CONSTRUCTION MATH

a training course developed by the

FLORIDA DEPARTMENT OF TRANSPORTATION

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CONSTRUCTION MATH

CONTENTS

DIRECTIONS TO COURSE USERS

INTRODUCTION

This is Construction Math -- a course of training in basic mathematics for highway inspection personnel. It can also be used by materials testing, design and other technical employees.

TRAINING TECHNIQUE

Construction Math has been designed for self-instructional training:

► You can work alone.
► You can make as many mistakes as are necessary for learning -- and correct your own mistakes.
► You can finish the training at your own speed.

Some space has been provided for you to make calculations in this book -- but use a scratch pad too. You will then be able to work faster, make mistakes without a lot of erasing, and make up and solve problems of your own.

You will keep this book as your personal copy, so work neatly any problems you may want to use for reference.

EXAMINATION

An examination has been developed for Construction Math. It contains questions and problems only -- no answers. So that you can prepare yourself properly for that examination, Construction Math contains quizzes and highway problems. If you can handle the quizzes and highway problems in Construction Math, you will have no difficulty with the examination.

INSTRUCTIONS

This is not an ordinary book. You can't read it from page to page as you do other books.

Most chapters begin with STARTING POINTS FOR TRAINING. If you already know how to make certain calculations you will be advised to skip the respective training sections.

The answers to all problems follow their respective chapter.

Go on to Chapter One.

CHAPTER ONE

Basic Calculating

CONTENTS

1

BASIC CALCULATING

This chapter has been written to provide a review-- a brief retraining in adding, subtracting, multiplying and dividing. It covers both whole and decimal numbers.

Much of the training involves things you have known before, and probably know now. Other discussions will make clear things you have misunderstood about basic calculations.

Experience with persons who have taken this training indicates that you should review this chapter -- even if you need little practice in the basic calculations.

STARTING POINTS FOR TRAINING

Different persons need to start their training in different places. Work the following problems.

PROBLEMS

1. $503.1 \div 29.25 =$ ____________
2. $1163.4375 \div 42.5 =$ ____________

Right? Work the "HIGHWAY PROBLEMS" on page 1-35. If you have difficulty with those problems, study the appropriate sections in this chapter.

Wrong? Try Problems 3 and 4.

3. $27.5 \times 36.5 =$ ____________
4. $414.5 \times 21.89 =$ ____________

Right? Fine! You can skip to the section titled "DIVIDING WHOLE AND DECIMAL NUMBERS" on page 1-24. If you would rather review "BASIC CALCULATING," scan the earlier sections.

Wrong? Work Problems 5 and 6.

5. $4151 - 393.5 =$ ____________
6. $51.68 - 42.9579 =$ ____________

Right? Start with "MULTIPLICATION AND DIVISION" on page 1-17. Review "ADDITION" and "SUBTRACTION" if you like.

Wrong? Try Problems 7 and 8.

7. $2150.5 + 377.1 =$ ____________
8. $64.54 + 975.2376 =$ ____________

Right? Start studying "SUBTRACTION," page 1-12. You probably should study "ADDITION" for a quick review.

ADDITION

ADDING WHOLE NUMBERS

You probably will remember everything about adding whole numbers -- but let's see.

Add these numbers: 26, 70, 2 and 36.

One -- Set the problem up -- like this:
-- Line up your columns from the right.

$$\begin{array}{r} 26 \\ 70 \\ 2 \\ \underline{+\ 36} \end{array}$$

Two -- Start at the top of the right column and add down.
-- In this case, 6 + 2 + 6 = 14

Add down

$$\begin{array}{r} 26 \\ 70 \\ 2 \\ \underline{+\ 36} \\ 4 \end{array}$$

Three -- Write 4 below the column you added, as shown above.
-- Write 1 above the next column, as shown below.
-- The "1" is a "carry digit." It is "carried" from the answer in the first column and added with the digits in the second column.

Four -- Add the second column and write the answer in the answer space as shown.
-- In this case, the digits 1, 2, 7 and 3 add to 13

Carry digit

$$\begin{array}{r} \mathbf{1} \\ 26 \\ 70 \\ 2 \\ \underline{+\ 36} \\ 134 \end{array}$$

PROBLEMS

9. Add the following whole numbers:

   3 + 31 + 430 + 27 = _______________

10. Add the following whole numbers:

    11 + 273 + 200 + 35 = _______________

Right? On both problems? Study "ADDING DECIMAL NUMBERS."

Get one wrong -- or both? Study "Discussion on Problems 9 and 10" on the next page.

Discussion on Problems 9 and 10

You made a mistake in Problem 9 or Problem 10 -- or both. Let's look at them in more detail.

- Did you line up your columns from left to right?

- Make sure the right-hand column is straight.
- Make all other columns just as straight.

3	11
31	273
430	200
27	35

- Did you add the <u>right</u> columns first?

- Ignore the other columns while doing this step.
- Pretend they are not there.

3	1
1	3
0	0
7	5
11	9

- Did you get "11" when you added 3,1 and 7? Did you get "9" when you added 1, 3 and 5? If not you just made a mistake adding. You will learn how to check addition later in this chapter.

- Did you place the right-hand digits in your answer space, and the left-hand digit above the next column? (In the second problem, you only have a right-hand digit. Place a zero up there if you like.)

Discussion on Problems 9 and 10,continued

```
        0
  3    11     Add the next column
 31   273      Ignore the rest
430   200
 27    35
---   ---
  1     9
```

- Did you get "9" when you added 1, 3, 3 and 2? Did you get "11" when you added 0, 1, 7, 0 and 3? If not, look at these columns again.

- Did you place the right-hand digits in your new answers below the second columns and use the left-hand digits as carry digits?

```
01     10
  3    11
 31   273
430   200
 27    35
---   ---
 91    19
```

- Did you add the third columns? If not, do it now.

  0 + 4 = 4. Place 4 in the answer space.
  1 + 2 + 2 = 5. Place 5 in the answer space.

- Your final work should look like this:

```
01     10
  3    11
 31   273
430   200
 27    35
---   ---
491   519
```

Now, try Problems 11 and 12.

PROBLEMS

11. 17 + 22 + 347 + 9 = ____________   12. 151 + 9 + 70 + 2411 = ____________

Calculations. Problems 11 and 12

02	**11**
17	**151**
22	**9**
347	**70**
+ 9	**+ 2411**
395	**2641**

Right? You knew it all the time?

Wrong? You will find your problem as you go on with the training. Study "ADDING DECIMAL NUMBERS".

ADDING DECIMAL NUMBERS

Adding decimal numbers is the same as adding whole numbers -- except that you have to watch the decimal points. Decimal numbers are lined up by the decimal points. The decimal points must form vertical lines. 103.2, 21.7, 45.6, and 7.5 should be lined up as shown below:

```
Like this:   103.2     Not like this:  103.2       Nor like this:  103.2
              21.7                     21.7                        21.7
              45.6                     45.6                        45.6
               7.5                     7.5                            7.5
```

Vertical Line

Place the number having the most decimal places at the TOP of the column. Attach zeros to make each number have the same number of decimal places as the top one. If the top number has three decimal places, all numbers should have three decimal places.

87.5 + 102.567 becomes $\begin{array}{r} 102.567 \\ \underline{+\ \ 87.500} \end{array}$ Not $\begin{array}{l} \ \ 102.567 \\ \underline{+87.5\quad\ \ } \end{array}$

PROBLEMS

13. 542.1 + 127.22 + 6.345 + 5842.01 = ____________________

14. 127.52 + 87.967 + 1035.6 + 0.007 = ____________________

Right? -- Even the decimal points? Go on to "Carry Digits".

Wrong? -- Study "Discussion on Adding Decimal Numbers".

Carry Digits

Carry digits are used to improve accuracy. Some persons feel they should not use carry digits -- only kids do. Maybe so -- but the kids make fewer mistakes than most adults.

It doesn't make sense to add a long column of digits and -- in a few seconds -- lose track of the answer.

```
 3 3
 193
 241
 739
8165
 437
  21
1238
 544
----
  78
```

Can you add the first and second columns and, after a distraction, remember your carry digit for the third column? Almost no one can. So why not write it down? In the most logical place:

AT THE TOP OF THE NEXT COLUMN OF DIGITS.
- It makes it easier to add long columns.
- You can work faster.
- You can re-add quickly.

Obviously, when adding only a few numbers, you sometimes can work faster without carry digits. Go on to "Discussion on Neatness and Accuracy".

Discussion on Neatness and Accuracy

Calculations should be neat in order to ensure correct answers are reached.

To get the right answers:

- ► Practice working neatly -- using clearly written digits.
- ► Line up columns of whole numbers from the right
- ► Line up columns of decimal numbers by the decimal points.
- ► Put your answers in the correct places.
- ► Use lined or cross-section paper to help.

Neat Work

```
 613        14.56
+ 34      + 11.71
```

Columns in line

Decimal points in line

Careless Work

```
613       14.56
+  34     +  11.71
```

Columns out of line

Decimal points out of line

Answers in Place

```
 613        14.56
+ 34      + 11.71
 647        26.27
```

Answers Out of Place

```
613       14.56
+  34     +  11.71
647             26.27
```

Study "SUBTRACTING WHOLE NUMBERS".

SUBTRACTION

SUBTRACTING WHOLE NUMBERS

You know how to subtract, of course. 9 - 3 = 6, 10 - 4 = 6 and 15 - 6 = 9. Still, some review might help.

Sample problem: 5765 - 394 = __ ?

The solution steps are as follows:

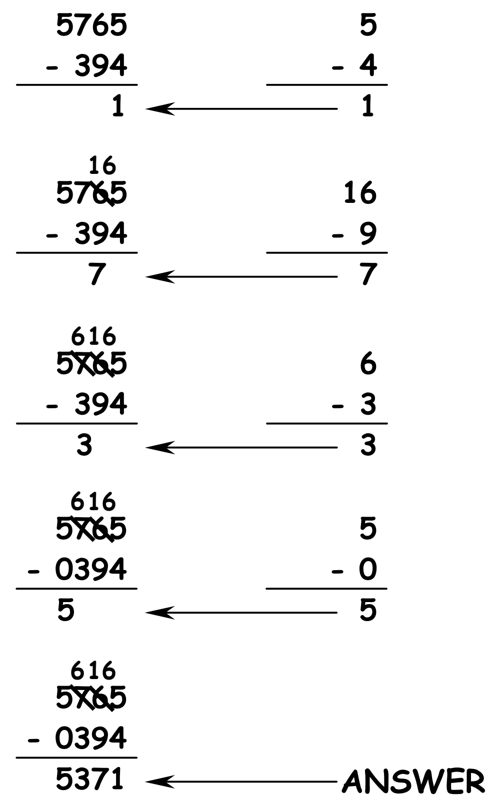
One -- Line up whole numbers from the right.

Two -- Subtract from right to left -- in this case, 4 from 5, 9 from 6, 3 from 7, O from 5 -- and put your answers below each column.

Three -- When you can't subtract -- as you can't when you get to the "9 from 6" change the 6 to 16. Subtract 9 from 16 and put your answer below the 9.

Four -- To change a 6 to 16, you have to take the digit "1" from the next digit left -- in this case, the 7. This is known as borrowing digits. Change the 7 to 6 and subtract the 3.

Five -- Subtract "0" from the 5. All blank spaces are zeros in subtractions and addition.



PROBLEM

15. Make these calculations:

1367 - 265 = _______________

1439 - 749 =________________

2315 - 316 =________________

Get them all right? Study "SUBTRACTING DECIMAL NUMBERS".

Get one or more wrong? Did you:

- Work them in your head? If so, set them up and work them longhand.
- Work them longhand? Check your work against the calculations below and find your mistakes.

$$\begin{array}{r} 1367 \\ -\ 265 \\ \hline 1102 \end{array} \qquad \begin{array}{r} {}^{13} \\ {}^{0\,\cancel{3}\,13} \\ \cancel{1}\cancel{4}\cancel{3}9 \\ -\ 749 \\ \hline 690 \end{array} \qquad \begin{array}{r} {}^{12\,10} \\ {}^{1\,\cancel{2}\,\cancel{0}\,15} \\ \cancel{2}\cancel{3}\cancel{1}\cancel{5} \\ -\ 316 \\ \hline 1999 \end{array}$$

NOTE: In the second problem, the 4 was changed to 3 then to 13. In the third one, the 1 became 0 and then 10, and the 3 became 2 then 12.

SUBTRACTING DECIMAL NUMBERS

Subtracting decimal numbers is the same as subtracting whole numbers -- except that you line up the columns by the decimal points.

Subtract 5.011 from 45.42

```
   45.42
 -  5.011        Decimal points in line
```

One -- Line up the decimal points

```
   45.420   <--- Add zeros
 - 05.011
```

Two -- Add zeros

```
        1 10
   45.4~~2~~~~0~~   <--- Show Borrowed digits
 - 05.011
```

Three -- Subtract as though you were working with whole numbers

```
        1 10
   45.4~~2~~~~0~~
 - 05.011
   40.409   <--- Subtract
```

PROBLEM

16. 29.53 - 4.1 = _____________

    3.154 - 0.264 = _____________

    417.1 - 17.901 = _____________

Right? Even on that last one? You can pass any subtraction test.

Study "CHECKING ADDITION - SUBTRACTION".

Wrong? Go back and review.

CHECKING ADDITION - SUBTRACTION

CHECKING ADDITION

All mathematical calculations should be checked for accuracy. Add upwards to check the accuracy of addition. Follow these four steps:

One -- Add downwards first, as usual.

Two -- Draw a line above the problem and erase or cross out the old "carry" digits.

Three -- Add each column of digits upwards to check your answers; put the new "carry" digits at the bottom.

Four -- Compare the original answer with the new answer. If a difference exists between your original answer and the new answer, repeat the addition procedure in Steps 2 and 3 above until the answers match.

Draw Line

Erase or cross out

$$\begin{array}{r} \hline \cancel{11} \\ 127.1 \\ 54.3 \\ +\ 26.1 \\ \hline 207.5 \end{array}$$

New Answer → 207.5

Compare

Original Answer → 207.5

$$\begin{array}{r} 207.5 \\ \hline \cancel{11} \\ 127.1 \\ 54.3 \\ +\ 26.1 \\ \hline \cancel{11} \\ 207.5 \end{array}$$

CHECKING SUBTRACTION

Add again to check the accuracy of subtraction.

One -- Set up the answer and the subtracted value as a problem in addition, then add.

Subtraction		**Checking by Addition**	
214.76	Original Value	190.66	Answer
- 24.10	Subtracted Value	- 24.10	Subtracted Value
190.66	Answer	214.76	Original Value

Two -- Compare the new answer to the original value. If it matches, your work should be right.

214.76		190.66
- 24.10	**Compare**	- 24.10
190.66		214.76

Go on to "MULTIPLICATION AND DIVISION."

MULTIPLICATION AND DIVISION

MULTIPLYING WHOLE AND DECIMAL NUMBERS

Terms and Symbols

Three terms are used in multiplication -- "original value," "multiplier" and "answer."

Four symbols are used to indicate that numbers should be multiplied:

1. The times sign -- x -- as in 2 x 3, meaning 2 times 3.
2. Parentheses -- ( ) -- as in 3(4), meaning 3 times 4.
3. A dot -- • -- as in 2 • 3 meaning 2 times 3.
4. Letters placed side-by-side -- LW -- meaning Length times Width.

Alignment

Numbers are lined up from the right for multiplication -- without regard to decimal points.

Use the longer number as the original value, the shorter as the multiplier.

47 x 381 becomes

$$\begin{array}{r} 381 \\ \underline{\times\ 47} \end{array}$$

Use the longer number as the original value, the shorter as the multiplier.

863 x 24 becomes

$$\begin{array}{rl} 863 & \leftarrow \text{Original value} \\ \underline{\times\ 24} & \leftarrow \text{Multiplier} \end{array}$$

Alignment, continued

941.4 x 22	becomes	941.4 x 22	No decimal points or zeros are used to change whole numbers into decimal numbers for multiplication.
91.11 x 0.211	becomes	91.11 x 0.211	Always line up the numbers from the right without regard to decimal points

Steps in Multiplication

The steps in multiplying are presented below. Read each and follow the work done in the example below.

One -- Line up the numbers by the right-hand digits.

Step One

$$\begin{array}{r} 59 \\ \times\ 2.3 \\ \hline \end{array}$$

Two -- Start with the right-hand digit in the multiplier and multiply each digit in the original value by it. Work from right to left. Insert the necessary "carry digits~ above the digits to the left in the original value as shown:

3 x 9 = 27. Place the 7 in the answer. Carry the 2.
3 x 5 = 15. 15 + 2 = 17. Place the 17 in the answer. Keep your columns straight as shown in the example.

Step Two

$$\begin{array}{r} {}^{2} \\ 59 \\ \times\ 2.3 \\ \hline 7 \end{array} \qquad \begin{array}{r} 59 \\ \times\ 2.3 \\ \hline 177 \end{array}$$

Three -- Repeat Step Two for EACH digit in the multiplier, working from right to left. Insert new "carry digits." 2 x 9 = 18. Place the 8 as shown -- below the 2 and use the 1 as a new carry digit. 2 x 5 = 10. And 10 + 1 = 11. Place the 11 as shown.

Step Three

$$\begin{array}{r} {}^{2} \\ 59 \\ \times\ 2.3 \\ \hline 177 \\ 8 \end{array} \qquad \begin{array}{r} {}^{1} \\ \cancel{2} \\ 59 \\ \times\ 2.3 \\ \hline 177 \\ 118 \end{array}$$

Four -- Add the two columns -- pretending a zero has been placed as shown. Your answer is 1357.

Step Four

$$\begin{array}{r} {}^{1} \\ \cancel{2} \\ 59 \\ \times\ 2.3 \\ \hline {}^{1}177 \\ 118 \\ \hline 1357 \end{array}$$

Five -- Count the decimal places in the original value and the multiplier. In this case, there is one decimal place in the multiplier, none in the original value. Count-off decimal places in the answer equal to those in the original value and multiplier. Place a decimal point left of 7 -- to provide one decimal place in this answer.

Step Five

$$\begin{array}{r} 59 \\ \times\ 2.3 \\ \hline 177 \\ 118 \\ \hline 135.7 \end{array}$$

PROBLEM

17. 22 x 3.6 = ____________

0.46 x 1.8 = ____________

Right? Skip to "Discussion on Multiplication".

Wrong? Review again the five steps together with the calculations below:

Step One

$$\begin{array}{r} 3.6 \\ \times\ 22 \\ \hline \end{array} \qquad \begin{array}{r} 0.46 \\ \times\ 1.8 \\ \hline \end{array}$$

Step Two

$$\begin{array}{r} {}^{1} \\ 3.6 \\ \times\ 22 \\ \hline 2 \end{array} \qquad \begin{array}{r} 3.6 \\ \times\ 22 \\ \hline 72 \end{array} \qquad \begin{array}{r} {}^{4} \\ 0.46 \\ \times\ 1.8 \\ \hline 8 \end{array} \qquad \begin{array}{r} 0.46 \\ \times\ 1.8 \\ \hline 368 \end{array}$$

Step Three

$$\begin{array}{r} {}^{1} \\ 3.6 \\ \times\ 22 \\ \hline 72 \\ 2 \end{array} \qquad \begin{array}{r} {}^{1} \\ \cancel{1} \\ 3.6 \\ \times\ 22 \\ \hline 72 \\ 72 \end{array} \qquad \begin{array}{r} {}^{4} \\ 0.46 \\ \times\ 1.8 \\ \hline 368 \\ 6 \end{array} \qquad \begin{array}{r} \cancel{4} \\ 0.46 \\ \times\ 1.8 \\ \hline 368 \\ 46 \end{array}$$

Step Four

$$\begin{array}{r} 3.6 \\ \times\ 22 \\ \hline 72 \\ 72 \\ \hline 792 \end{array} \longrightarrow 79.2$$

$$\begin{array}{r} 0.46 \\ \times\ 1.8 \\ \hline {}^{1}368 \\ 46 \\ \hline 828 \end{array} \longrightarrow 0.828$$

<u>Discussion on Multiplication</u>

Persons who have difficulty multiplying usually make one or more of these mistakes:

1. They set up the problems improperly -- or even carelessly,
2. They multiply one digit by another incorrectly,
3. They add incorrectly, or
4. They place the decimal point improperly.

1. Set Up - As indicated above, select the longer numbers as the original values, the shorter as the multipliers and line up the problems from the right:

42.13 x 0.102 → 42.13 —— Right digits lined up
<u>x 0.102</u>

Keep putting the decimal points where they belong in the original values and the multipliers -- but line up the numbers by the digits, not the decimal points.

2. Multiplying -The digits that have to be multiplied are shown at the right:

As you can see, 9 x 9 = 81, 8 x 8 = 64 and 7 x 7 = 49.

As can also be seen, 9 x 8 = 72 and 9 x 7 = 63.

The answers to any multiplied pair of digits can be found in the table.

