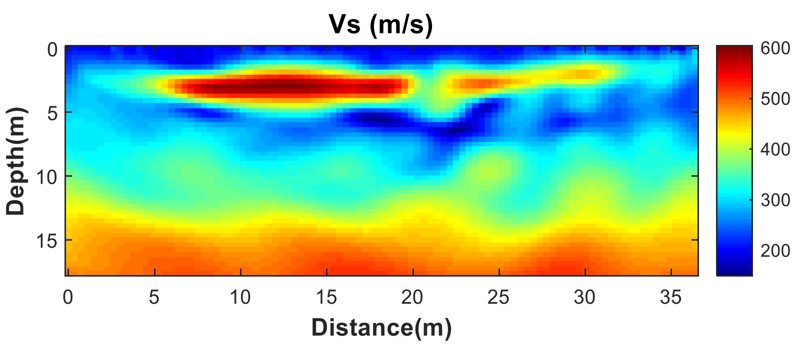
Bell result: line 1

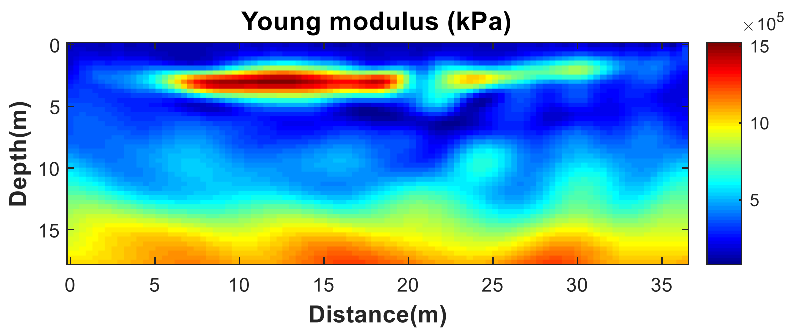


Bell result: line 2

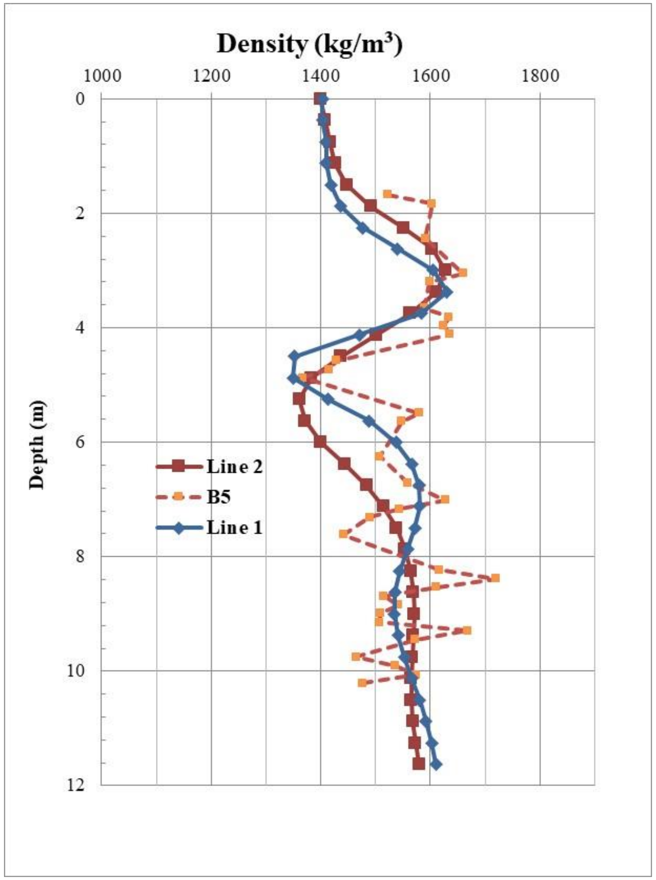








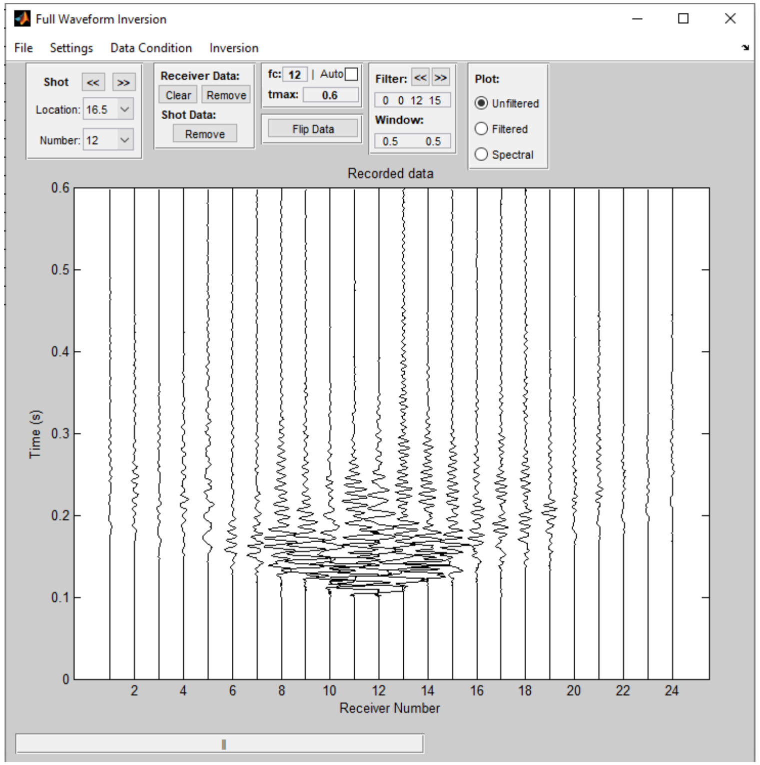
Bell site

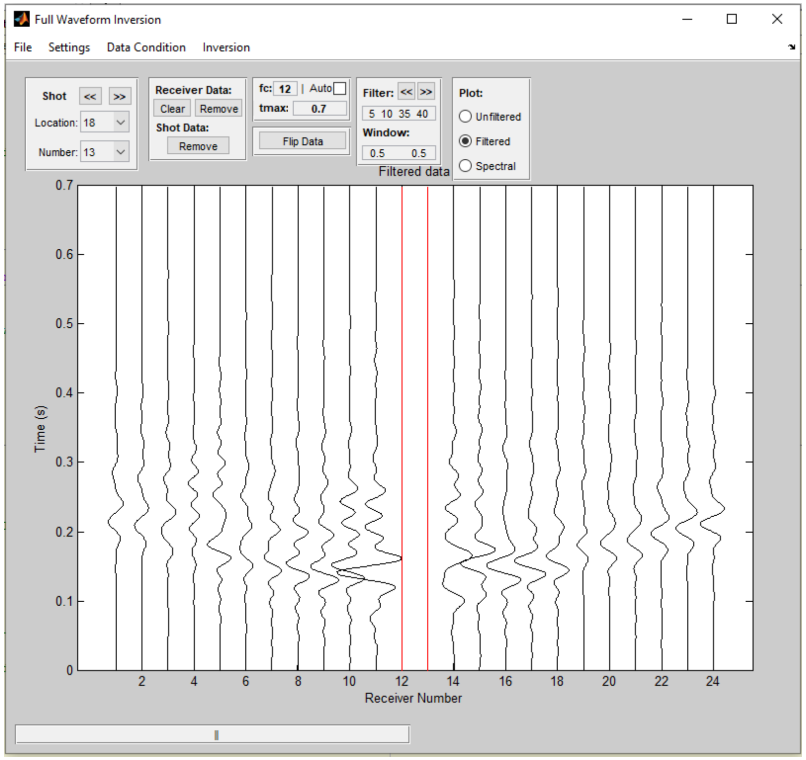


Comparison of inverted densities and coring sample's density

Task 4: Develop GUI software

- GUI for SH-Love FWI will be done in Matlab and converted to executable file
- Users can graphically input receivers/sources, condition and analyze data





Conclusion

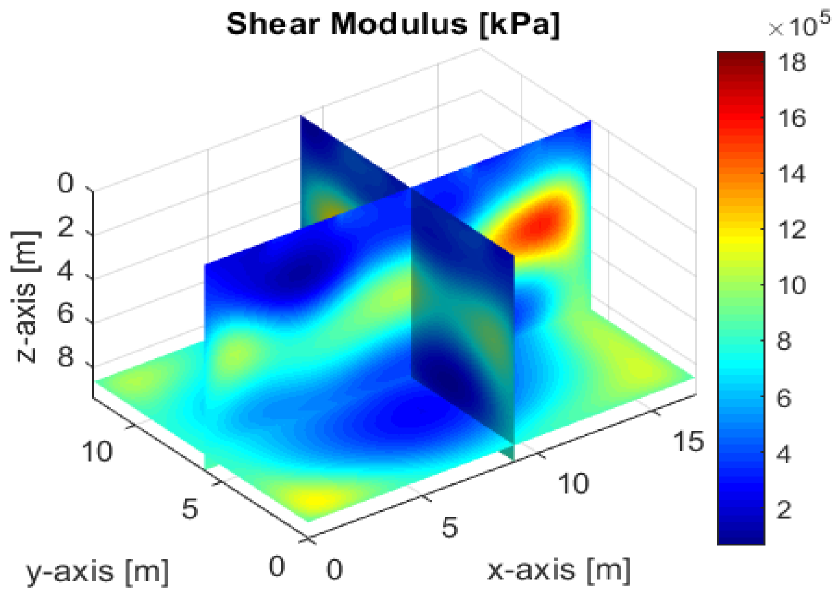
- A new 2D SH-Love FWI method has been developed for characterization of rock density and strength.

- Synthetic results show that the method can characterize both mass density and Vs.

- Field results confirm that the rock density and strength can be characterized by the SH-Love FWI method. Soil/rock density from seismic testing seems consistent with those from coring samples.

- The method is expected to provide new capabilities of rock characterization for entire material volume supporting shallow foundations at high resolutions (foot pixels to 30 ft depth).

Thank You!



Shear Modulus [kPa]
×10^5
18
16
14
12
10
8
6
4
2
0
2
4
6
8
z-axis [m]
10
5
0
y-axis [m]
0
5
10
15
x-axis [m]