## Sinkhole Detection with 3-D Full Elastic Seismic Waveform Tomography

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## Need for 3D sinkhole detection

> Sinkholes can cause infrastructure collapses that lead to significant property damage and even fatalities
> Typical invasive testing SPT, CPT - tests $<.1 \%$ of material
> Need for NDT/geophysical testing over large volume of material
> Image vertical and lateral extents of 3D voids


Sinkhole collapses

## 3D FWI Motivation

> 3D FWI is wave-equation based and has the potential to

- use full information content (waveforms), both phase and magnitude
- characterize both Vp and Vs of 3D test domain at high resolution (ft pixel)
- provide 3 dimensions of a buried void



## 3D FWI method

> Forward modeling by 3-D wave equations
$\rho \frac{\partial v_{i}}{\partial t}=\frac{\partial \sigma_{i j}}{\partial x_{j}}+f_{i}$ where $i, j=1,2,3$
$\frac{\partial \sigma_{i j}}{\partial t}=\lambda \frac{\partial v_{k}}{\partial x_{k}}+2 \mu \frac{\partial v_{i}}{\partial x_{j}} \quad$ if $i \equiv j$
$\frac{\partial \sigma_{i j}}{\partial t}=\mu\left(\frac{\partial v_{i}}{\partial x_{j}}+\frac{\partial v_{j}}{\partial x_{i}}\right) \quad$ if $i \neq j$


PML is used at bottom and 4 vertical boundaries.

## 3D FWI method

> Model updating by Gauss-Newton

- Velocity residual: $\quad \Delta \mathbf{d}_{i, j}=\mathbf{F}_{i, j}(\mathbf{m})-\mathbf{d}_{i, j}$
- Misfit function:
- Model updating:

$$
\begin{aligned}
& \mathrm{E}(\mathbf{m})=\frac{1}{2} \Delta \mathbf{d}^{t} \Delta \mathbf{d} \quad \overbrace{\sqrt{n}}^{\text {Filter, focus, balance gradient vector }} \underbrace{n+1} \\
& \mathbf{m}^{n+1}=\mathbf{m}^{n}-\alpha^{n}\left[\mathbf{J}^{t} \mathbf{J}+\lambda_{1} \mathbf{P}^{t} \mathbf{P}+\lambda_{2} \mathbf{I}^{t} \mathbf{I}\right]^{-1} \mathbf{J}^{t} \Delta \mathbf{d}, \\
& \mathbf{J}_{i, j}=\frac{\partial \mathbf{F}_{i, j}(\mathbf{m})}{\partial m_{p}}
\end{aligned}
$$

- Gauss-Newton inversion is done in frequency domain to reduce RAM

$$
\tilde{u}(\mathbf{x}, \omega)=\sum_{l=1}^{n t} \exp (\sqrt{-1} \omega l \Delta t) u(\mathbf{x}, l \Delta t) \Delta t
$$

## Data Analysis

> Start analysis at lowest frequencies and move up
> Low frequencies (large wavelengths) require less detailed information of initial model
> Adding high frequency data gradually helps increase resolution of entire domain and resolve small features

Misfit function $E(\mathbf{m})$


## Synthetic test on void




> $24 \times 36 \times 18 \mathrm{~m}$ model, $4.5 \times 4.5 \times 4.5 \mathrm{~m}$ at 9 m depth
$>$ Test configurations

- $8 x 12$ (96) receivers at 3 m spacing
- $9 x 13$ (117) shots at 3 m spacing



## Synthetic result: 3D view

> 2 inversion runs at 15 and 25 Hz central frequencies
> 36 hours on a desktop computer ( 32 cores of 3.46 GHz each and 256 GB of memory)


Initial model


Inverted result

## Synthetic result: plane view at void center




Vertical view

Horizontal view

## Gainesville site

- dry retention pond in Gainesville
- test area of $36 \times 9 \mathrm{~m}$ ( $120 \times 30 \mathrm{ft}$ )
- 96 receivers located in $24 \times 4$ grid
- 52 shots located in $13 \times 4$ grid
- 48 geophones twice
- PEG active source


Stage 1


## Sample data

> measured data combined from the two stages for 96 -channel shot gather
> consistent wave magnitudes and propagation pattern



## Data analysis

- Test domain of 60 ft . depth $\times 120 \mathrm{ft}$. length $\times 30 \mathrm{ft}$. width is divided into 13,824 cells of $2.5 \times 2.5 \times 2.5 \mathrm{ft}$.

- Power spectrum
- 2 inversion runs at 12 and 22 Hz central frequencies
- 26 hours of computer time

- Initial model


## Gainesville site




Waveform comparison for 2 sample shots

## Gainesville site results



## Gainesville site: results at planes



Views of Different Distance (y axis) Note change in Vs $(0<y<9 m(30 f t))$

## Gainesville site: Vs vs. SPT



## Newbery site

> dry retention pond in Newberry
> 25 lines (A to Y ) at 3 m spacing
> tested on a known void identified by 2D FWI in 2011
$>$ test area of $36 \times 12 \mathrm{~m}$ ( $120 \times 40 \mathrm{ft}$ )
> 48 receivers located in $12 \times 4$ grid
$>65$ shots located in $13 \times 5$ grid
> 24 geophones twice
> PEG source


## Data analysis

- Test domain of 60 ft . depth $x 120 \mathrm{ft}$. length x 40 ft . width is divided into 18,432 cells of $2.5 \times 2.5 \times 2.5 \mathrm{ft}$.
- 2 inversion runs at 12 and 22 Hz central frequencies
- 44 hours of computer time

- Power spectrum


Initial model

## Newbery site



Waveform comparison for 2 sample shots

## Newberry site results



## Newberry site: results at vertical planes



## Conclusion

> Both Vs and Vp could be characterized at high resolution (ft pixel) to 60 ft in depth by the 3-D FWI method.
> Buried voids could be identified to 3-diameter depth with surface measurement.
> 30-40 hours of computer time for each test area of $120 \times 40 \mathrm{ft}$

## Thank You!