<u>TABLE OF MULTIPLIED NUMBERS</u>

	9	8	7	6	5	4	3	2	1
9	81	72	63	54	45	36	27	18	9
8		64	56	48	40	32	24	16	8
7			49	42	35	28	21	14	7
6				36	30	24	18	12	6
5					25	20	15	10	5
4						16	12	8	4
3							9	6	3
2								4	2
1									1

3. Adding - Adding the columns in multiplication is the same as in any other work. Just be sure your columns are straight.

$$\begin{array}{r} 0.119 \\ \underline{\times\ 0.19} \\ 1071 \\ \underline{119} \\ 2261 \end{array}$$

Use carry digits if necessary.

4. Placing Decimal Points. Count the decimal places in the from the right in the answer.

$$\begin{array}{rl} 0.119 & = 3 \text{ decimal places} \\ \underline{\times\ 0.19} & = \underline{2} \text{ decimal places} \\ 1071 & \ 5 \\ \underline{119} & \\ \underbrace{2261}_{\text{4 digits}} & \end{array}$$

There are 4 digits in the answer – but you need 5 decimal places. So, add a zero to the left.

$$0.\underbrace{02261}_{\text{5 decimal places}}$$

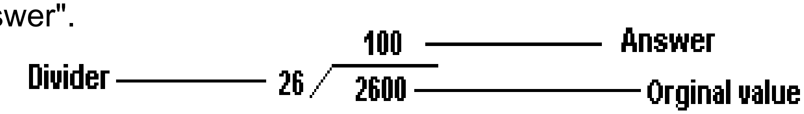
0.119 = 3 decimal places
x 0.19 = 2 decimal places
1071 5
119
2261
4 digits
There are 4 digits in the answer -- but you need 5 decimal places. So, add a zero to the left.
0.02261
5 decimal places

Go to "DIVIDING WHOLE AND DECIMAL NUMBERS". If you don't fully understand multiplication, you will get more practice later in this chapter.

DIVIDING WHOLE AND DECIMAL NUMBERS

Terms and Symbols

The terms used in division are "original value", "divider" and "answer".



The symbols used are:

1. The division sign ÷ as in 2600 ÷ 26, meaning twenty-six hundred divided by twenty-six.

2. The fraction signs —— and / as in $\frac{2600}{26}$ and 2600/26, each meaning twenty-six hundred divided by twenty-six.

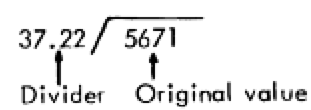
3. The division box $\overline{)\quad}$ as in $\overline{)2600}$

Steps in Setting Up for Division

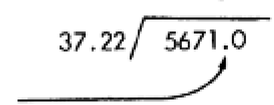
The five steps used to set up problems for division are listed below. Each step is presented with an example shown on the right. Study each step and follow the changes made in the examples.

Problem: Divide 5671 by 37.22.

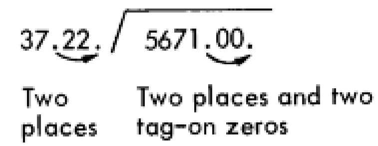
One -- Place the numbers as shown at the right:



Two -- If there is no decimal point in the divider, skip to Step Four. If there is a decimal point in the divider but none in the original value, place a decimal point to the right of the last digit in the original value.

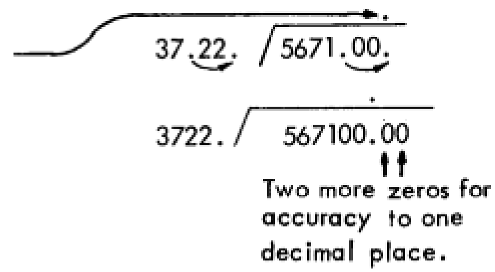


Three -- Move the decimal point in the divider to the right of the last digit. Move the decimal point in the original value also to the right, by the same number of decimal places as you moved the decimal point in the divider. Attach zeros if necessary places tag-on zeros.



Four -- Place a decimal point in the answer space directly above the decimal point in the original value.

Five -- Add zeros to the original value as needed to obtain required accuracy.



PROBLEM

18. Set up the following problems for division. Place all decimal points in their proper positions and attach zeros as needed to obtain required accuracy. DO NOT DIVIDE.

Problem	Required Accuracy
318 ÷ 49.28 = ____________	One decimal place
235.739 ÷ 217.8 = ____________	Three decimal places
78.359 ÷ 6.02 = ____________	Four decimal places
14 ÷ 2.3 = ____________	Two decimal places

NOTE: When you divide one number by another, the accuracy of your answer depends on how far you carry your answer. If your answer has to be accurate to two decimal places, your calculation must be carried to three decimal places. You can attach zeros to the original value until this happens. This will become clear in the practice work on the following pages.

Dividing -- Six Steps

Dividing is finding out how many times the divider will go into the original value. Dividing involves six steps.

Divide 578 by 9 -- to one decimal place.

One -- Determine whether the divider is equal to or smaller than the first digit in the original value. In this case, 9 is larger than the first digit -- 5.

$$\begin{array}{l} . \\ 9\overline{)\underline{5}78.00} \end{array}$$

Now look at the first two digits. Is the divider equal to or smaller than the first two digits in the original value -- the number 57? It is smaller, of course.

$$\begin{array}{l} . \\ 9\overline{)\underline{57}8.00} \end{array}$$

Two -- Since 9 will go into 57 six times, place a 6 in the answer space as shown. Multiply 6 x 9 and place your answer as shown.

$$\begin{array}{l} 6. \\ 9\overline{)578.00} \\ 54 \end{array}$$

Three -- Subtract the multiplied value from the first two digits in the original value.

$$\begin{array}{l} 6. \\ 9\overline{)578.00} \\ \underline{-54} \\ 3 \end{array}$$

```
     6 .
9 ) 578.00
   - 54
     38
```

Four -- Bring the next figure in the original value down and place it to the right of the remainder.

```
     64.
9 ) 578.00
   - 54
     38
    - 36
      2 0
```

Five -- Determine how many times the divider will go into the digit you brought down. In this case, 9 will go into 38 four times. Place a "4" in the answer. Multiply 4 x 9 = 36. Place as shown, then subtract. Bring down the 0.

```
     64.2            64.22
9 ) 578.00      9 ) 578.00
   - 54            - 54
     38              38
    - 36            - 36
      2 0             2 0
     - 1 8           - 1 8
        20              20
                       - 18
                          2
```

Six -- Now 9, will go into "20" two times. So place a 2 in the answer and multiply it times 9. Place the answer 18, as shown and subtract again.

Repeat Step Six until you keep getting the same answer. In this case, you will keep getting the answer "2."

Stop dividing when your answer is as "accurate" as you need it to be -- in this case, one decimal place.

PROBLEM

19. Divide 646 by 8, to an accuracy of one decimal place.

Divide 5.005 by 0.12, to an accuracy of two decimal places.

<u>Calculations, Problem 19</u>

$$
\begin{array}{rll}
 & 80.75 & \\
8 & \overline{)\,646.00} & \\
 & \underline{-64} & \leftarrow 8 \times 8 \\
 & 06\,0 & \\
 & \underline{-5\,6} & \leftarrow 7 \times 8 \\
 & 40 & \\
 & \underline{-40} & \leftarrow 5 \times 8 \\
 & 00 &
\end{array}
$$

$$
\begin{array}{rl}
 & 41.708 \\
0.12. & \overline{)\,5.00.500} \\
 & \underline{-4\,8} \\
 & 20 \\
 & \underline{-12} \\
 & 8\,5 \\
 & \underline{-8\,4} \\
 & 10 \\
 & \underline{-0} \\
 & 100 \\
 & \underline{-96} \\
\text{Remainder} & = 4
\end{array}
$$

If you stopped dividing too soon, turn to page 1-25 and read the Note again.

Dividing with Two - and Three-Digit Dividers

Dividing with two or three digits in the divider is the same as dividing with only one digit -- except that it's more difficult.

444 ÷ 33, to one decimal place:

One -- Set up the problem.

```
33 √ 444.00
```

Two -- How many times will 33 go into 44? -- Once. Show 1 in the answer space, multiply 33 x 1, write it down and subtract.

```
       1 .
33 √ 444.00
    - 33  ← 1 x 33
      11
```

Three -- Bring down the 4. How many times will 33 go into 114? Try 3. Multiply and subtract again.

```
       13.
33 √ 444.00
    - 33
      114
     - 99  ← 3 x 33
       15
```

Four -- Bring down a 0. How many times will 33 go into 150? About 4 times. (If you have to, try 5. Since 5 x 33 = 165, and since you can't subtract 165 from 150, try 4 again.)

Five -- Bring the other 0 down. How many times will 33 go into 180? Now try 5, multiply and subtract.

```
       13.45
33 √ 444.00
    - 33
      114
     - 99
       15 0
     - 13 2  ← 4 x 33
         1 80
       - 1 65  ← 5 x 33
Remainder =  15
```

Regardless of the size of the divider, you have to divide it into the original value.

PROBLEM

20. Divide the numbers below. Show all work. Carry all answers to three decimal places.

$72.46 \div 0.354 =$ _______________

$6.3079 \div 1.32 =$ _______________

Did you get them right? Only a minor mistake or two? Do you know how to divide? If so, study "CHECKING MULTIPLICATION AND DIVISION".

Make several mistakes? Do you have difficulty with division? Go back and review.

CHECKING MULTIPLICATION AND DIVISION

CHECKING MULTIPLICATION

Divide the answer by the multiplier to check the accuracy of multiplication. Your work checks if your new answer is the original value.

Multiplication Calculation

```
   7.62   Original value
 x 43.5   Multiplier
 ------
   3810
  2286
 3048
 ------
 331470   Answer
```

Checking Calculation

```
                      7.62   Original value
Multiplier  43.5. √ 331.4.70   Answer
                    304 5
                    -----
                     26 9 7
                     26 1 0
                     ------
                        8 70
                        8 70
                        ----
```

In the calculations above, 7.62 is multiplied by 43.5. The answer, 331.470, is then divided by the multiplier. The answer obtained by dividing is the same as the original value -- so the multiplication is accurate.

PROBLEM

21. Try it yourself. Multiply the following numbers and check your answers by dividing.

    1.41 x 2.2 = _______________
    23.7 x 0.12 = _______________

Did you get the original values when you divided? Then you know how it's done. Study "CHECKING DIVISION". ~ Did you get different original values? Go back and review.

CHECKING DIVISION

Multiply the answer by the divider to check the accuracy of division. Add any remainder. Your new answer should match the original value in the division problem. Study these examples:

Division Calculation | **Checking Calculation**

```
                 204.689    Answer
Divider  0.354. √72.460.000
                 70 8
                  1 660
                  1 416
                    244 0
                    212 4
                     31 60
                     28 32
                      3 280
                      3 186
                         94  Remainder
```

Original Value

```
  204.689   Answer
  x 0.354   Divider
   818756
 1023445
 614067
72.459906
     + 94   Remainder
72.460000   Original Value
```

```
                064.806    Answer
Divider  4.6.  √298.1.100
                276
                 22 1
                 18 4
                  3 7 1
                  3 6 8
                    0 300
                      276
                       24  Remainder
```

Original Value

```
   64.806   Answer
    x 4.6   Divider
   388836
  259224
298.1076
     + 24   Remainder
298.1100   Original Value
```

PROBLEM

22. Divide the following numbers and check your answers by multiplying.

    14.513 ÷ 2.3 = ________________   49.7 ÷ 7.1 = ________________

Did you get the original values when you divided? If you did, you know how to check your own division. Then go on to "HIGHWAY PROBLEMS" to complete this chapter. If you made mistakes, go back and review.

HIGHWAY PROBLEMS

A section on practical highway inspection problems is presented at the end of most chapters. The problems presented in this chapter are concerned with addition, subtraction, multiplication and division of whole and decimal numbers.

Some of the problems require you to "think them through" before making calculations. If you have difficulty remembering particular steps, review the proper sections in this chapter.

23. 549.3 lbs + 317.6 lbs + 103.7 lbs + 298.6 lbs + 138.7 lbs = ________________
    30 lbs + 17 lbs + 20 lbs + 41 lbs + 59 lbs = ________________
    320.71 cu. yds. + 126.1 cu. yds. + 529.37 cu. yds. = ________________

24. Determine the number of miles of project length yet to be graded:

Total length of project	= 4.321 miles
Miles graded to date	= 2.381 miles
Miles yet to be graded	= _____ miles

25. 2292 kg (5,052 lbs.) of reinforcing steel were delivered to a job site.
    1738 kg (3,832 lbs.) of reinforcing steel were used in various portland cement concrete structures.
    How many lbs. of reinforcing steel should be on hand? __________ lbs.

HIGHWAY PROBLEMS, continued

26. One truck can haul 10 batches. Each batch weighs 4,000 lbs.

    How many lbs. can be hauled in the truck? _______________ lbs.

27. The data below are from the Bill of Reinforcing Steel and the Reinforcing Bar Table. Use this data to determine the total weight of reinforcing steel required.

    First, for each size, multiply the number of pieces required by the lengths. Your answers will be the total lengths needed.

    Next, multiply each total length by the weight per foot to arrive at the weights of each size.

    Last, add the weights.

    O.K.! Fill in the blanks.

Size	Number required	Length	Total Length	Weight per foot	Total item weight
4	13	42 feet		0.668 lbs.	
4	19	8 feet		0.668 lbs.	
5	90	20 feet		1.043 lbs.	

Weight _____________________lbs.

HIGHWAY PROBLEMS, continued

28. An application rate for liquid asphalt is determined by dividing the square yards of area covered into the number of gallons of asphalt used. The answer is expressed in 0.01 of a gallon per one square yard.

    Example: .26 gal/sq. yd.

    NOTE: The symbol "/" means "per." "0.26 gals/sq. yd" means 0.26 gallons per square yard".

If 975 gallons of liquid asphalt are used to cover 3,850 square yards -- what is the application rate in gallons/square yard to the nearest 0.01? ________________

This concludes Chapter One. The calculations taught in this chapter are used repeatedly in all remaining chapters. If you have made a few mistakes, don't worry. Most persons do. Your skill will develop as you practice. If you really don't understand one or more sections in Chapter One, study Chapters Two and Three. Then study Chapter One again, from the beginning -- but let a few days pass first.

ANSWERS TO PROBLEMS

Page 1-3
1. 17.2
2. 27.375
3. 1,003.75
4. 9,073.405
5. 3,757.5
6. 8.7221
7. 2,527.6
8. 1,039.7776

Page 1-5
9. 491
10. 519

Page 1-8
11. 395
12. 2,641

Page 1-9
13. 6,517.675
14.

Page 1-13
15. 1,102
690
1,999

Page 1-14
16. 25.43
2.89
399.199

Page 1-20
17. 79.2
0.828

Page 1-24
18. 31800.00 ÷ 4928
2357.3900 ÷ 2178
7835.90000 ÷ 602
140.000 ÷ 23

Page 1-27
19. 80.75
41.708

Page 1-29
20. 204.689
4.779

Page 1-30
21. 3.102
2.844

Page 1-32
22. 6.31
7
23. 1,407.9 lbs.
167 lbs.
976.18 cu. yds.
24. 1.94 miles
25. 554 kg (1,220 lbs.)

Page 1-33
26. 40,000 lbs.
27. 546’ 364.728 lbs.
152’ 101.536 lbs
1,800’ 1,877.4 lbs.
2,343.664 lbs.
28. 0.25 gal./sq. yd.

CHAPTER TWO

Rounding

CONTENTS

2

ROUNDING

Numbers are rounded for at least two reasons:

1. To provide a stopping place in calculating -- particularly in dividing some calculations that otherwise would go on indefinitely.

2. To make numbers easier to use -- without sacrificing the degrees of accuracy needed.

To round a decimal number is to reduce the number of digits used.

The number "21.6666" can be rounded to three degrees of decimal accuracy or to the nearest whole number -- as follows:

Original Number	Degree of Accuracy	Rounded Number
21.6666	0.1	21.<u>7</u>
21.6666	0.01	21.6<u>7</u>
21.6666	0.001	21.66<u>7</u>
21.6666	Whole Number	22

The digits following the last digit used are dropped. In the first three examples, the last digit used was increased to 7. The decision to increase or not to increase the last digit used is based on three "RULES FOR ROUNDING DECIMAL NUMBERS."

ROUNDING DECIMAL NUMBERS

Rounding is done by following three rules:

Rule One -- Determine the LAST DIGIT TO BE USED -- the last digit needed for accuracy.

If the number 15.528 is to be rounded to two decimal places -- the LAST DIGIT TO BE USED is the "2." The 8 will be dropped.

Rule Two -- If the digit following the last digit to be used is 0, 1, 2, 3 or 4 -- drop it and all that follow it. DO NOT CHANGE the last digit to be used.

5.90
5.91
5.92
5.93
5.94

The following numbers are rounded to one decimal place:

All are rounded to 5.9 -- the 0, 1, 2, 3 and 4 are dropped.
The 9 remains unchanged.

5.901
5.9167
5.9499

All are rounded to 5.9. The digits in the second, third, fourth and all following decimal places are dropped.

Rule Three -- If the digit following the last digit to be used is 5, 6, 7, 8 or 9 -- drop it and all digits that follow it. Add 1 to the last digit to be used.

The following numbers are rounded to one decimal place:

3.45
3.46
3.47
3.48
3.49

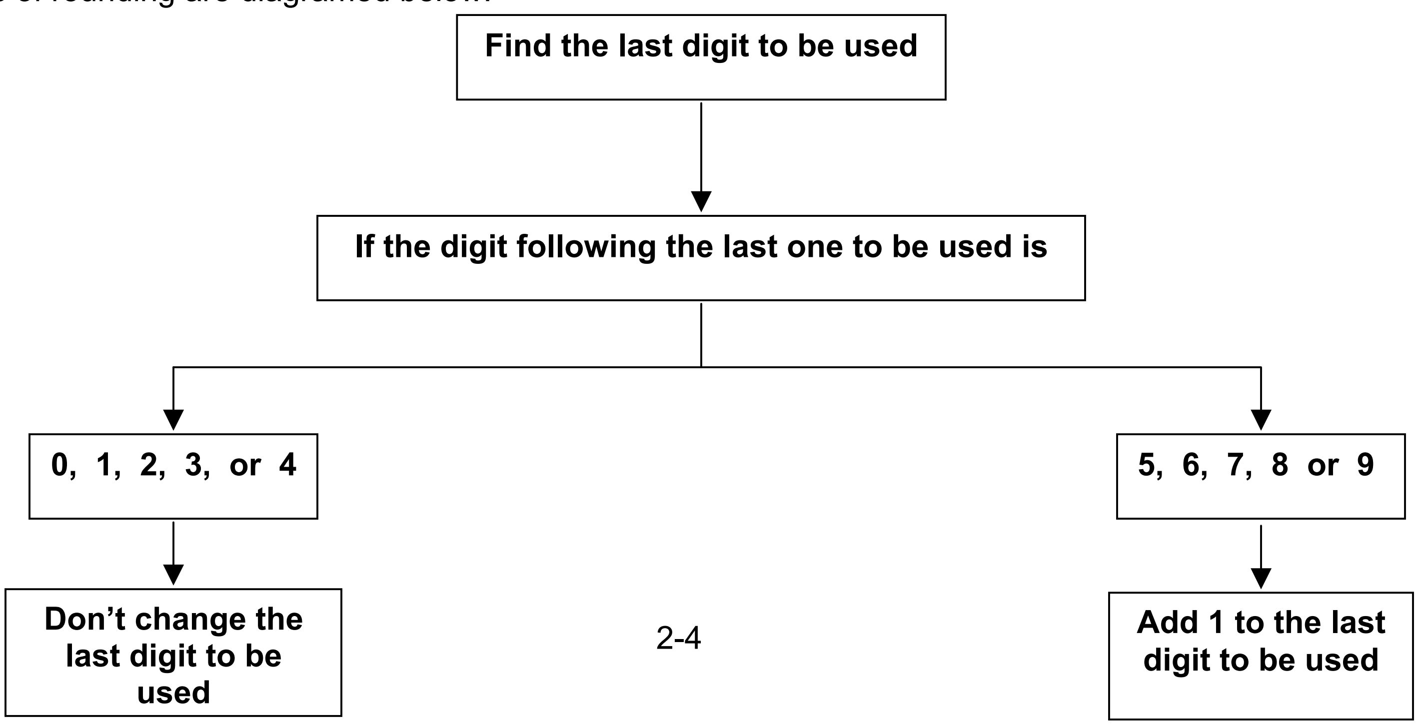
All of these numbers are rounded to 3.5 – the 5, 6, 7, 8, and 9 are dropped and the value “1” is added to the “4” – the last digit used.

3.456
3.4791
3.49899

All are rounded to 3.5. The digits in the third, fourth, and following decimal places are ignored.

Summary

The rules of rounding are diagramed below:



PROBLEMS

1. Round the following numbers:

From	To
997.487	Nearest whole number = ___________________
63.7458	Two decimal places = _____________________
92.55	Nearest whole number = ___________________
436.488 53	Three decimal places = ____________________

2. Round these numbers to two decimal places:

   84.375 = ___________________

   9.4656 = ___________________

   321.3849 = _________________

   0.9993 = ___________________

3. Round these numbers to three decimal places:

   10.7555 = ___________________

   0.019 500 = ___________________

   1.998 501 = ___________________

   99.9985 = ___________________

Get all answers right? Skip to "ROUNDING IN A SERIES OF CALCULATIONS." Mistakes? Go back and review

ROUNDING IN A SERIES OF CALCULATIONS

GENERAL RULE

When it is necessary to make one or more calculations to obtain a final answer, all preliminary answers should be carried out and rounded to one decimal place more than needed in the final answer.

NOTE: All answers calculated prior to the final answer are called "preliminary answers."

An example of the general rule is shown below -- where the final answer is to be reported in a whole number:

Areas paved:	Square Yards	Preliminary answers will be
	Square Yards	carried out and rounded to
	+ Square Yards	0.1 (tenths).
Total area paved:	Square Yards	

Final answer will be rounded to a whole number.

Additional examples are provided on the following pages to show how the general rule is applied when calculating final answers to the nearest whole numbers, 0.1 (tenths), 0.01 (hundredths), and 0.001 (thousandths).

CALCULATING TO THE NEAREST WHOLE NUMBERS

When final answers must be in whole numbers, all preliminary answers will be carried out and rounded to 0.1 (tenths). Final answers will be rounded to the nearest whole numbers.

Example when Multiplying

Preliminary Calculation A

10.5 x 2.3 = 24.15
Rounded to: 24.2

Preliminary Calculation B

12.5 x 4.6 = 57.50
Rounded to: 57.5

Final Calculation

24.2 x 57.5 = 1391.50
Rounded to: 1392

PROBLEM

4. Round the answers in preliminary calculations A and B, below, to 0.1 (tenths). Round the final answers to the nearest whole number.

A	=	5.4 x 27.3	=	147.42	rounded to:	________________
B	=	15.6 x 72.8	=	1 135.68	rounded to:	________________
Final Answer	=	A x B	=	167 402.18	rounded to:	________________

Study "CALCULATING TO 0.1 (TENTHS)."

CALCULATING TO 0.1 (TENTHS)

When final answers must be in 0.1, all preliminary answers will be carried out and rounded to 0.01. Final answers will be rounded to 0.1.

Example when Multiplying

Preliminary Calculation A

25.52x 13.14 = 35.3328

Rounded to: 335.33

Preliminary Calculation B

9.45 x 3.2 = 30.240

Rounded to: 30.24

Final Calculation

335.33 x 30.24 = 10,140.3792

Rounded to: 10,140.4

PROBLEM

5. Round the answers in the preliminary calculations below to 0.01. Round the final answer 0.1.

A =	12.34 x 7.63	=	94.1542	rounded to:	____________________
B =	7.15 x 1.12	=	8.0080	rounded to:	____________________
Final Answer =	A x B	=	754.1415	rounded to:	____________________

Study "CALCULATING TO 0.01 (HUNDREDTHS)."

CALCULATING TO 0.01 (HUNDREDTHS)

When final answers must be in 0.01 (hundredths), all preliminary answers must be carried out and rounded to 0.001 (thousandths). Final answers will be rounded to 0.01 (hundredths).

Preliminary Calculation A

542.15 x 28.47 = 19.0428

Rounded to: 19.043

Preliminary Calculation B

138.375 x 3.2 = 11.4170

Rounded to: 11.417

Final Calculation

19.043 x 11.417 = 1.667

Rounded to: 1.647

PROBLEM

6. Round the answers in preliminary calculations below to 0.001. Round the final answer to 0.01.

A	=	137.29) 25.16	=	5.4566	rounded to:	__________
B	=	547.15) 89.28	=	6.1284	rounded to:	__________
Final Answer	=	A X B	=	1.123	rounded to:	__________

Right? Excellent! Skip to "CALCULATING TO 0.001 (THOUSANDTHS)."
Wrong? Go back and review.

CALCULATING TO 0.001 (THOUSANDTHS)

When final answers must be in 0.001 (thousandths), all preliminary answers will be carried out and rounded to 0.0001 (ten-thousandths). Final answers will be rounded to 0.001 (thousandths).

Example when Adding

Preliminary
Calculation A

```
  122
  4.468 91
  2.156 72
  1.120 13
+ 0.018 82
  --------
  7.764 58
```

Rounded to: 7.7646

Preliminary
Calculation B

```
  1 2 12
  3.971 63
  1.058 72
  2.182 91
+ 1.505 62
  --------
  8.718 88
```

Rounded to: 8.7189

Final Calculation

```
  1 11
  7.7646
+ 8.7189
  -------
 16.4835
```

Rounded to: 16.484

PROBLEM

7. Round preliminary answers A and B to 0.0001 (ten-thousandths). (Enter the last digit used in the spaces provided.) Subtract the rounded answers: A - B = C. Round the final answer to 0.001 (thousandths).

Preliminary Calculation A

15.447 92
- 12.396 27
3.051 65

Round to: 3.051_ = A (Preliminary answer)

Preliminary Calculation B

10.523 46
- 8.327 61
2.195 85

Round to: 2.195_ = B (Preliminary answer)

Final Calculation: A - B

3.051_ Enter last digit here --
- 2.195_ from answers A and B above.

___.____ C Final answer before rounding

Rounded to: ___.____ Final answer

Right? -- Excellent! Go on to the next page.
Wrong? -- A brief review of this Chapter should clear up any problem.

ROUNDING IN CONSTRUCTION MATH

In this course, we will use the rules of rounding set forth in this chapter. In practice, you should follow these same rules unless you are given other instructions.

For Pi (π), use 3.14 or 3.142 or 3.1416
as required for the desired degree of accuracy.

ACCURACY

The need to calculate to 0.1 (tenths), 0.01 (hundredths) or 0.001 (thousandths) depends on the significance of each calculation. In practice, you will learn the accuracy required for specific calculations and situations. If all persons involved in the inspection, payment accounting and auditing functions of the Department use identical practices with regard to rounding and degrees of accuracy, they will get identical answers in all calculations.

ANSWERS TO PROBLEMS

Page 2-5

1. 997
   63.75
   93
   436.489

2. 84.38
   9.47
   321.38
   1.00

3. 10.756
   0.020
   1 .999
   99.999

Page 2-7

4. 147.4
   1135.7
   167 402

Page 2-8

5. 94.15
   8.01
   754.1

Page 2-9

6. 5.457
   6.128
   1.12

Page 2-11

7. 3.0517 = A
   2.1959 = B
   0.8558 = C
   0.856 = Final answer

QUIZ ON CHAPTERS ONE AND TWO

The training covered in Chapters One and Two has been used to develop the quiz below. Only a few actual calculations are required to answer the questions -- but detailed knowledge of terms and procedures is needed.

You should be able to answer 90 percent of the questions correctly without going back. Try it. (The numbers in parentheses represent the numbers of answers to be counted right or wrong in the quiz.)

1. What are the symbols for "add," "subtract," "multiply," and "divide"? List seven symbols:
   ____ (7)

2. How many decimal places are contained in the number 128.0596? _______________ (1)

3. If you were to calculate to "thousandths," how many decimal places would you have?
   _______________ (1)

4. Change the whole number 257 to a decimal number containing one decimal place:
   _______________ (1)

5. Check addition accuracy by ____ (1)

6. Check subtraction accuracy by ____ (1)

7. Check multiplication accuracy by ______________________________________ (1)

QUIZ, continued

8. Check division accuracy by ____ (1)

9. Line up the decimal points for addition and ______________________________ (1)

10. Line up the right-hand digits for _____________________________________ (1)

11. The original value is 0.241. The multiplier is 0.105. The answer is "25 305."
    Place the decimal point: 25305 (1)

12. The original value is 72.05. The divider is 7.564. The answer must be carried to 0.01 (hundredths). Set the problem up and add zeros: __________________________ (1)

13. Round to 0.01 (hundredths): 29.805 _______ 4.7749 _______ (2)

14. Round to whole numbers: 0.5001 _______ 13.499 _______ (2)

15. If a final answer is to be rounded to 0.01 (tenths) the preliminary answers must be calculated to and rounded to ________________________. If a final answer is to be rounded to 0.01 (hundredths) the preliminary answers must be calculated to and rounded to __________________________. (2)

QUIZ, continued

16. Reinforcement steel is to be calculated to the nearest whole kilogram (pound). Fifteen pieces weigh 3.18 kg (7.02 pounds) each, 47.70 kg (105.30 pounds) total. Nine pieces weigh 2.67 kg (5.89 pounds) each, 24.03 kg (53.01 pounds) total. Should 47.70 (105.30) and 24.03 (53.01) be rounded before adding or after adding? ___________ If the total weight is 71.7 kg (158.3 lbs.), what should be the final answer? __________(2)

17. Change the numbers below to three-decimal-place numbers: (4)

41.2	______________	26	________________
0.0401	______________	0.01	________________

HOW DID YOU DO?

There are 30 answers in the quiz. If you made more than four mistakes, review the chapters. If you made four or fewer mistakes, go to Chapter Three.

ANSWERS TO QUIZ

Page 2-14

1. +, -, x, ( ), B,), / , square root, --
2. 4
3. 3
4. 257.0
5. adding upwards
6. adding subtracted value and answer
7. dividing answer by multiplier

Page 2-15

8. multiplying answer by divider
9. subtraction
10. multiplication
11. 0.025 305
12. $7564 \quad \sqrt{72050.00}$
13. 29.81 4.77
14. 1 13
15. 0.01 (hundredths)
    0.001 (thousandths)

Page 2-16

16. before
    72 (158)
17. 41.200 26.000
    0.040 0.010

CHAPTER THREE

Symbols - Squares - Cubes - Equations - Formulas

CONTENTS

3

SYMBOLS - SQUARES - CUBES - EQUATIONS - FORMULAS

STARTING POINTS FOR TRAINING

PROBLEM

1.

$A = \pi r^2$ $\pi = 3.142$ $r = 2.5$

$A =$ ___________________________ Round to 0.01.

2.

$X = \dfrac{Y^3}{Z^2}$ $X = 27.3$ $Z = 8.5$

$Y^3 =$ ___________________________ Round to 0.1.

Right? On both? Scan Chapter Four and review the sections you need to review. Then work the "HIGHWAY PROBLEMS" section.

Wrong? Work Problem 3.

PROBLEM

The answer to the multiplications and division below is the value of "X".

3. Set up as an equation and solve for "X". Round to 0.01.

$X = (8.35 \times 4.2) \div (5.1 \times 3.2)$

Right? Scan "STANDARD TERMS, ABBREVIATIONS AND SYMBOLS" and review "EQUATIONS" if you like. Then start studying "FORMULAS."

Wrong? Study the calculations below -- then work Problem 4.

Calculations, Problem 3

$$X = \frac{8.35 \times 4.2}{5.13 \times 2}$$

$$X = \frac{35.07}{16.32}$$

X = 2.148 rounded to <u>2.15</u>

PROBLEM

4. Solve these problems. Round to 0.01.

$13.5^2 =$
$9.03^2 =$
$5.81^3 =$
$20.05^3 =$

Right? Start studying "EQUATIONS." Scan "STANDARD TERMS, ABBREVIATIONS AND SYMBOLS" if needed.

Wrong on any of them? Try to get the right answers by checking your work.

STANDARD TERMS, ABBREVIATIONS AND SYMBOLS

Abbreviations and symbols are used to simplify mathematical calculations. But terms, abbreviations and symbols should be used with consistency. If individuals use various terms, abbreviations and symbols to mean different things, mistakes are made.

Several symbols already have been used in this course:

- Letters, such as **A**, **B**, **C** and **X** have been used to represent numbers and values.
- Signs, such as +, -, x, and ÷ have been used to represent calculations.
- Signs, such as ( ) ( ) and • also have been used to mean "multiply", and $\frac{A}{B}$ or A/B to mean "divide".

Many of the customary unit standard terms, abbreviations and symbols used in highway construction are shown on the next page.

STANDARD TERMS, ABBREVIATIONS AND SYMBOLS

Term	Abbreviation	
Inches	in. or "	2 in. or 2"
Feet	ft or '	2 ft. or 2'
Yards	yds.	5 yds.
Miles	mi.	10 mi.
Grams	gms.	3.47 gms.
Pounds	lbs	173.5 lbs.
Tons	Tns	21.4 Tns
Hours	hrs.	7.4 hrs.
Gallons	gals.	1248.3 gals.
Square Inches	sq. in.	2 sq. in.
Square Yards	sq. ft. or S.F.	7 sq. ft. or 7 S.F.
Square Feet	sq. yds. or S.Y.	5 sq. yds. or 5 S.Y.
Square Miles	sq. mi.	6 sq. mi.
Acres	Ac.	3 Ac.
Average	Avg.	------
Cubic Inches	cu. in.	1 cu. in.
Cubic Feet	cu. ft. or C.F.	8 cu. ft. or 8 C.F.
Cubic Yards	cu. yds. or C.Y.	9 cu. yds. or 9 C.Y.

Term	Symbol
Area	A
Volume	V
Length	L or l
Width	W or w
Height	H or h
Diameter	Dia.
Circumference	C
Radius	R or r
Pi	π
Percent	%

NOTE

There are two abbreviations each for square feet, square yards, cubic feet and cubic yards.

Other abbreviations often are used. Learn to interpret all of the abbreviations equally well. But always use one of the abbreviations shown here for

PROBLEM

5. A number of terms, symbols and abbreviations are listed in the left-hand column below. For each item in that column, show the term, symbol or abbreviations that can be used. The first one is done for you.

	Item	Term Represented	Symbol	Abbreviation
1.	Yd.	Yard		
2.	Percent		______	
3.	cubic yard		______	
4.	Acre		______	
5.	Ton	______________		
6.	Gallons			__________
7.	square yard			__________

Problem 5. continued

	Item	Term Represented	Symbol	Abbreviation
8.	Gal	____________		
9.	Yard		_____	
10.	Average			________
11.	r	____________		
12.	^{o}F	____________		
13.	H	____________		
14.	Width		_____	
15.	Pound			________

Right? Sure. Go on to "SQUARED AND CUBED NUMBERS," next page.

Wrong? Fill in the right answers above and memorize as many as you can. Go on to "SQUARED AND CUBED NUMBERS". You will be tested again later in this chapter.

SQUARED AND CUBED NUMBERS

Squared numbers are numbers multiplied ONCE by themselves. Cubed numbers are numbers multiplied TWICE by themselves. A small "2" indicates that a number should be multiplied by itself. A small "3" indicates that it should be multiplied by itself twice.

5^2 means 5 "squared" -- or 5 times 5 or 25.

5^2 means 5 "cubed" -- or 5 times 5 times 5 or 125.

In highway work, the symbols for squaring and cubing numbers are used often in formulas. The training in this section is meant to show you how squaring and cubing are done -- not how to use squared and cubed numbers as final answers.

CALCULATIONS

The numbers below have been squared or cubed:

Number	Calculation	Value
10^2	10 x 10	100
12^2	12 x 12	144
4^3	4 x 4 x 4	64
9^3	9 x 9 x 9	729

PROBLEM

6. Square or cube the following numbers as indicated:

4^2 = ______________

12.3^2 = ______________

7^3 = ______________

2.5^3 = ______________

Right? Go on to "EQUATIONS."

Wrong? Check your multiplications by dividing. You probably used the right procedure, but made mistakes in calculating. If you did, go on to "EQUATIONS".

EQUATIONS

The term "equation" means "things equal to each other."

$$10 + 10 = 5 + 15$$

$$(6 \times 2) + 8 = 30 - 10$$

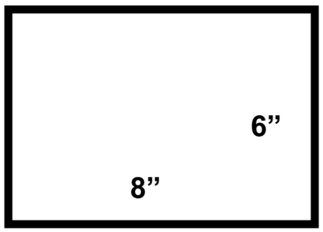
$$(14 \div 2) + 13 = (8 \times 2) + 4$$

Each mathematical expression above is an equation -- since the calculation left of the equal sign always results in the same answer as the calculation on the right. In this case, all calculations result in the answer "20". Each equation is the same as saying 20 = 20.

PURPOSE OF EQUATIONS
In the example above, all the values are known and all calculations can be made quickly. In highway work, many values are unknown. Equations are used to find the values of unknowns. Each equation represents the values and calculations that will result in the value of the unknown.

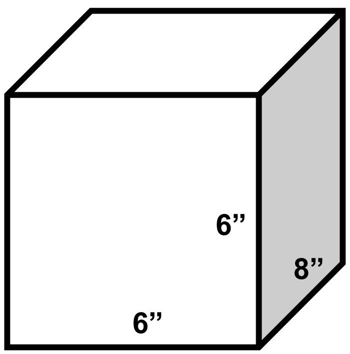
LETTERS AS UNKNOWNS

In most equations, letters are used to represent unknown values:



You know the length and width of the rectangle at the left.
So, set up an equation:

A = 8" x 6".
The letter A represents the unknown Area.



You know the length, width and height of the rectangular solid at the left. Volume = Length x Width x Height.
So, set up an equation:

V = 8' x 6' x 6'
The letter V represents the unknown Volume.

It usually is best to use specific letters to represent unknowns: C for Circumference. A for Area. V for Volume. L for Length. W for Width. H for Height.
The letter X can be used to represent unknown numbers that have no specific designation.

PROBLEMS

7. Area of triangle = 360 S. F.

   Length of base = 32'

   Height = ________________

8. Volume of rectangular solid = 1,232 C. F.

   Area of base = 176 S.F.

   Height of solid = ________________

Right? Go on to Problems 9-11.

Mistakes? Try again.

PROBLEMS

Set up the equations needed for the problems below and solve for the unknowns.

9.

Width	= 12 ft.
Length	= 10 ft.
0.5 of the area	= ________________

10.

V = 1000 C.F. (Triangular solid)

H = 10 ft.

A = ________________

11 .

V = 1,600 C.Y. (Rectangular solid)

L = 40 yds.

W = 4 yds.

H = ________________

Right? Go on to "FORMULAS."

Mistakes? Check your equations and then your calculations.

FORMULAS

WHAT THEY ARE

The term "formulas" is the plural of "formula." You also may hear the term "formulae." It also is correct -- but not often used. You have been using formulas all through this chapter:

$$A = LW \qquad V = AH \text{ or } LWH \qquad A = \frac{LH}{2}$$

FORMULAS COMMONLY USED BY THE DEPARTMENT

$A = LW$	**Area**	=	**Length times Width ---- squares and rectangles**
$A = \frac{LW}{2}$	**Area**	=	**Length times Width divided by 2 ---- triangles**
$A = LWH$	**Volume**	=	**Length times Width times Height ---- rectangular solids**
$A = \frac{LWH}{2}$	**Volume**	=	**Length times Width times Height divided by 2 ---- triangular solids**
$A = \pi r^2$	**Area**	=	**Pi times the radius squared ---- circles**
$C = \pi D$	**Area**	=	**Pi times Diameter ---- circles**
$D = \frac{W}{V}$	**Density**	=	**Weight divided by Volume**

HOW THEY ARE USED

As indicated earlier, formulas are used to plan calculations. They represent mathematical procedures. Cases 1 and 2 below demonstrate how formulas are used in calculating.

Case 1 -- when the formula can be used as is.

Step One -- REPLACE symbols with known values -- as shown below:

If: $A = LW$

And: $L = 5', W = 10'$

L has been replaced by 5'.
W has been replaced by 10'.

Then: $A = 5' \times 10'$

Step Two -- CALCULATE the unknown value -- as shown:

If: $A = 5' \times 10'$

Then: $A = 50$ sq. ft.

Some formulas are easy to solve. Others are more difficult.

Case 2 -- when the formula must be changed.

Step One -- CHANGE the formula so that you can calculate the unknown value:

If: A = LW -- and you must calculate for L,
Then: A/W = L

or: L = A/W

Step Two -- REPLACE symbols with known values:
If: L = A/W

And you know that A = 50 S.F. and W =10 ft.

Then: $L = \frac{A}{W} = \frac{50\ \text{S.F.}}{10\ \text{ft.}}$

The known values have now replaced two symbols, A and W.

Step Three -- CALCULATE the unknown value:

$L = \frac{A}{W} = \frac{50\ \text{S.F.}}{10\ \text{ft.}} = 5\ \text{ft.}$

You can always check your new formula by using simple values and making quick calculations. For instance: You know

A = LW. Make up some values for L and W, say 3 ft. and 6 ft.
Now, A = LW = 3 ft. x 6 ft.= 18 sq. ft.

To find a formula with which you can calculate for L, find a calculation using the values for A and W that will give 3 ft. as the answer. You have to divide 18 sq. ft. by 6 ft. to get the answer 3 ft.

So $L = \frac{18\ \text{sq. ft.}}{6\ \text{ft.}} = \frac{A}{W}$ or $L = \frac{A}{W}$

PROBLEMS

12. Convert the formula H = V/A-- to solve for A and V:

A =

V =

13. Solve for W -- using the formula A = LW . Round to 0.01.

If: A = 37.5 sq. ft., L = 21.3 ft.

Then: W = _____________ ft.

Right? Good Work! Go on to "HIGHWAY PROBLEMS".

Wrong? Try again.

PRACTICE

Any formula can be converted.
This is good practice in logic alone.

$M = D \div G$ ... Therefore: $D = M \times G$ and $G = D \div M$

M =miles per gallon
D = distance traveled, miles
G =gallons of gas used

Suppose: M = 16 mpg.
D = 320 mi
G = 20 gal .

Then: 16 mi./gal. =320 mi ÷ 20 gal.

And: 320 mi = 16 mi./gal. x 20 gal.

Yet: 20 gal. = 320 mi ÷16 mi./gal.

= 320 mi x 1 gal. /16 mi.

There's a way of doing this involving substitutions and calculations. It is too complex for discussion here and of little or no value to inspectors. Practice a number of formulas of your own, and use numbers to make them work out. You soon will catch on to the relationships.

Go on to "HIGHWAY PROBLEMS."

14. The following data were collected for computing the unknown value R for a power hammer. What is R? Round to a whole number.

$$R = \frac{2\,E}{S + 0.1}$$

E = 7.5 ft-tons

S = 0.5 in.

R = ________________ tons
whole number

15. Using this density formula, D = W / V, solve for the unknowns in each of the following. Round to 0.01.D is in units of lbs/ft□, W is units of lbs, and V in units of ft□.

W = 4.95, V = 0.04

D = ________________

D = 129.31, W = 4.66

V = ________________

D = 127.89, V = 0.03

W = ________________

ANSWERS TO PROBLEMS

Page 3-2
1. 19.64
2. 1972.4

Page 3-3
3. X = 2.15

Page 3-4
4. 182. 25
   81.54
   196.12
   8060.15

Page 3-7
5. 2. %
   3. C.Y. or cu. yd.
   4. Ac.
   5. Ton
   6. gals.
   7. S.Y. or sq. yd.

Page 3-8
8. Gallon
9. yd.
10. avg
11. radius
12. Fahrenheit
13. height
14. W
15. lb.

Page 3-10
6. 16
   151.29
   343
   15.625

Page 3-13
7. 22.5 ft.
8. 7 ft.

Page 3-14
9. 60 sq. ft.
10. 100 sq. ft.
11. 10 yd.

Page 3-18
12. A = V/H , V = AH
13. 1.76

Page 3-20
14. 25
15. 123.75
    0.04
    3.84

CHAPTER FOUR

Units of Measurement

CONTENTS

4

UNITS OF MEASUREMENT

STARTING POINTS FOR TRAINING

Determine the space occupied by the concrete structures in Problems 1, 2 and 3 -- then check your answer.

PROBLEM

1. V = L W H

A box that measures 15.60 ft. x 11.40 ft. x 6.30 ft.

V = ________________ cu.yds.
0.01

> The degrees of accuracy needed in the final answers are noted under the blanks for the answers.

PROBLEMS, continued

2. A portland cement concrete footing that measures:

2'3" x 2 1/10' x 9.3'

V = ________________ cu.yds.
0.01

3. A concrete structure that measures:

0.90 ft. x 18.60 in. x 2.70 ft.

V = ________________ cu.yds.
0.01

Right on Problems 1, 2 and 3? Excellent! Skip to "WEIGHT MEASUREMENTS."

Wrong? Start with "TERMS AND MEASURES" on the next page.

TERMS AND MEASURES

You already know much or all of the information discussed in this chapter relative to measurement -- BUT, let's review.

TERMS USED

The term "measure" refers to standard values: 12 inches per foot, 16 ounces per pound, 43,560 square feet per acre, 27 cubic feet per cubic yard.

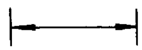
The term "measurement" means the actual measure of something -- length, weight, area or volume.

TYPES OF MEASURES

The six basic types of measures are:

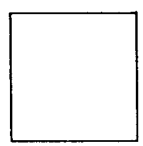
- Length
- Area
- Space volume
- Liquid volume
- Weight (mass)
- Rate -- as in gallons per square yard.

LENGTH, AREA AND VOLUME MEASUREMENTS



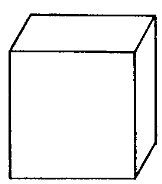
LENGTH

Measurements of length, width, height and distance are linear measurements. Examples of linear measures are inches, feet, yards and miles.



AREA

Measurements of area are in square measurements and hectares. Examples of the term "square" are square inches, square feet, square yards, and square miles.



VOLUME

Measurements of volume are cubic measurements. An example of the term "cubic" are cubic inches, cubic feet, and cubic yards.

CALCULATING LENGTH, AREA AND VOLUME MEASUREMENTS

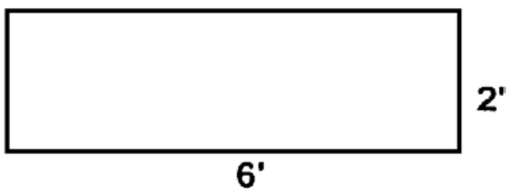
Length

Add or subtract lengths and distances to obtain measurements:

550 ft. + 475 ft. + 370 ft. = 1395 ft.

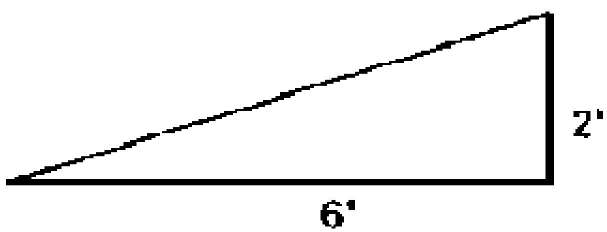
Area

One -- Multiply length times width to calculate areas of squares and rectangles:



6' x 2' = 12 sq.ft.

Two -- Divide by 2 to calculate areas of triangles:

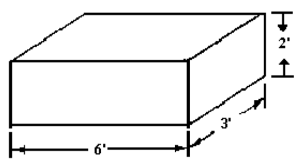


[6' x2'] / 2 = 6 sq.ft.

Right triangles are 0.5 of a rectangle.

Volume

Multiply length time’s width time’s height to calculate volumes of cubes and rectangular solids:



6 ft. x 3 ft. x 2 ft. =36 cu.ft.

The term "volume" represents quantity of space measured in cubic inches, cubic feet or cubic yards.

Note:

Linear times linear = square	Linear = Length	Cubic divided by square = linear
Square times linear = cubic	Square = Area	Square divided by linear = linear
Cubic divided by linear = square	Cubic = Volume	

PROBLEM

4. 1 acre = ____________ sq ft.

1 square yard = ____________ sq. ft.

1 mile = ____________ ft.

1 cubic yard = ____________ cu. ft.

1 cubic foot = ____________ cu. in.

PROBLEM

5. square foot multiplied by foot results in what? ______________________________________

cubic yard divided by square yard results in what? __________________________________

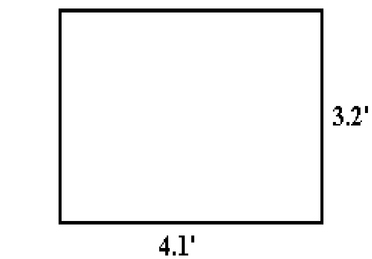
inch multiplied by inch results in what? ____

inch times inch times inch results in what? ______________________________________

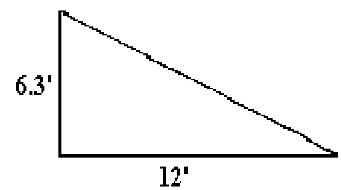
Area times height results in what? ____

PROBLEM

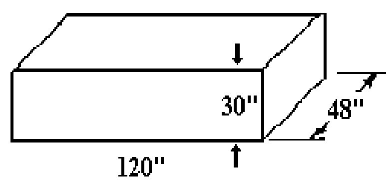
6. Calculate the areas and volumes shown below:



= ________________ sq.ft.
0.01



= ________________ sq.ft.
0.01



= ___________________ cu.ft.
nearest whole number

Right? Study the "REVIEW OF LENGTH, AREA AND VOLUME MEASUREMENTS" on page 4-11.

Mistakes? Check your multiplications by dividing. If you still have errors, check the conversion tables.

REVIEW OF LENGTH, AREA AND VOLUME MEASUREMENTS

The whole business of length, area and volume measurements is summarized below:

Length means distance -- how long something is or how far it is from one point to another, as the length of a pipe culvert or the distance from one side of a road surface to the other.

Measurements of length, width, height, depth, diameter, radius and distance are linear or lineal measurements.

Area means surface size. The surface sizes of squares and rectangles are obtained by multiplying the lengths by the widths.

If the surface is vertical, such as a wall, multiply length by height. If the surface is a triangle, multiply the length by the height and divide by two.

Volume means capacity -- how much space is contained in the object. Volumes of cubes and rectangular solids are obtained by multiplying one surface area by height or depth -- length x width x height or depth.

Standard length, area and volume values are shown in the tables at the beginning of this chapter. Length, area and volume measurements involve common calculations -- inches can be added to, subtracted from and multiplied by inches, but not by other units.

Go on to "WEIGHT MEASUREMENTS."

WEIGHT MEASUREMENTS

TABLE OF MEASURES

The weight measures used by the Department are shown below:

WEIGHT MEASURES

Measure	Symbols (Abbreviation)	Equivalent
GRAM	G	---------------
POUND	lb.	16 oz.
TON	Tn	2000 lbs.

PROBLEM

7. Convert the following measurements as indicated. Round to 0.01.

1,650 lbs. = _______ tons

14.78 tons = _______ lbs.

57.8 oz. = _______ lbs.

2.96 lbs. = _______ oz.

PROBLEMS

8. Convert the following and round:

1,250 lbs. = ________________ tons
0.01

27.3 oz. = ________________ lbs.
0.1

9. Calculate the following and round:

2.8 lbs. +12.2 oz. + 38.5 oz. = ________ oz.
0.1

12.9 lbs. + 62.3 oz. + 5.8 lbs. = __________ lbs.
whole number

FORMULA DATA

As an inspector, you will be given many formulas by the bridge and road design engineers, the materials testing engineers and the construction engineers.
For instance, a formula for calculating the bearing value on a test piling is:

$$R = \frac{167\ WH}{S + 25.4} \text{ for gravity hammers}$$

Customary Units:

$$R = \frac{2\ WH}{S + 1.0} \text{ for gravity hammers}$$

R = Safe bearing value in tons

W = Weight of striking part of hammer in tons.

H = Height of hammer fall to the nearest 0.1 foot.

S = Average penetration per blow to the nearest 0.01 inch for the last 10 to 20 hammer blows.

Suppose these are your data:

Weight of hammer	= 1.08 tons.
Height of fall	= 12 feet
Average penetration	= 0.50 inches

There you are! Several items of raw data and a formula. And you don't even understand the formula. Well, you don't have to understand it. You have to be able only to work with it. You can start anywhere. The sequence of the next page is only one of several possible sequences.

First -- Add the known value to the formula:

$$R = \frac{2\,WH}{S + 1.0} = \frac{2(1.08 \times 12)}{0.50 + 1.0}$$

<u>Note</u>: The units of measurement, i.e.: tons, feet, etc. are not used in this formula; only the numerical values are calculated. Study carefully the formula above and the values used in place of the symbols.

Second -- Calculate as the formula procedure indicates:

$$R = \frac{2(1.08 \times 12)}{0.50+1.0} = \frac{25.92}{1.50} = 17.28 \text{ tons}$$

R (rounded to 0.1) = <u>17.3 tons</u>

PROBLEM

10. In the formula, $R = \frac{2\,WH}{S + 1.0}$

What would be the bearing value if the raw data were these?

Weight of hammer = 1.22 tons
Height of fall = 10 ft.
Average penetration = 0.40 in.

$R = \frac{\rule{3cm}{0.4pt}}{0.1}$ tons

LIQUID MEASUREMENTS

The only liquid measurement used in highway construction work is gallons. Capacities in gallons often are calculated in cubic feet:

<u>7.48 gallons = 1 cubic foot</u>

-- To convert cubic feet to gallons, <u>multiply</u> by7.5.
-- To convert gallons to cubic feet, <u>divide</u> by 7.5.
However, don't forget the rules of rounding and degrees of accuracy.

PROBLEM

11. How many gallons of liquid can be placed in a container having a 283.5 cubic foot capacity?

_______________ gals.
whole number

Calculation, Problem 11

283.5 cu. ft. x 7.5 gals. per cu. ft. = 2,126.25 gals
Rounded to: 2,126 gals.

Go on to "RATE MEASUREMENTS."

RATE MEASUREMENTS

CALCULATING RATES

Highway personnel often must be able to calculate rates -- gallons per square yard, cubic feet per minute, cubic yards per hour.

Two measures are involved in every rate calculation -- such as gallons and yards, feet and minutes, or cubic yards and hours. Of these, one is a fixed value, the other a variable.

Liquid asphalt application rates are expressed in □gallons per square yard□. The square yard is a fixed value. It never changes. The number of gallons applied is a variable value. It changes.

Rates of all kinds can be expressed as "formulas." For example, asphalt application rates can be expressed as follows:

$$\text{Asphalt application rate} = \frac{\text{gallons applied}}{\text{area covered}}$$

$$\text{(or) rate} = \frac{\text{gals.}}{\text{sq.yds.}}$$

Rule: Divide variable values by the fixed values to get rates.

Work the problem on the next page. The relationships existing in all forms of rates should become clear.

CALCULATING RATES (continued)

Sample Problem

Quantity of asphalt used: 1,474 gal.
Width of spray bar: 21 feet
Length of application:1,755 feet

What is the application rate -- in gallons per square yard?

Solution

You already know the rate "formula:" Application rate = $\dfrac{\text{gallons applied}}{\text{area in sq.yds.}}$

(or) Rate = $\dfrac{\text{gals.}}{\text{sq.yds.}}$

The liters (gallons) used are known, but the area is unknown. So, compute the "unknown" quantity -- which is the area covered -- as follows: A = LW = 1755 ft. x 21 ft. = 36,855 sq. ft.

Now, you can "plug-in" the known quantities -- into the application rate formula.

Application rate = $\dfrac{\text{1,474 gals.}}{\text{4,095 sq.yds.}}$ = $\dfrac{\text{0.36 gals.}}{\text{1 sq.yd.}}$ = 0.36 gals. per sq.yd

Note: The "rate" itself must be expressed as gallons per ONE square yard." The quantities in the rate formula are calculated by DIVIDING the yards into the gallons as indicated above. The variable value is divided by the fixed value -- the gallons by the square yards.

Remember, a "rate" is an expression of a variable value per one unit of a fixed value. Above, the fixed unit is one square yard.

PROBLEM

12. A contractor placed 1,000 gallons of asphalt on 2,700 square yards of road surface. What was the application rate?

Right? Keep going. Wrong? Keep going too. Rates will soon become clear.

<u>Application Rates Problem</u> Using an application rate of 0.38 gals./sq. yd., how many gallons of asphalt will be needed to cover an area 22' wide by 11.2 miles?

<u>Calculation</u>

11.2 miles = 5,280 ft. per mile x 11.2 mi. = 59,136 ft.

59,136 ft. x 22 ft. = 1,300,992 sq. ft. (1,300,992 sq. ft. ÷ 9 sq. ft. = 144,554.67 sq. yds.)

144,554.67 sq. yds. can be rounded to 144,555 sq. yds.
144,555 sq.yds. x 0.38 gals. per sq. yds. = 54,930.9 gals.

(54,930.9 can be rounded to 54,931 gals.)

PROBLEM

13. An asphalt plant produces 12.35 tons of hot asphalt every 5 minutes. What is the production rate in tons per hour?

Other Rates Problem

A batch plant produced the portland cement concrete needed in laying 3,800 linear feet of 9" pavement, 12' wide in a 12-hour day. What was the productive capacity of the plant in cubic yards per hour?

Calculation
3,800 lineal feet times 12 lineal feet
= 45,600 square feet.
9 inches = 0.75 feet
45,600 square feet times 0.75 lineal feet
= 34,200 cubic feet.
(34,200 cubic feet divided by 27 cubic feet per cubic yard = 1,266.67 cubic yards).
1,266.67 cubic yards divided by 12 hours
= 105.56 cubic yards per hour.
As you can see:
Feet multiplied by feet provides answers in square feet.
Square feet multiplied by feet provides answers in cubic feet.
Cubic feet divided by the fixed value of 27 cubic feet per cubic yard provides answers in cubic yards. Total production divided by hours of production provides answers in terms of productivity per hour.

Go on to "HIGHWAY PROBLEMS."

HIGHWAY PROBLEMS

14. How many acres are there in a parcel of land 145' wide and 240' long?

A = __________ acres
0.001

15. How much does the following water tank hold? [1 ft □ =7.48 gals.]

L = 10.5'0
H = 5.2'   V = _____________ gals.
W = 5.1'   whole number

16. A section of portland cement concrete box culvert measures 44 feet 8 inches by 4 feet 6 inches by 9 inches. How much concrete is needed?

V = _______________ cu. yds.
0.1

17. An asphalt distributor is fitted with a (an) 18-foot spray bar. During a shot, the distributor traveled 1500 linear feet. 1,153.8 gallons of liquid asphalt were used.

What was the application rate?

_________ gals./sq. yd.
0.01

18. A Contractor drove a timber piling with a single-acting air hammer. Compute the bearing capacity (R) in tons based on the data below:

$$R = \frac{2\,E}{S + 0.1 + 0.01P}$$

Where:

R = Safe Bearing value in tons

S = Average penetration per blow, in inches, as recorded for the last 10 to 20 blows

E = WH = the energy that the hammer delivers, in foot-tons

W = Weight of striking part of hammer in tons

H = Height of hammer fall in feet

<u>Test Data:</u>

Weight of hammer = 1.5 tons

Height of hammer = 13 ft.

Average penetration per blow for the last <u>10</u> blows = 0.25”

$$R = \frac{\rule{4cm}{0.4pt}}{0.1}\ \text{tons}$$

HIGHWAY PROBLEMS, continued

18. Surface treatment is planned for 12,000 square yards on S.R. 435. The application rate for aggregate is 0.25 cubic feet per square yard. How many cubic yards of aggregate are needed?

________________________ C.Y.
whole number

19. A 216-cubic foot truck hauled 11 loads of limerock material to your job site. How many cubic yards of material is this?

________________________ C.Y.
whole number

ANSWER TO PROBLEMS

Page 4-2
1. 41.50

Page 4-3
2. 1.63
3. 0.14

Page 4-8
4. 43,560
   9
   5,280
   27
   1,728

Page 4-9
5. cubic feet
   yard
   square inches
   cubic inches
   volume

Page 4-10
6. 13.12
   37.80 sq. ft.
   100

Page 4-12
7. 0.83
   29,560.00
   3.61
   47.36

Page 4-13
8. 0.83
   1.7
9. 95.5
   23

Page 4-16
10. 17.4

Page 4-17
11. 2,126

Page 4-20
12. 0.37 gals/sq. yds.

Page 4-21
13. 148.20 tons/hour

Page 4-22
14. 0.799 acres
15. 2,083 gals.
16. 5.6 cu. yds
17. 0.38 gals./sq. yds.

Page 4-23
18. 70.9 tons

Page 4-24
19. 111 C.Y.
20. 88 C.Y.

CHAPTER FIVE

Averages- Percents- Ratios- Proportions

CONTENTS

Editors Note: This chapter introduces the term “slope”. It has been traditional in road building organizations to refer to slope a “run over rise”. In other words the first number indicated the length of a slope and the second number indicated the height. When the Department changed to metric in 1992, the metric version of slope was adopted. The Metric version of slope uses “rise over run”. The Departments Metric to English Conversion Committee decided in 1998 to keep this feature of the metric system while returning back to English units. This was done to keep the Departments slope reference more in keeping with today’s accepted standards.

This manual uses slope as rise over run (i.e.,1:3 slope means that the slope has a foot “rise “or “height” for every three feet of “run” or “length”. During this transition period, the Inspector will encounter both systems in the field. While in the field, it is important for the Inspector to determine which system the Contractor has marked on the lay out stakes and which system is being used in the plans for estimating purposes. The mathematics used with slope in this chapter and throughout the rest of this text does not change.

5

AVERAGES- PERCENTS- RATIOS- PROPORTIONS

STARTING POINTS FOR TRAINING

PROBLEMS

1. The results of the Marshall Stability Test for three pills are

Stability Test	Pounds
1	662
2	660
3	665

Average = ________________ pounds
whole number

Make a mistake? Skip Problems 2, 3 and 4, and start studying "AVERAGES."

Get the right answer? Try Problem 2.

PROBLEMS, continued

2. Write each of the following as percentages:

0.30 ___________ %

0.10 ___________ %

0.965 ___________ %

Calculate the following percentage:

27.5% of 123___________

If you made mistakes, skip Problems 3 and 4 study "WORKING WITH PERCENTAGES."

If not, try Problem 3.

PROBLEMS, continued

3. A slope has a 1:3 slope ratio and a vertical distance of 6.5 ft. What is the horizontal distance?

Horizontal distance = ____________ft.
(0.1)

A slope has a 1:3.5 ratio and a horizontal distance of 26 ft. What is the vertical distance?

Vertical distance = _____________ft.
(0.01)

If you made mistakes, skip Problem 4 and study the ratio section of "WORKING WITH RATIOS AND PROPORTIONS." Review "WORKING WITH PERCENTAGES" first, if you like.

If you made no mistakes, try Problem 4.

4. A portland cement concrete mix design contains 6 bags of cement per 1.11 cu. yds. of mixed aggregate. How many bags of cement will be needed for16.2 cu. yds. of mixed aggregate?

___________ bags of cement
tenths

If you made a mistake, start studying the proportion section of "WORKING WITH RATIOS AND PROPORTIONS." Study "WORKING WITH PERCENTAGES" and the complete section "WORKING WITH RATIOS AND PROPORTIONS" first if you like.

If you did not make a mistake, work the "HIGHWAY PROBLEMS" at the end of this chapter for practice. If you have difficulty with those problems, study the appropriate sections in this chapter.

AVERAGES

Averages are computed by:

- Adding the series of numbers like this:

$$\begin{array}{r} 12 \\ 27 \\ 18 \\ 21 \\ \underline{+19} \\ 97 \end{array}$$

- Dividing them by the number of tems in the series -- 5 items:

$$\begin{array}{r} 19.4 \\ 5 \overline{)\ 97.0} \\ \underline{5} \\ 47 \\ \underline{45} \\ 20 \\ \underline{20} \end{array}$$

The average of this series is 19.4 -- the number obtained by dividing.

Have you written a formula for finding averages? Go ahead. Try. Then check it against this one:

$$\text{Average} = \frac{\text{sum total of the numbers when added}}{\text{the numbers of numbers added}}$$

With abbreviations and symbols you can write the formula like this:

$$\text{Avg} = \frac{X_1 + X_2 \ldots . X_n}{n}$$

PROBLEM

5. Calculate the averages of the following series of numbers:

   431 + 500 + 439 + 414

   2011 + 1991 + 2181

Right? Try Problem 6.

Wrong? You must have missed the point. Adding all of the numbers and dividing by the number of numbers in the series obtain the average. The two calculations from Problem 5 are shown below:

Add		Divide	
$\begin{array}{r} 431 \\ 500 \\ 439 \\ \underline{+\ 414} \\ 1784 \end{array}$	Number of Items = 4	$\begin{array}{r} 446 \\ 4\overline{)1784} \\ \underline{16} \\ 18 \\ \underline{16} \\ 24 \\ \underline{24} \end{array}$	= Avg.

Add		Divide	
$\begin{array}{r} 2011 \\ 1991 \\ \underline{+\ 2181} \\ 6183 \end{array}$	Number of Items = 3	$\begin{array}{r} 2061 \\ 3\overline{)6183} \end{array}$	= Avg.

-- Be sure you add correctly. Check your addition if necessary.
-- Divide your answer by the number of items added. Check your division.

Try Problem 6.

PROBLEM

6. What is the average of 8 + 8.25 + 7.875 + 8.125 + 8?

Go to "WORKING WITH PERCENTAGES."

WORKING WITH PERCENTAGES

Percentages are expressed in 0.01. For example 0.125, which can be written as 12.5% or 12.5 percent. Here are some other examples:

0.1	= 10% or 10.0 percent	0.667	= 66.7% or 66.7 percent
0.25	= 25% or 25.0 percent	0.75	= 75% or 75.0 percent
0.333	= 33.3% or 33.3 percent	1	= 100% or 100.0 percent
0.5	= 50% or 50.0 percent	1.5	= 150% or 150.0 percent

A ten-percent increase from 90 is 99. A ten-percent decrease from 99 is 89.1. Seventy-five percent of 200 is 150. The number 150, when increased by 25 percent, is 187.5.

CALCULATING PERCENTAGES

To calculate a percentage, divide the numerator by the denominator and move the decimal point two places to the right

PROBLEM

7. Calculate the following percentages:

0.30=_________ %

0.80 =________ %

0.40 = ________ %

Right? Study "WHAT TO CALCULATE."

Wrong? Go back and review.

WHAT TO CALCULATE

Suppose you want to know the percentage of moisture in a sample, based on the dry mass (weight) of the sample. which number is divided by which?

Divide the amount of moisture by the dry weight and change the answer to a percentage. $\mathbf{\frac{M}{D} = \% \, M}$

First --Subtract dry weight from wet weight to find the amount of weight loss.

7.14 g	wet weight
- 6.79 g	dry weight
0.35 g	weight loss

Second -- Divide the amount of weight loss by the dry weight.

$$6.79 \overline{\smash{)}\,0.350000} = 0.0515 \quad (\text{rounded to } 0.001) = 0.052$$

Third --Change the decimal answer to a percentage. 0.052 = 5.2%

The moisture content is 5.2% (rounded) based on dry weight.

If the weight of the moisture had been divided by the wet weight, the calculation would have shown a moisture content of 4.9% based on the wet weight.

Check your calculations:
6.79 x 0.052 = 0.353 80 or 0.35 g
Dry weight x percent moisture as a decimal = weight loss in grams

READING DECIMAL NUMBERS AND PERCENTAGES

Calculating percentages is easy -- but reading them gives some people difficulty.

-- Move the decimal point two places right to read a decimal number as a percentage.
-- Move the decimal point two places left to read a percentage as a decimal number.

0.333 = 33.3%
33.3% = 0.333

Decimal Number	Percentage	Percentage	Decimal Number
0.100	10.0	15.0	0.15
0.970	97.0	2.230	223.0

And

0.02%	= 0.02 of 1% --(Two one-hundredths of one percent)	= 0.0002
0.20%	= 0.2 of 1% -- (Two-tenths)	= 0.0020
2.00%	= 2% -- Two percent	= 0.02
20.00%	= 20% -- Twenty percent	= 0.20

PROBLEM

8. Read the following numbers as percents. The first one has been completed to show how the answers should be written.

0.007 = <u>0.7%, or zero point seven percent</u>
0.094 = ____________________________
0.87 = ____________________________
1.36 = ____________________________

MOVING DECIMAL POINTS

Percents often contain decimal points -- but they are not ordinary decimal numbers.

-- The decimal number 0.10 must be changed to 10.0 to be used as a percent -- 10.0%.

-- The decimal number 0.000 613 must be changed to 0.0613 to be used as a percent -- 0.0613%

-- Also, 0.92 = 92%, and 0.9271 = 92.71 %.

And: $0.09\% = \frac{0.09}{100} = 0.0009$ as a decimal number.

$\frac{125}{100}$ of $1\% = \frac{125}{100}\% = 1.25\% = \frac{125}{100} = 0.0125$ as a decimal

PROBLEM

9. A sample of dried aggregate weighs 2.95 lbs. 10% does not pass the No. 4 sieve. 14% passes the No. 4 sieve but does not pass the No.10 sieve. The balance passes the No. 10 sieve.

What is the weight of the aggregate that passes the No. 10 sieve?

For Problem 9, find what percentage of the aggregate passed the No.10 sieve.

10% + 14% = 24% -- percentage not passing No.10 sieve.

100% - 24% = 76% -- percentage passing No.10 sieve.

Then change the percentage to a weight value, as shown:

2.95 lbs. x 76% = 2.24 lbs. weight of aggregate passing the No. 10 sieve.

Try Problem 10.

PROBLEMS, continued

10. If the percent moisture, based on dry weight, is 4.6% -- how much would 3,000 lbs. of dry aggregate weight wet?

Wet weight = ____________ lbs.

Right? Of course! You had to add the weight of the moisture [4.6% of the dry weight] to the dry weight. You could have done it two ways: adding percents or adding weights. Calculations are shown below.

If the dry weight of the aggregate is 100.00% of the 3,000 lbs. and the weight of the moisture is 4.6% of the 3,000 lbs., then the wet weight of the aggregate can be found by adding.

Adding Percents

(Wet weight) = dry weight + moisture weight

= 100% + 4.6% of 3,000 lbs.

= 104.6% of 3,000 lbs.

But 104.6% = 1.046

Wet weight = 1.046 x 3,000 lbs. = 3138 lbs.

Adding weights

Moisture weight = 4.6% of 3,000 lbs

But 4.6% = 0.046

Moisture weight = 0.046 x 3,000 lbs.
= 62.6 kg (138 lbs.)

Wet weight = dry weight + moisture weight
= 3,000 lbs.+138 lbs.
= 3,138 lbs.

PROBLEMS, continued

11. Move the decimal points to change the decimal calculations to percentage readings:

$$\begin{array}{r} 0.333 \\ 120\overline{)40.000} \\ \underline{360} \\ 400 \\ \underline{360} \\ 400 \end{array} = \underline{\qquad\qquad} \%
\qquad
\begin{array}{r} 0.050 \\ 70\overline{)3.500} \end{array} = \underline{\qquad\qquad} \%
\qquad
\begin{array}{r} 0.30 \\ 21\overline{)6.30} \end{array} = \underline{\qquad\qquad} \%$$

Study "MEANING OF 100 PERCENT" below.

MEANING OF 100 PERCENT

One hundred percent of a unit means the WHOLE THING. Fifty percent of the unit means half of it. And two hundred percent of a unit is just another way of saying 2 units. Five hundred percent -- five units, etc.

WORKING WITH RATIOS AND PROPORTIONS

Ratios express relationships between values:

-- A ratio of 1 engineer to 4 technicians means that one engineer is employed for every four technicians employed. The ratio is shown as 1:4.

-- (A ratio of 1 to 5 -- written 1:5 -- can be used to mean a 1-foot vertical distance to 5-foot horizontal distance).

Proportions express equality between ratios. Expressions using two ratios -- as in 3:1 = 6:2 -- are called proportions.

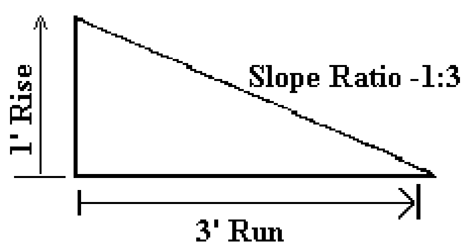
-- The proportion 3:1 = 6:2 is read as "three to one equals six to two" or "three is to one as six is to two."

-- A proportion of 4:1 = 16:4 can be used in calculating a mix.

SLOPE RATIOS

Slope ratios are expressed as 1: 3 -- meaning 1 foot to 3 feet.

The first number represents vertical distance -- distance up or down -- and the second represents horizontal distance -- distance out:



CALCULATING RATIOS FROM VALUES

Only one step is needed to work out a ratio from a given set of values: Divide one value by the other.

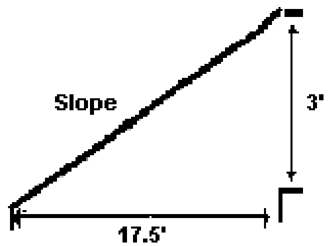
-- Horizontal distance	Unknown	
-- Vertical distance	65 feet	-- Multiply 65 by 6 -- 390 ft.
-- Slope ratio	1:6	
-- Horizontal distance	390 feet	
-- Vertical distance	Unknown	-- Divide 390 by 6 -- 65 ft.
-- Slope ratio	1:6	

CALCULATE VALUES FROM RATIOS

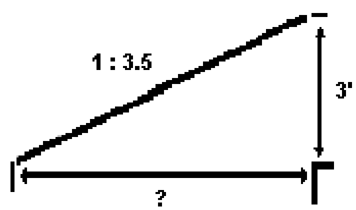
Given one value and a ratio, the other value can readily be calculated. Either multiply or divide.

-- Horizontal distance	42 feet
-- Vertical distance	10.5 feet -- Divide 42 by 10.5 -- 1:4 slope ratio
-- Slope ratio	Unknown
-- Coarse aggregate	720 cubic yards
-- Fine aggregate	120 cubic yards -- Divide 720 by 120 -- 6:1 ratio
-- Ratio	Unknown

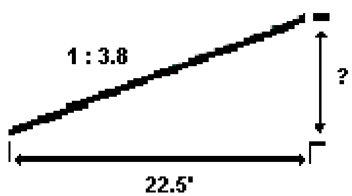
PROBLEMS



12. Compute the slope ratio: Round to 0.1.



13. Determine the horizontal distance: Round to 0.1.



14. Determine the vertical distance: Round to 0.1.

Right? On all three? Study "CALCULATING VALUES USING PROPORTIONS."

Mistake? Go back and review.

CALCULATING VALUES USING PROPORTIONS

Proportions are used to find needed quantities.

If coarse aggregate and fine aggregate are to be mixed in a ratio of 2.5: 1, and you have 2000 cubic yards of fine aggregate, how many cubic yards of coarse aggregate will be needed?

-- 2.5:1 = Unknown: 2,000 cu. yds. -- or, 2.5:1 = X: 2,000 cu. yds.

Since, in the first ratio, the first value is 2.5 times as much as the second value, the comparable value in the other ratio must be 2.5 times as much as the second value.

-- 2.5 times 2,000 cu. yds. is 5,000 cu. yds.

So 2.5:1 = 5,000 cu. yds. :2,000 cu. yds. -- or the coarse aggregate needed = 5,000 cubic yards

Any combination of ratios can be used. Here are three:

6:1 = 18,600: something 5:4 = X: 4,420 7:6 = 7,770:X

6:1 = 18,600: $\frac{18,600}{6}$

5:4 = $\frac{5}{4}$ of 4,420:4,420 7:6 = 7,770: $\frac{6}{7}$ of 7,770

6:1 = 18,600:<u>3,100</u> Since $\frac{5}{4}$ of 4,420 = 5,525 $\frac{6}{7}$ of 7,770 = 6,660

5:4 = 5,525:4,420 7:6 = 7,770:6,660

ANOTHER PROPORTION

If fill from a borrow pit shrinks 20% after it is compacted, how much will be needed to fill 4,000 cubic yards? Since one cubic yard of material from the pit yields? yards of fill, the problem readily can be set up.

1C.Y. :0.80C.Y.

0.80: ? C.Y. = 1 C.Y. :4,000 C.Y. and $\dfrac{1}{0.80} = \dfrac{X}{4000}$

Since 1: 0.80 = 1.25, and X = 4,000 times 1.25

Then 1: 0.80 = 5,000 C.Y. :4,000 C.Y.

Check

20% of 5,000 C.Y. = 1,000 C.Y.

5,000 C.Y. - 1,000 C.Y. = 4,000 C.Y.

If sand contains 17% water based on dry weight, how much wet sand is needed to provide 3,450 lbs. of dry sand? Since dry weight is the base value -- dry weight = 100%. If the total weight is 17% water -- the wet weight is 117% of the dry weight as shown below:

Dry weight	= 100%
Water	= 17%
Total Weight [Wet weight]	= 117%

So -- Wet weight	= 117% of 3,450 lbs. (dry sand)
	= 1.17 x 3,450 lbs.
	= 4,036.5 lbs.

PROBLEMS

Set these problems up as proportions. Work them any way you like.

15. A sample of sand has a wet weight of 11 lbs. The sample contains 1.2 lbs. of water. How much water will 1,600 lbs. of wet sand contain?

_______ lbs. water
0.1

16. A portland cement concrete mix design specified 6 bags of cement and 1,284 lbs. of fine aggregate per cubic yard of concrete. How many lbs. of fine aggregate will be required for a 28-bag mix?

________ lbs.

17. 58 gallons of asphalt were used during a 100' seal coat test run. At this rate, how many gallons of asphalt will be used in a 1,250' run?

________ gals.

Right? Work the "HIGHWAY PROBLEMS."

Mistakes? Go back and review.

HIGHWAY PROBLEMS

18. A gradation test was run on 5 samples of Grade No. 3 aggregate. The percentages found passing the 1" sieve are listed below. What would be the average percent passing?

75.3
70.4
72.6
73.1
74.9

_______________ %
whole number

19. Determine the average pavement thickness based on these core samples:

Depth
Core # 1 - -9 $\frac{5}{8}$ inches deep
Core # 2 - 9 inches deep
Core # 3 - -9 $\frac{1}{2}$ inches deep
Core # 4 - -9 $\frac{1}{8}$ inches deep
Core # 5 - -9 $\frac{5}{8}$ inches deep
Average thickness = _________________ in.

HIGHWAY PROBLEMS, continued

20. A sample was taken from a soil cement job. The wet weight of the material was 603.13 g and the dry weight as 565.79 g.

    What was the moisture content (%) at this sample on dry mass?

    Moisture Content = _________ %
    0.1

21. At station 5 + 02.92 the P.I. elevation is 95.67 ft. At station 8 + 68.60 the P.I. elevation is 104.39 ft.

    What is the grade?

    $$\text{Percent grade} = \frac{\text{Vertical rise or fall}}{\text{Horizontal distance}} \;\square\; 100$$

    At station 5 + 02.92 the P.I. elevation is 95.67 ft.
    At station 8 + 68.60 the P.I. elevation is 104.39 ft.

    What is the % grade?

    Percent grade = ______________%
    0.01

HIGHWAY PROBLEMS, continued

22. Percent moisture = $\dfrac{\text{Wet weight - Dry weight}}{\text{Dry weight}} \times 100$

    A 116.6 g sample of fine aggregate is dried. The dry weight is 108.3 g. What is the percent moisture?

    Percent moisture = __________%
    0.1

23. A total run of asphalt mix weighs 40,000 lbs. If 6.5% of this weight is the asphalt, what is the weight of the aggregate?

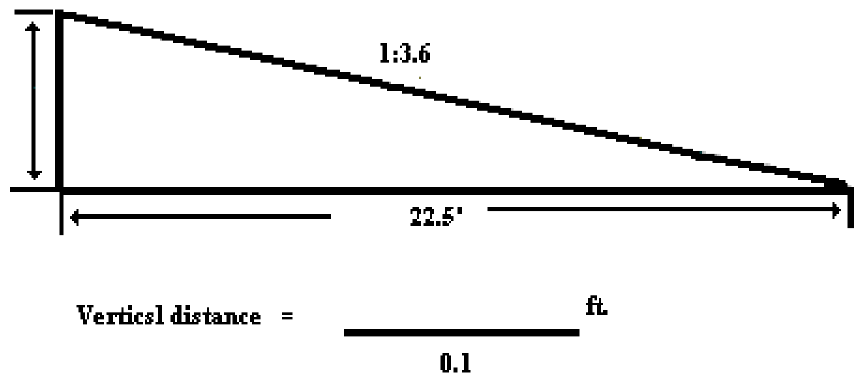
    Weight of aggregate = _____________ lbs.
    whole number

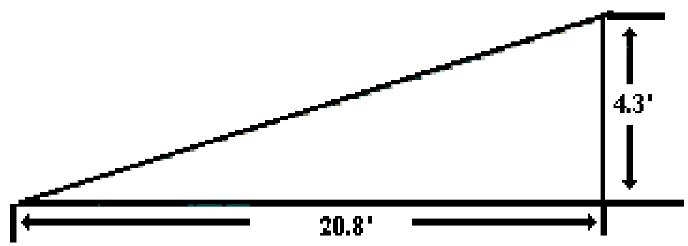
24. A 20 lb. sample was taken from a stockpile. 3.1 lbs. pass the #10 sieve.

    What percentage of the total sample passed the #10 sieve?

    ____________ %
    whole number

25. Compute the vertical distance:





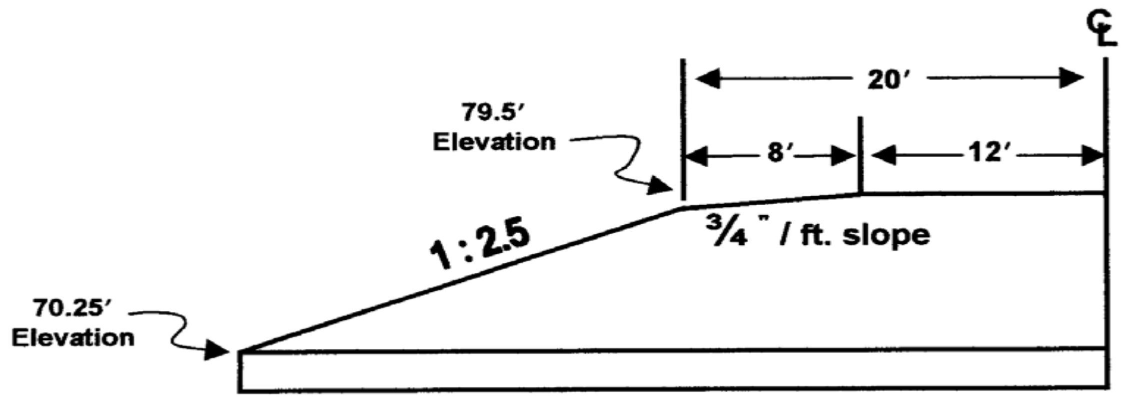
26. Determine the slope ratio:

Slope ratio = ________________1

HIGHWAY PROBLEMS, continued

27. What length of 36" - diameter pipe is needed for the culvert below? (Disregard the slope of the pipe itself.)

Pipe length = ________ ft.



28. It takes 4.7 sq. yds. of portland cement concrete pavement per linear foot of ditch pavement. How many sq. yds. of concrete ditch pavement will be necessary on a 4,550 project?

____________ sq. yds.
whole number

HIGHWAY PROBLEMS, continued

Work Problems 29 and 30 as proportions:

29. If shrinkage is 15%, how many cubic yards of material will be needed for a base course 1.520 miles long, 25 feet wide and 6 inches deep?

_________ cu.yds.

30. 6 bags of cement are used for 1 cubic yard of concrete. How many pounds of cement will you need for 2 1/4 yards? 1 bag cement = 94 lbs.

___________ lbs.
whole number

ANSWERS TO PROBLEMS

Page 5-3
1. 662

Page 5-4
2. 30
   10
   96.5
   33.83

Page 5-5
3. 19.5
   7.43
4. 87.6

Page 5-7
5. 446
   2,061

Page 5-8
6. 8.05

Page 5-10
7. 30
   80
   40

Page 5-12
8. 9.4%, or nine point four percent
   87%, or eighty-seven percent
   136%, or one hundred thirty-six percent

Page 5-13
9. 2.24

Page 5-14
10. 3,138

Page 5-15
11. 33.3
    5
    30

Page 5-18
12. 1:5: 8
13. 10.5'
14. 5.9'

Page 5-21
15. 174.5
16. 5,992
17. 725

Page 5-22
18. 73
19. 9 3/8 or 9.375

Page 5-23
20. 6.6
21. 2.38

Page 5-24
22. 7.7
23. 37,400
24. 16

Page 5-25
25. 6.3
26. 0.2

Page 5-26
27. 43.1
28. 21,385

Page 5-27
29. 4,272.9
30. 1,269

QUIZ ON CHAPTERS THREE, FOUR AND FIVE

You should be able to answer 85% of the questions correctly without going back to Chapters Three, Four and Five. Try it.

1. What are the symbols for the following terms? (3)

   Average ________ Pi ________ Ratio _______________

2. To square a number is to ______________ (1)

3. To cube a number is to ______________ (1)

4. Feet times feet = ______________ feet (1)

5. Cubic feet divided by feet = ______________ feet (1)

6. Square feet times feet = ______________ feet (1)

7. Cubic feet divided by square feet = _____________ feet (1)

QUIZ, continued

8. Is "X = 25.3 □ 3.87" a ratio or an equation? ____________ (1)

9. In equations, what you do to one side must be ________________________________ (1)

10. Write this problem as a line equation:

$$\frac{27.54 + 33.07}{} \times \frac{98.10}{21.50}$$

________________________ (1)

11. Develop formulas for the following: (5)

-- The area of a rectangle
-- The volume of a rectangular solid
-- The length of a rectangle in which the area and width are known:
-- The base area of a rectangular solid in which volume and height are known:
-- The height of a rectangular solid in which volume and base area are known:

12. How many even digits are there? ________ (1)

QUIZ, continued

13. Write the following as two-place decimal numbers: (4)

211.1	________	99 + 0.2
27 + (33 ÷ 100)	________	0.6

14. Calculate the following equation: (1)

0.508 x 0.562 x 0.875 = ______________________

15. Convert the following:

7.48 gallons to________ cu. ft.
13.7 tons to__________ lbs.
5,000 lbs. to__________ tons
2 cu. ft. to___________ gals.
2 cu. yds. to_________ cu. ft.
2 sq. ft. to___________ sq. in.
2 mi. to_____________ feet

QUIZ, continued

16. Using an application rate of 0.33 gals./sq. yd., how many gallons of asphalt are needed to cover 100 sq. yds.? ___________ (1)

17. A distributor pump discharges 350 gallons of asphalt in 3.5 minutes. What is the rate of discharge? ________________ (1)

18. Aggregate being used for an asphalt mix loses eight percent of its weight in the dryer -- based on DRY weight. How much does 200 pounds of wet aggregate weigh when it comes out of the dryer? _________________ (1)

19. If 40 lbs. of wet aggregate contains 8 lbs. of water, what is the percent moisture based on wet weight? _________________ (1) Dry weight? _____________ (1)

20. One cubic foot is what decimal part of a cubic yard? _________________ (1)

21. If the slope ratio is 1 :4.5 and the horizontal distance is 45 feet, what is the vertical distance? _________________ (1)

22. If 4.5:1 = X : 5, what is X? _____________(1)

QUIZ. continued

23. A mix design specifies 28.7 bags of cement per 6.5 cubic yards of concrete. How many bags of cement will be required to produce 650 cubic yards of concrete? Round to the nearest whole bag. _______________ (1)

24. A slope has a vertical distance of 7.75'. The horizontal distance is 38.75'. The slope ratio is _______________ (1)

25. You check the accuracy of multiplication by _______________ (1)

26. Show seven decimal places in this multiplication answer: 73914 __________________ (1). Round the answer to four decimal places: ______________________________________ (1)

27. Round the final answer to 0.1: Preliminary answer 90.1499 + Preliminary answer 90.0049 = final answer__________ ______________(1)

28. The symbols "r" and "%" represent ____________ and ____________.

29. Ac. represents ___________, and S.F. represents ____________. (2)

30. The symbols V, L and W represent _____________, _____________, and ___________. (3)

QUIZ, continued

31. 9^2 ________ 10^3 ________ (2)

32. If A = LW, L = _______ (1)

33. If V = AH, H = _______. (1)

34. Complete the following tables: (6)

LINEAR MEASURES

Foot	Yard	Mile
12 in.	_____ ft.	_____ ft.

AREA MEASURES

Square Inch	Square Foot	Square Yard	Acre
1 sq. in.	1 sq. ft.	1 sq. yd.	
	144 sq.in.	_____ sq. ft.	_____ sq.ft.

VOLUME MEASURES

Cubic Inch	Cubic Foot	Cubic Yard
1 cu. in.	1 cu. ft.	1 cu. yd.
	_____ cu. in.	_____ cu. ft.

QUIZ, continued

35. Tons multiplied by pounds per ton will give answers in ____________________ (1)

36. Eight inches expressed in 0.01 foot is __________________________________ (1)

37. 0.01 expressed as a percentage is ____________________________________ (1)

38. 0.50 is how many percent? ____ (1)

There are 66 answers in the Quiz. Did you get 60 or more right?

If so, you should be able to answer the questions on the final examination without difficulty.

Did you get 59 or less right? You may have studied too fast -- or you may have taken the Quiz too fast. Erase your answers and take the Quiz again in a few days. Study Chapters 3, 4 and 5 as necessary to get as least 60 right next time.

ANSWERS TO QUIZ

Page 5-29

1. Avg., π , :
2. Multiply it once by itself
3. Multiply it twice by itself
4. square
5. square
6. cubic
7. linear

Page 5-30

8. equation
9. done equally to the other side
10. X = (27.54 + 33.07) $\underline{98.10/21.5}$
11. A = LW
    V = LWH

    L = A/W

    A = V/H
    H = V/A

12. Five

Page 5-31

13. 211.10, 99.20, 27.33, 0.60
14. 0.508 x 0.562 x 0.875 = 0.250
15. 1
    27,400
    2.5
    14.96
    54
    288
    10,560

Page 5-32

16. 33
17. 100 gals./min.
18. 185.2 lbs.
19. 20.0%, 25.0%
20. .037 C.Y.
21. 10 ft.
22. 22.5

Page 5-33

23. 2,870 bags
24. 1 : 5
25. dividing the answer
    by the multiplier
26. 0.0073914, 0.0074
27. 180.2
28. radius and percent
29. Acres and square feet
30. Volume, Length, Width

Page 5-34

31. 81 and 1,000
32. A / W
33. V /A
34. 3 ft. and 5,280
    9 sq.ft. and 43,560 sq. ft.
    1,728 cu. in. and 27 c u. ft.

Page 5-35

35. pounds
36. 0.67 ft.
37. 1%
38. 50%

CHAPTER SIX

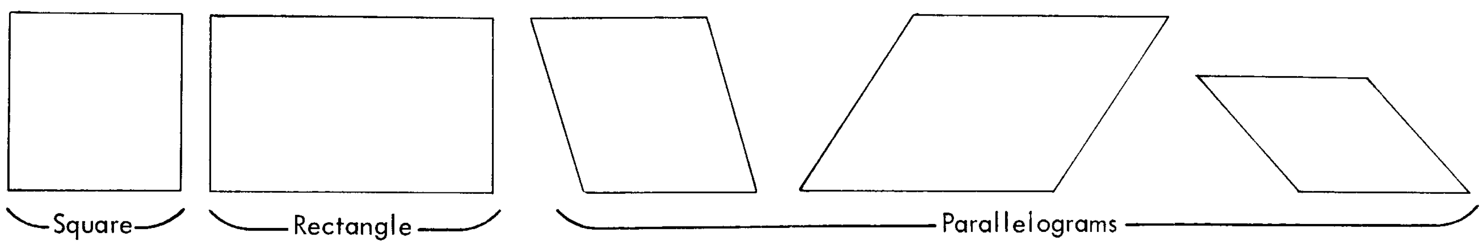
Calculating Areas

CONTENTS

6

CALCULATING AREAS

SQUARES, RECTANGLES AND OTHER PARALLELOGRAMS



Each figure above has four sides. All opposite sides are parallel.

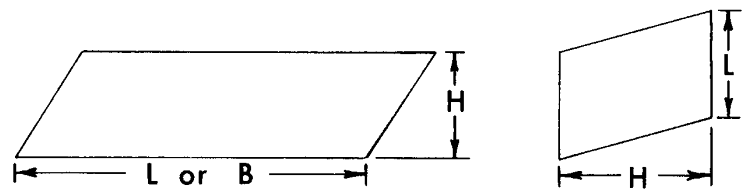
AREA CALCULATIONS

Calculating the areas of squares and rectangles was discussed briefly in Chapter Four.

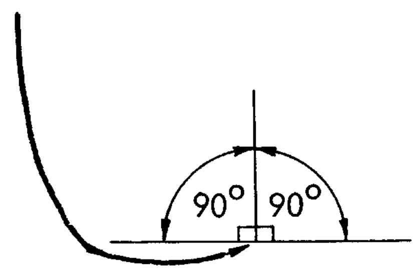
Area = Length times Width A = L W

Calculating the areas of parallelograms involves one additional step -- finding the height. A = LH

Measure the height along a line perpendicular to the length. Multiply length by height to get area. The side a figure rests o n is the BASE. Some persons call this the base length and use a B instead of an L. Then, A = BH



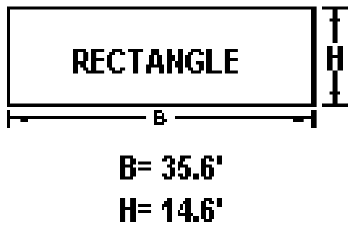
The term "perpendicular" means at a right angle: This little box tells you that an angle is 90° -- a right angle. And, the lines that form the angle are perpendicular.



PROBLEMS

1. Calculate the areas shown below:

A.

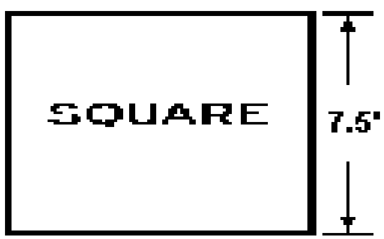


Area = ______________ sq. ft.
0.01

C.

Area = ______________ sq. ft.
whole number

B.

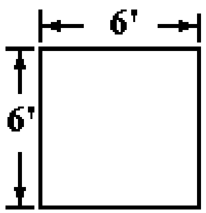


Area = ______________ sq. ft.
0.01

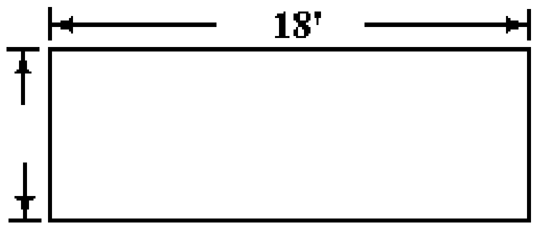
DISCUSSION OF PARALLELOGRAM AREAS

Calculating the areas of squares and rectangles is easy. Calculating the areas of parallelograms is also easy.

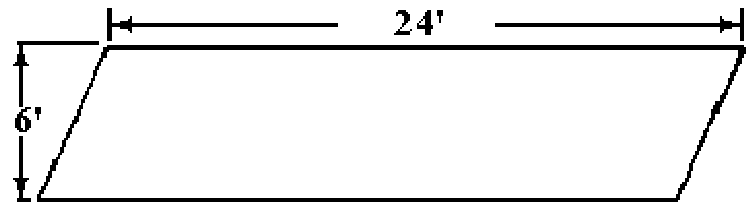
> <u>Multiply</u> the length of one side by the length of a perpendicular side to get the areas of squares and rectangles.
>
> <u>Multiply</u> the length by the height to get the areas of parallelograms.
>
> Formula: A = L W and A = L H



6' x 6' = 36 sq. ft.

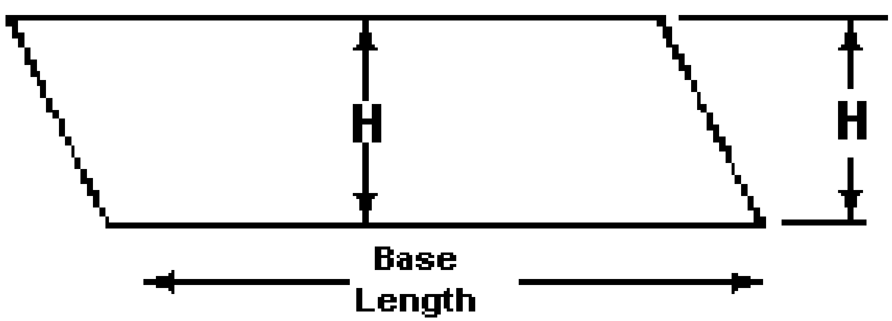


6' x 18' = 108 sq. ft.



6' x 24' = 144 sq. ft.

Just be careful when
measuring the heights of parallelograms. Since the opposite sides are always parallel, measure the <u>perpendicular distance from the side used as the length to the opposite</u>.



Both H's are perpendicular distances for this parallelogram.

PROBLEM

2. Calculate the following areas and check your calculations by working backwards. No diagrams are shown.

L = 21'	W = 9.1'	L = 7.6’
W = 27'	L = 10.0'	H = 7.2'
A = ______ sq. ft.	A = ______ sq. ft.	A = ______ sq. ft.

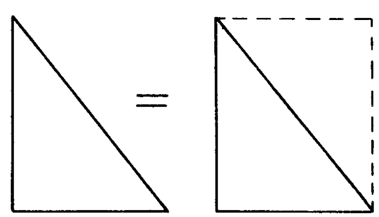
TRIANGLES

CHARACTERISTICS OF TRIANGLES

There are three types of triangles: right, acute and obtuse.

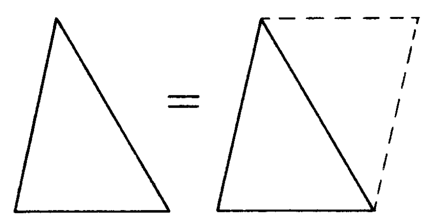
- ➢ A right triangle has one 90 degree angle.
- ➢ An acute triangle has three interior angles of less than 90 degrees each.
- ➢ An obtuse triangle has one interior angle larger than 90 degrees.
- ➢ The interior angles of any triangle must add to 180 degrees.
- ➢ Any triangle is half of a parallelogram!

<u>RIGHT</u>



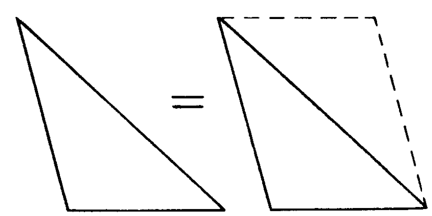
Right Triangle = Half of a Rectangle.

<u>ACUTE</u>



Acute Triangle = Half of a Parallelogram.

<u>OBTUSE</u>



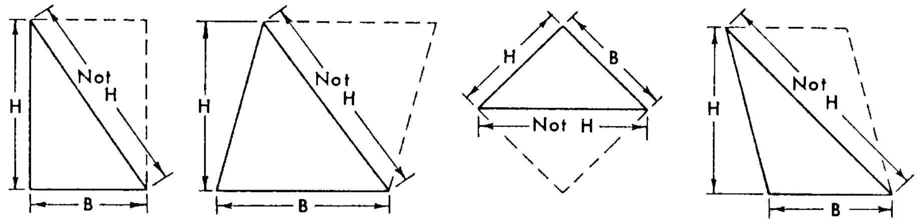
Obtuse Triangle = Half of a Parallelogram.

AREA CALCULATIONS

"Average-Base-Height" Method

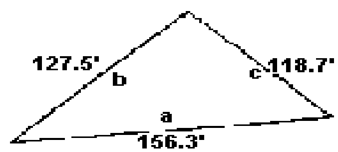
The area of any triangle is half the area of the parallelogram it would form.

$A = \frac{BH}{2}$ Area equals Base times Height, divided by 2



Be sure to use the right dimensions for the base and height values! As can be seen in the diagrams above, any line can be selected as a base line -- but visualize the parallelogram accordingly so that you use the proper height value.

"Lengths-of-Sides" Method



The area of any triangle can be calculated by using the formula shown below -- if the lengths of all sides are known!

$$Area = \sqrt{S\ (S - a)\ (S - b)\ (S - c)} \quad \text{....where....} \quad S = \frac{a + b + c}{2}$$

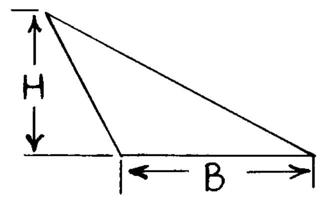
The symbol √ means to square root a number.
The symbol "S" represents the sum of the lengths of all sides, divided by 2.
The symbols "a, " "b, " and "c" represent the lengths of the individual sides -- as shown in the diagram.

PROBLEMS

Compute the areas of the following triangles: Calculate to 0.01(hundredths). Check your calculations.

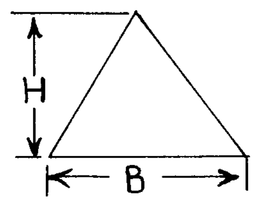
3.

B = 7.8'
H = 11.3'
A = ____________ sq. ft.



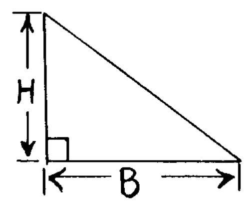
4.

B = 8.3"
H = 5"
A = ____________ sq. in

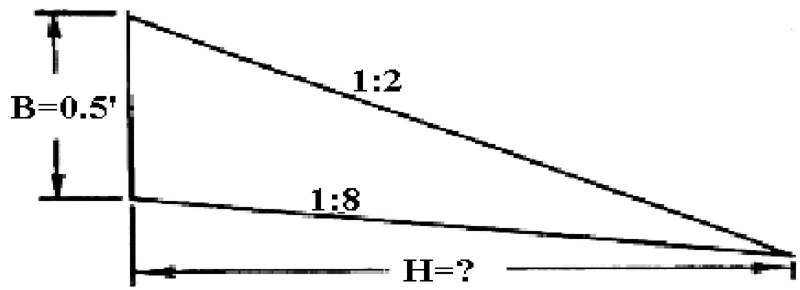


5.

B = 18.9'
H = 15.2'
A = ____________ S. F.



Occasionally, you may have to calculate the area of a triangle when you know only the length of the base and the slopes of two sides:



To calculate the area, first calculate the height -- then use the formula: $A = \frac{BH}{2}$

The height can be calculated with this formula: $H = \frac{B}{\text{Difference between the slopes}}$

To find the difference between the slopes, set up each slope as a percent, with B as the vertical dimension. Then subtract the smaller percent from the larger. For example, in the triangle above, the difference between the slopes is $(1 \div 2) - (1 \div 8) = 0.5 - 0.125 = 0.375$

The height of the triangle above is found like this:

$$H = \frac{B}{\text{Difference between the slopes}}$$

$$= \frac{B}{(1 \div 2) - (1 \div 8)} = \frac{0.5'}{0.375'} = 1.333 \text{ ft}$$

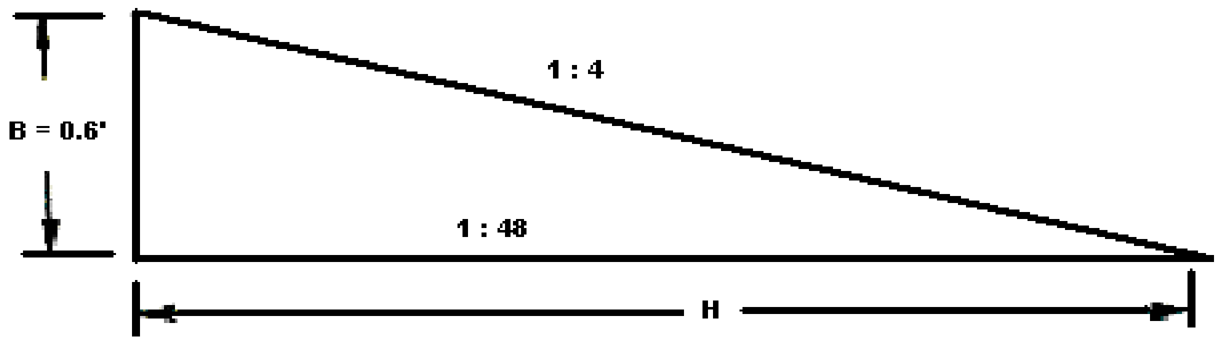
Once you know the height the area is calculated:

$$A = \frac{BH}{2} = \frac{0.5' \times 1.333'}{2} = \frac{0.667}{2} = 0.33 \text{ sq. ft.}$$

PROBLEM

6. Calculate the area of this shoulder section:

NOTE: Dimensions are vertical and horizontal.



H = _______________________ ft.
thousandths

A = _______________________ sq. ft.
hundredths

QUIZ

Inches times inches = ________________ inches.

Feet times feet = ________________ feet.

Feet times feet, divided by two = ________________ feet.

Square yards divided by yards = ________________

The areas of squares and rectangles are found by multiplying ________________ by ________________.

The areas of parallelograms are found by multiplying ________________ by ________________.

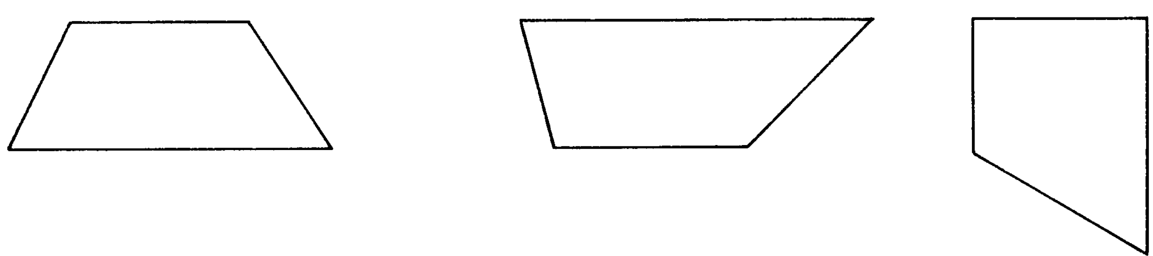
The areas of triangles where the lengths of all sides are not known are found by multiplying ________________ by ________________ and dividing by 2.

The accuracy of area calculations can be checked by dividing the ________________ by the ________________ to find ________________ or ________________.

Go on to "TRAPEZOIDS".

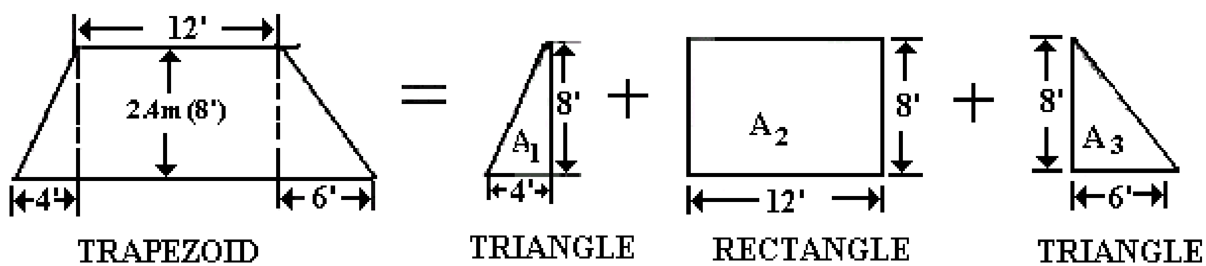
TRAPEZOIDS

Trapezoids are 4-sided figures -- havng two parallel sides and two non-parallel sides.



CALCULATING AREAS BY PARTS METHOD

Most trapezoids can be broken up into two triangles and a rectangle -- and solved part by part:



The area of each of the three parts can be calculated separately and added. A = A1 + A2 + A3 = Area of triangle + Area of rectangle + Area of triangle. The area of the above trapezoid can be calculated as follows:

$$A \; = A_1 + A_1 + A_3 = 4' \times 8' + [3.7\ 12' \times 8'] + \frac{6' \times 8'}{2}$$

= 16 *sq. ft* + 96 *sq. ft*. + 24 *sq. ft*. = 136 *sq. ft*.

CALCULATING AREAS BY AVERAGE-BASE-LENGTH METHOD

The formula for calculating the areas of trapezoids is:

$$A = \frac{B + b}{2} \times H$$

"B" is the long base line, "b" is the short one.

Multiply the height by the <u>average</u> of the two base-line lengths.

B = 30' b =20' H = 10'

<u>Calculation</u>

$$A = \frac{B + b}{2} \times H$$

$$A = \frac{30' + 20''}{2} \times 10 = 25' \times 10'$$

$$A = 250 \text{ sq. ft.}$$

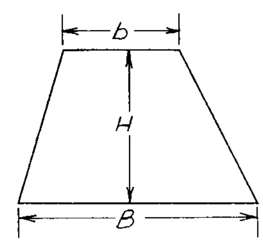
PROBLEM

7. Calculate the area of the following trapezoid using the Average-Base-Length Method:

b = 26.2”
B = 38.5”
H = 18.7”
A = ___________ sq. ft.
0.1

8. Calculate the area of the following trapezoid using trapezoid using both the Parts Method and the Average-Base-Length Method.

b = 17.3”
B = 40.1”
H = 21.5"
A = ___________ sq. in.
tenths

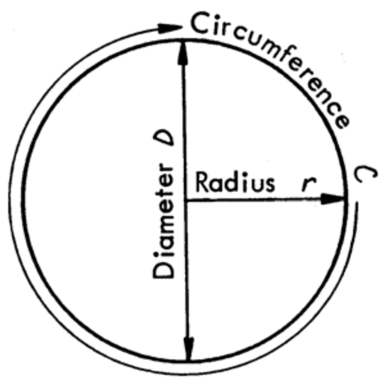


CIRCLES

CHARACTERISTICS OF CIRCLES

- The circumference of a circle is the length of the line that makes the circle.
- The diameter of a circle is the straight-line distance across the center of the circle.
- The value of pi -- π -- is the same for all circles. It is the result of dividing the circumference by the diameter.
- The radius of a circle equals half its diameter.
- The circumference of a circle is equal to the diameter times pi.
- The area of a circle is equal to pi times the squared length of the radius: $A = \pi r^2$
- The inside diameter of a pipe culvert is the longest measurement across the pipe opening.
- The outside diameter is the inside diameter plus twice the thickness of the culvert wall.
- The inside circumference is equal to the inside diameter times pi.
- The outside circumference is equal to the outside diameter times pi.

$$C = \pi D$$
$$D = 2r$$
$$\text{pi or } \pi = 3.14 = \frac{C}{D}$$
$$r = \frac{D}{2}$$
$$A = \pi r^2$$



The course material has been prepared using pi as 3.14 regardless of the degree of accuracy required. When the degree of accuracy for the answer is to 0.01, the value of pi should be 3.142.

All the problems in this section are based on using pi as 3.14.

CALCULATING AREAS OF CIRCLES

If the diameter of a circle is 10 inches, what are the circumference and the area?

$C = \pi D = 3.14 \times 10'' = 31.4''$

$A = \pi r^2 = (\frac{D}{2} \times \frac{D}{2}) \times 3.14 = (5'' \times 5'') \times 3.14 = \underline{78.50 \text{ sq.in.}}$

If the circumference of a circle is 62.83 inches, what are its diameter and area?

$C = \pi D$, so $D = \frac{C}{\pi} = \frac{62.83''}{3.14} = \underline{20 \text{ inches}}$

$A = \pi r^2$ and $r = \frac{D}{2}$, so $A = \frac{20''}{2} \times \frac{20''}{2} \times 3.14$

$A = (10 \text{ inches})^2 \times 3.14 = \underline{314.00 \text{ sq. ft.}}$

If the radius is 4.2 feet, what are the diameter, circumference and area?

$r = 4.2'$

$D = 2r = \underline{8.4'}$

$C = \pi D = 8.4' \times 3.14 = 26.376 \text{ ft.} = \underline{26.38 \text{ ft.}}$

$A = \pi r^2 = 3.14 \times 4.2 \text{ ft.} \times 4.2 \text{ ft.} = 3.14 \times 17.64 \text{ sq. ft.}$

$A = 55.3896 \text{ sq. ft.} = \underline{55.39 \text{ sq. ft.}}$

PROBLEMS

9. Round all the answers to 0.01(hundredths). Check your calculations by working backwards.

   C =157.00 feet

   D = ______ ft.

   r = ______ ft.

   A = ______ sq. ft.

10. r =16 inches

    D = ______ in.

    C = ______ in.

    A = ______ sq. in.

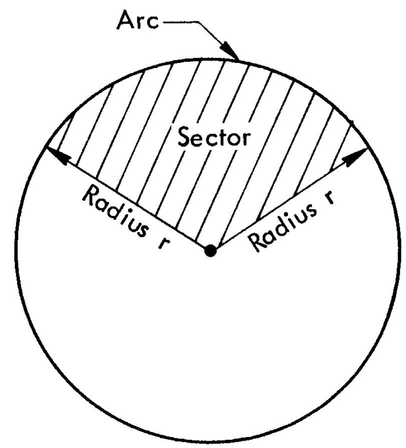
Right? Go on to "CIRCLE SECTORS AND CIRCLE SEGMENTS". Mistakes? Did you follow the right procedures and make mathematical errors? -- Or are the procedures confusing? If the procedures are difficult review the section on "CIRCLES" again.

CIRCLE SECTORS AND CIRCLE SEGMENTS

Some characteristics of circle sectors and circle segments are itemized below:
NOTE: The term "radii" is the plural of radius.

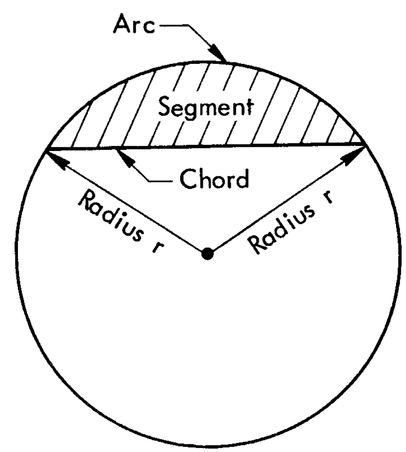
Circle Sectors

- A sector of a circle is the area between two radii and an arc.
- A arc of a circle is part of the circumference.



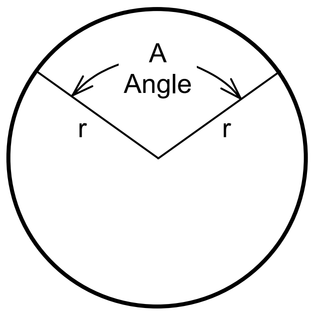
Circle Segments

- A segment of a circle is the area between an arc and its chord.
- A chord of a circle is the straight line between the ends of an arc.



Calculating Areas of Circle Sectors

To calculate the area of a circle sector, you must know the radius and the angle formed by the two radii. Then calculate the area using this formula:



$$A = \pi r^2 \times \frac{\text{angle}}{360°}$$

Angle = 100°
r = 5"

$$A = 3.14(5" \times 5") \times \frac{100°}{360°}$$

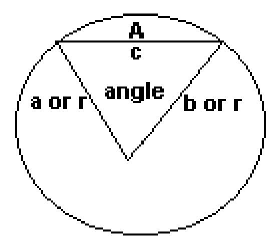
$$= 3.14(25 \text{ sq. in.}) \times 0.278$$

$$= 78.5 \text{ sq. in.} \times 0.278 = \underline{21.82 \text{ sq. in.}}$$

Calculating Areas of Circle Segments

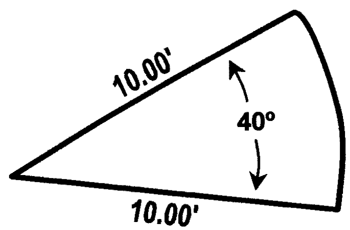
Calculating areas of circle segments is easy if you know how to calculate areas of circle sectors. The area of a segment is equal to the area of the sector minus the area of the triangle formed by the two radii and the chord. You can calculate the area of a segment using this formula:

$$A = \left(\pi r^2 \times \frac{\text{angle}}{360°}\right) - \sqrt{S(S-a)(S-b)(S-c)}$$



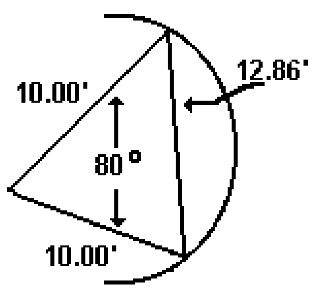
PROBLEMS

11. Find the area of this circle sector:



A = ______________.sq. ft.
0.1 (tenths)

12. Find the area of this circle segment:



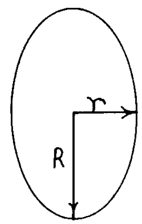
A = ______________.sq. ft.
0.1

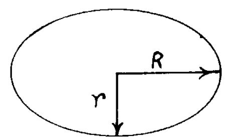
Right? Go on to "ELLIPSES". Mistakes? If you are having difficulty review the section on "CIRCLE SECTORS AND CIRCLE SEGMENTS" again.

ELLIPSES

CHARACTERISTICS OF ELLIPSES

Ellipses are oblong circles -- egg-shaped circles. The radii of ellipses vary depending on the measurements made:

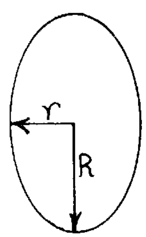




NOTE: The term "radii" is the plural of radius.

CALCULATING ELLIPTICAL AREAS

Elliptical areas are calculated by using the formula A = π(Rr). "R" represents the long radius and "r" the short one.



r = 10"
R = 15"

$$\begin{aligned} A &= \pi\,(Rr) \\ &= 3.14 \times 15" \times 10" \\ &= 3.14 \times 150 \text{ sq. in.} \\ &= \underline{471.0 \text{ sq. in.}} \end{aligned}$$

PROBLEMS

Round your answers to tenths. Check your answers by working backwards.

13. R = 31.5"

    r = 14.3"

    A = ____________ sq. in.

14. R =15.8'

    r = 9.3'

    A = ____________ sq. ft.

Right? Good! Go on to "FILLETS". Mistakes? Compare your calculations to those on the next page.

Calculations, Problems 13 and 14

13. R = 31.5"
r = 14.3"
A = π (Rr) = 3.14 (31.5" x 14.3") = 3.14 x 450.45 sq. in.
= 1414.4130 sq. in. = <u>1414.4 sq. in.</u>

14. R = 15.8'
r = 9.3'
A = π (Rr) = 3.14 (15.8' x 9.3') = 3.14 x 146.94 sq. ft.
= 461.3916 sq. ft. = <u>461.4 sq. ft.</u>

Checking Calculations

13. A = 1414.4130 sq. in.
r = 14.3 in.
R = ___?___

$$R = \frac{1414.4130 \text{ sq. inches}}{3.14} \div 14.3''$$

R = 450.45 sq.ft. ÷ 14.3" = <u>31.5"</u>

14. A = 461.3916 sq.ft.
R = 15.8 ft.
r = ___?___

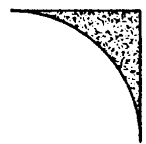
$$R = \frac{461.3916 \text{ sq. feet}}{3.14} \div 15.8'$$

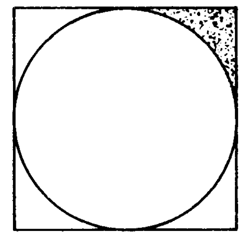
R = 146.94 *sq.ft.* ÷ 15.8' = <u>9.3'</u>

Go on to "FILLETS".

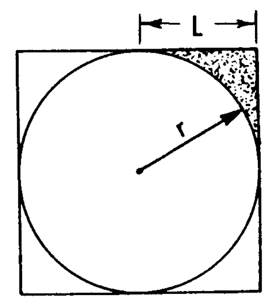
FILLETS

"Fill its" is how to pronounce -- fillets. Sometimes you will hear them called SPANDRELS. Fillet areas are leftovers when maximum circle areas are taken from square areas.





CHARACTERISTICS OF 90° FILLETS



- The length of the side of a 90° fillet is equal to the radius of the circle.
  r = radius of the circle or length of the 90° fillet side

- The length of the side of a fillet is also equal to half the length of the square that could be formed.
  L = length of the side of the fillet

- Length equals radius.
  $L = r$

- The area of a fillet = $\dfrac{(2L)^2 - (\pi r^2)}{4}$ or $(2r)^2 - \dfrac{(\pi r^2)}{4}$ or $\dfrac{(2L)^2 - (\pi L^2)}{4}$

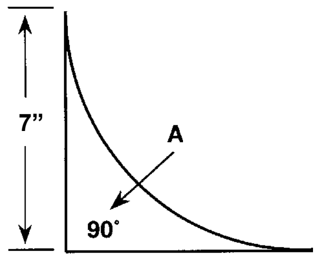
- The area of a fillet = $\dfrac{4r^2 - \pi r^2}{4} = \dfrac{(4 - 3.14)r^2}{4} = \dfrac{0.860r^2}{4} = 0.215r^2$

- The area of two fillets = $0.430r^2$ -- three fillets = $0.645r^2$ -- four fillets = $0.860r^2$.

PROBLEMS

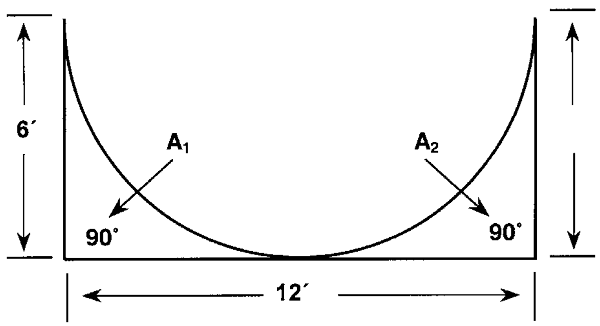
Round the following to 0.01.

15.



A = ________________ sq. in.

16.

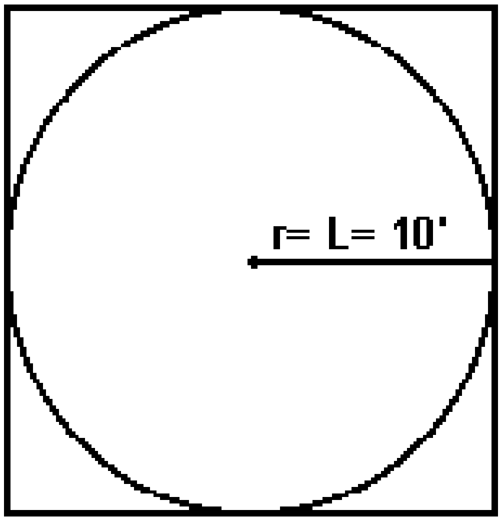


$A_1 + A_2 =$ ________________ sq. ft.

Right? Go on to "USING CONSTANTS IN FILLET AREA CALCULATIONS". Wrong? Go back and review.

USING CONSTANTS IN FILLET AREA CALCULATIONS

Do you remember the fillet constants? We used one at the bottom of the last page. They make fillet calculations easy. We can use this figure and calculate the area of one, then two, then three and finally all four fillets.



The area of TWO FILLETS is:

$$A = 0.430 \times r^2$$
$$= 0.430 \times 100 \text{ S.F.}$$
$$= \underline{43.0 \text{ S.F.}}$$

The area of THREE FILLETS is:

$$A = 0.645 \times r^2$$
$$= 0.645 \times 100 \text{ S.F.}$$
$$= \underline{64.5 \text{ S.F.}}$$

Finally, the area of FOUR FILLETS is:

$$A = 0.860 \times r^2$$
$$= 0.860 \times 100 \text{ S.F.}$$
$$= \underline{86.0 \text{ S.F.}}$$

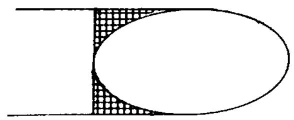
The area of ANY FILLET in this figure is

$$A = 0.215 \times r^2$$
$$= 0.215 \times 100 \text{ S.F.}$$
$$= \underline{21.5 \text{ S.F.}}$$

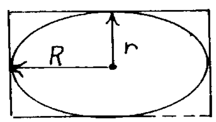
Go on to "IRREGULAR FIGURES".

IRREGULAR FILLETS

Suppose you run into an elliptical fillet?

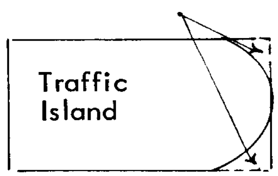


The shaded area at left has one long side and one short one. You already know the answer! Calculate the area of the rectangle that can be formed, subtract the area of the ellipse, and divide by four.



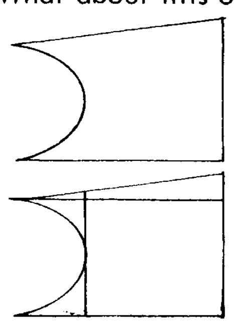
R = length of long side
r = length of short side

How about these?



They are done the same way, of course.
R = length of long side
r = length of short side
One -- Calculate the area of the total rectangle.
Two -- Deduct the areas of the two simulated fillets.

What about this one?

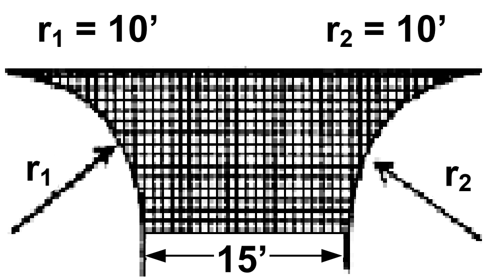


Divide the area as below, of course.
Calculate the four areas separately.
If you run into any fillet areas that involve other than 90° angles, work out ways of making them 90° angles. OR, Subtract the area of the circle or ellipse from the area of the rectangle and multiply that answer by the size of the angle in the fillet.

Go on to "Highway Problems".

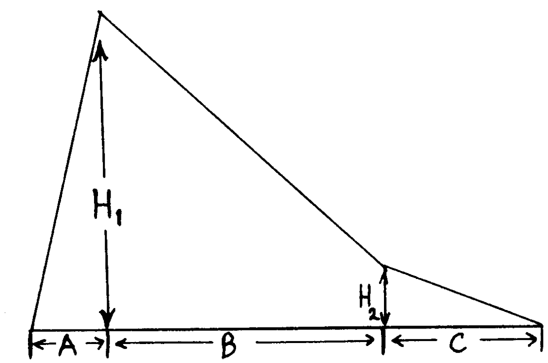
HIGHWAY PROBLEMS

17. Calculate the square yard area of the following driveway entrance: The fillets are 90°.



Area = ________________ sq. yds.
0.1

18. Calculate the area of this plot.

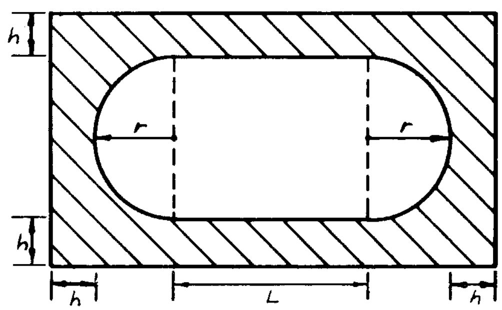


H_1 = 40'
H_2 = 8'
A = 10'
B = 35'
C = 20'

Area = ________________ sq. yds.
whole number

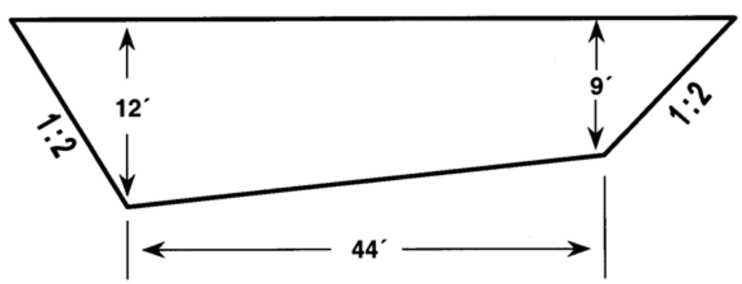
HIGHWAY PROBLEMS, continued

19. Calculate the area of the shaded portion in this drawing.



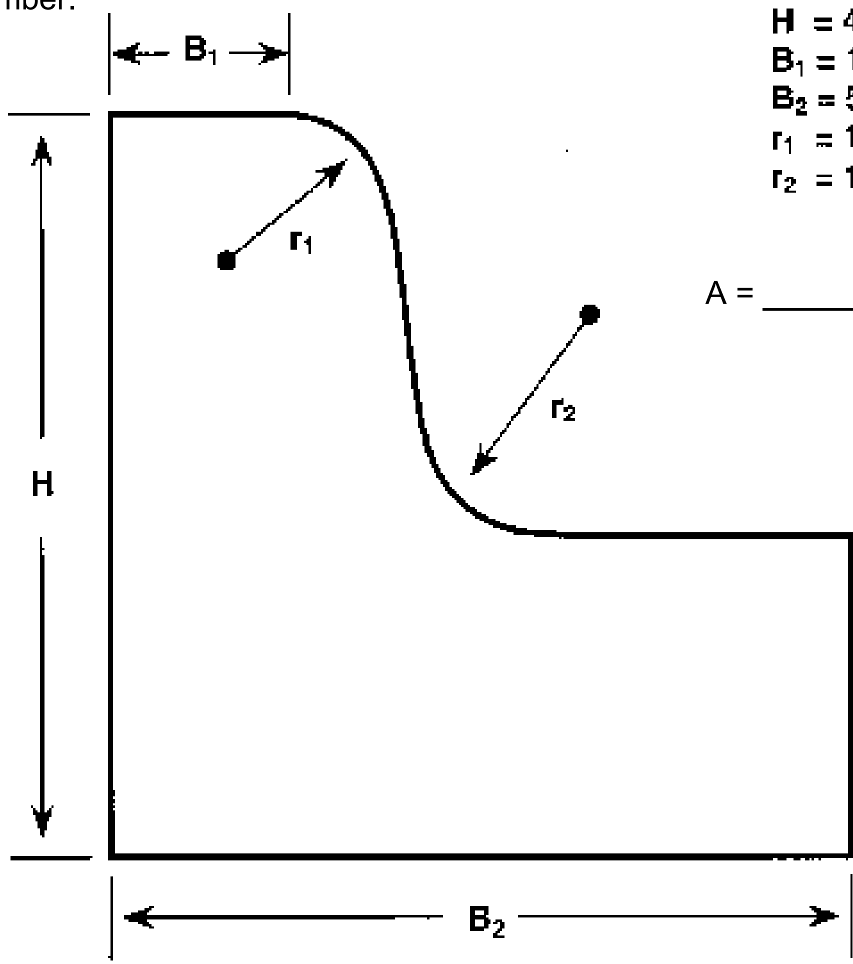
r = 5'
L = 10'
h = 2'
A = __________ sq.yds.
0.1

20. Find the cross-sectional area of this cut.



A = ____________ square feet

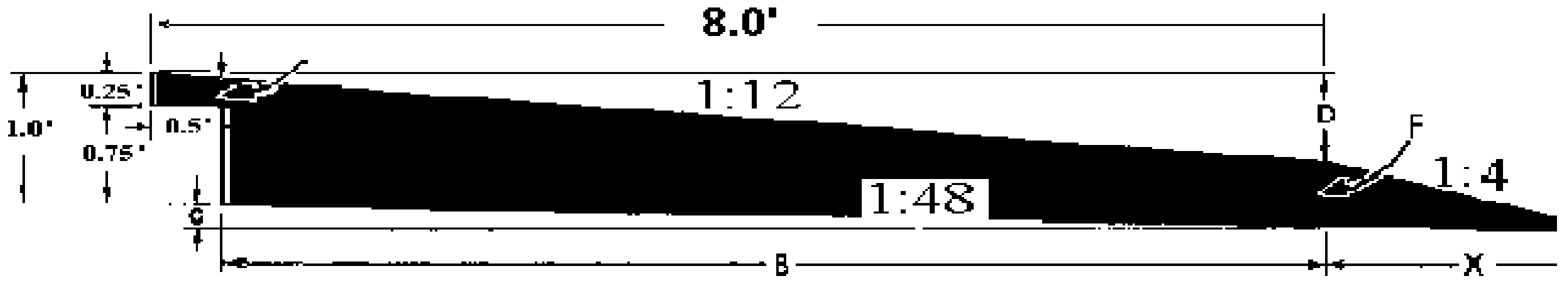
21. Calculate the area of the plot below. To calculate fillet areas, use the method shown on page 6-24. Round to whole number.



$H = 40'$
$B_1 = 10'$
$B_2 = 50'$
$r_1 = 10'$
$r_2 = 13' 4"$

A = _______________ sq yds.

Problems 22 and 23 refer to the shoulder cross-section below:



22. What are the dimensions of B, C, D, E, F and X? Round your answers to 0.001.

B = _________ ft. C = _________ ft.

D = _________ ft. E = _________ ft.

F = _________ ft. X = _________ ft.

23. What is the total cross-section area? Round your answer to 0.01.

A = _________ sq.ft.

ANSWERS TO PROBLEMS

Page 6-4

1. A. 526.88
   B. 56.25
   C. 275

Page 6-6

2. 567
   91.0
   54.72

Page 6-9

3. 44.07
4. 20.75
5. 143.64

Page 6-11

6. 2.618
   0.79

Page 6-12

- square
- square
- square
- yards
- length; width
- base; height
- base length; height
- area
  length
  width
  height

Page 6-14

7. 604.9
8. 617.1

Page 6-17

9. 50.00
   25.00
   1,962.50
10. 32.00
    100.48
    803.84

Page 6-20

11. 34.5
12. 20.5

Page 6-22

13. 1414.4
14. 461.4

Page 6-25

15. 10.54
16. 15.48

Page 6-28

17. 21.4
18. 124

Page 6-29

19. 17.5
20. 687

Page 6-31

21. 146

Page 6-32

22. B. 7.500
    C. 0.156
    D. 0.667
    E. 0.208
    F 0.489
    X. 2.134

CHAPTER SEVEN

Calculating Volumes

CONTENTS

7

CALCULATING VOLUMES

Most persons find volume calculations to be easier than area calculations -- after the needed areas are known. Volume calculations always involve multiplying the area, or an average area, by the length of the figure. The following formulas will be used for volumes:

1 $V = AH$ Volume = the Area times the Height (used for simple solids -- objects with parallel end areas of identical shape and equal area)

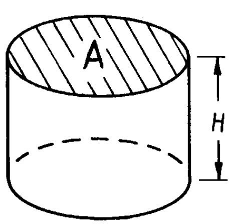
$V = (\frac{A_1 + A_2}{2})H$ Volume = Average Area times the Height (used mainly to approximate volumes) (Use for earthwork volumes)

$V = (\frac{A_1 + A_2 + \sqrt{A_1 A_2}}{3})H$ Volume = a sort of Average Area times the Height (used for pyramids, cones and frustrums)

$V = (\frac{A_1 + A_2 + 4\,A_m}{6})H$ Volume = a sort of Average Area times the Height (used with straight-edged shapes)(Use for concrete structure volumes)

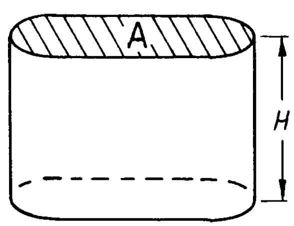
"Length" is substituted for "height" when the figure is laying down instead of standing up.

Figure One



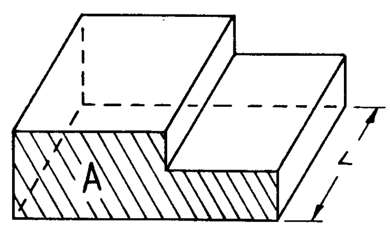
V = AH

Figure Two



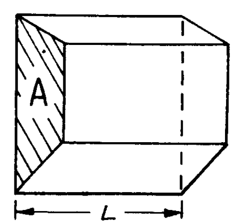
V = AH

Figure Three



V =AL

Figure Four



V = AL

BASIC SOLIDS

CONVERSIONS

As with areas that have to be converted from square inches to square feet, volumes often have to be converted from cubic inches to cubic feet. They sometimes have to be converted to gallons.

As a reminder:

-- One cubic foot = 1,728 cubic inches
-- One cubic foot = 7.48 gallons

PROBLEMS

Calculate the volume of the following shapes using the data:

1. Cube:

A = 144 sq. in.

V = _______ cu. in.

V = _______ cu. ft.

V = _______ gals.

PROBLEMS, continued

2. Cube:

   H = 3 ft.

   A = _______ sq. ft.

   V = _______ cu. ft.

   V = _______ cu. yds.

3. Rectangular solid:

   A = 400 sq. ft.

   H = 30 ft.

   V = _______ cu. ft.

4. Triangular solid:

   A = 72 sq. in.

   L =12 in.

   V = _______ cu. in.

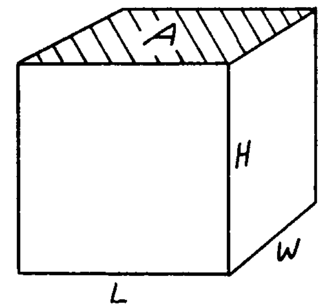
   V = _______ cu. ft.

Right on all four? Go on to Problems 5. Mistakes? Compare your calculations with those on the next two pages.

DISCUSSION OF BASIC VOLUME CALCULATIONS

You will recall from Chapter Five that the volumes of cubic and rectangular solids are calculated by using either of two formulas:

Figure Five



V = LWH or V = AH
V = Volume. L = Length. W = Width. H = Height. A = Area.

Problem 1 – Figure Five represents Problem 1.

Since A = 144 square inches and since V = AH, you need only to know the value of H! Since, in a cube, L, W and H are equal, H must be the square root of 144 square inches! The square root of 144 is 12, so H =12".

V = 144 square inches x 12 inches = 1728 cubic inches. That's one cubic foot and the equivalent of 7.5 gallons.

Problem 2

Essentially is the same as Problem 1. You know that you are working with a cube, so the length, height and width have to be equal -- and they have to be 3 feet each.

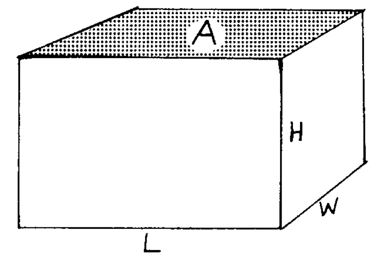
Since A= LW, A has to be 3' x 3' -- 9 sq.ft.

Since V = AH, V has to be 9 sq. ft. x 3' -- 27 C.F.

Since one cubic yard equals 27 cubic feet, this volume is one cubic yard.

Problem 3 -- Figure Six represents Problem 3.

Figure Six



Since V = AH, and A = 400 square feet, V = 400 sq. ft. x 30 ft. = 12000 cubic feet.

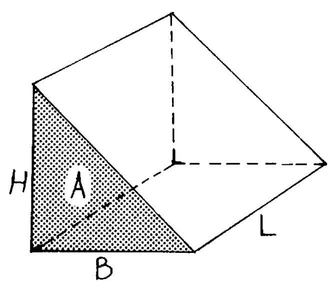
You don't have to know the L and W values of rectangular solids to calculate their volumes. You need only know (1) that it is a rectangular solid so that the opposite surface areas are equal, (2) the area of one surface and (3) the length of the dimension not used in calculating the area.

In Figure Six, for example, you can calculate the area of LH, LW or WH first. If the area is LH, the volume has to be LH x W. If the area is LW, the volume has to be LW x H, or if the area is WH, the volume has to be WH x L.

Problem 4

As you know, every triangle is equal to half the rectangle it can be used to form. Every triangular solid is equal to half the rectangular solid it can be used to form.

Figure Seven



Study Figure Seven. It depicts half of a cube. If you multiplied B times H times L, you would get the volume of a cube!

Assume B, H and L are 12 inches each. Then BHL would be 12" x 12" x 12" or one cubic foot. Since it is a triangular solid instead of a cube, the volume equals one half the value of BHL -- or one half the value of AL.

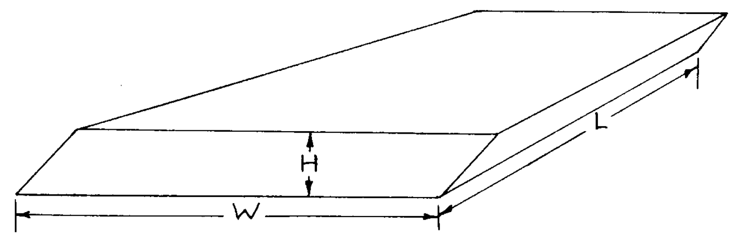
In Problem 4, you know (1) you are working with a triangular solid, and (2) the area of one end is 72 square inches.

Since V = AL, V = 72 sq. in. x 12 in. = 864 cu. in., and $\frac{\text{864 cu. inches}}{\text{1,728 cu. inches/cu. ft.}}$ = 0.5 cu. ft.

PARALLELOGRAM SOLIDS

5. The volumes of solids having equal parallelogram end areas are calculated the same way as other solids: V = AL or V = AH. But watch your end-area calculations.

L = 12'
W = 10'
H = 1.5'
V = ____________ cu.yds.
0.01



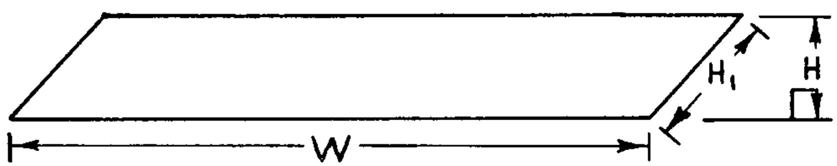
H must be vertical!

Calculation, Problem 5

V = AL

A = WH = 10' x l.5' = 15 sq. ft.

V =15 S.F. x 12' = 180 cu. ft. = <u>6.67 cu. yd.</u>



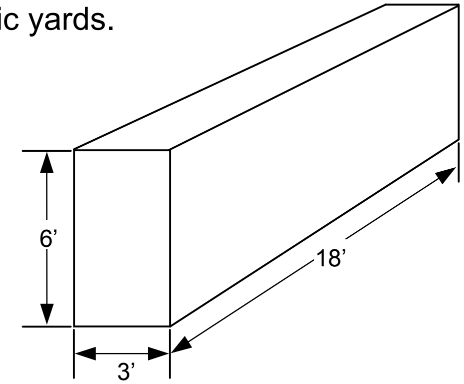
The product of the two adjacent sides, W and H, won't give the area. The area is the product of W and H, which are at right angles to each other.

Have trouble? Study areas of parallelograms again, Chapter Seven. Otherwise, go on to "Problems for Review".

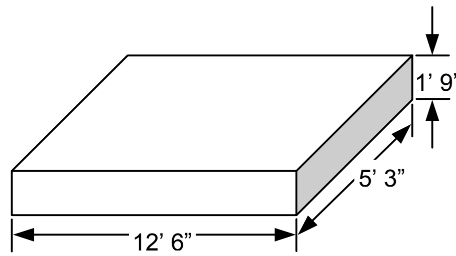
PROBLEMS

Calculate the volumes of these structures -- in cubic yards.

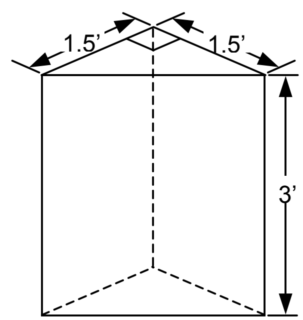
6. V = ________________ cu. yds.
whole number



7. V = ________________ cu. yds.
0.01



8. V = ________________ cu. yds.
0.01



Right? Go on to "TRAPEZOIDAL SOLIDS." Wrong? Go back and check.

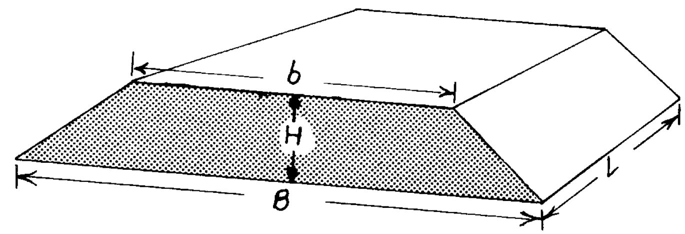
TRAPEZOIDAL SOLIDS

Trapezoidal solids are three-dimensional trapezoidal figures.

Formula:

$$A = \frac{B + b}{2} H$$

$$V = AL$$



Earlier, you calculated the area of a trapezoid. If both end areas are equal -- the volume calculation is simply

END AREA times LENGTH.

If: B = 50', b = 25', H = 10', L = 30'

END AREA $\quad A = \frac{B + b}{2} H$ $\quad$ and $\quad V = AL$

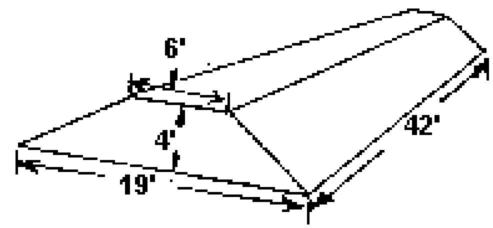
$$= \frac{50' + 25'}{2} 10' \qquad = 375 \text{ sq. ft.} \times 30' = 11{,}250 \text{ cu. ft.}$$

$$= 375 \text{ sq. ft.} \qquad = 416.67 \text{ cu. yds}$$

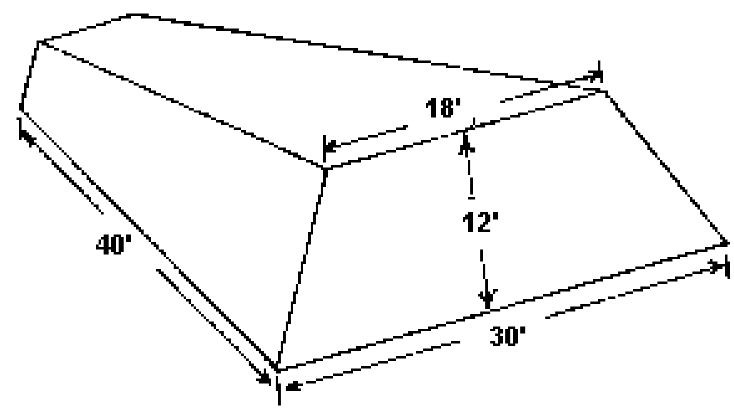
PROBLEMS

Compute the volumes of each of the following trapezoidal solids -- in cubic yards. Round to 0.01. Assume the opposing end areas are equal.

9. V = ________________ cu. yds.



10. V = ________________ cu. yds.



Discussion -- Problems 9 and 10

In calculating volumes of trapezoidal solids, multiply the END AREA times the LENGTH. That's all!

END AREA = $\frac{B + b}{2}$ H

VOLUME = END AREA x L

Problem 9

B = 19', b = 6', H = 4', and L = 42'

VOLUME = $\left(\frac{19' + 6'}{2} \times 4'\right) \times 42'$

= (12.5' x 4') x 42'

= (50.0 sq. ft.) x 42' = 2,100 cu. ft. = 77.78 cu. yds.

Problem 10

B = 30', b = 18', H = 12', and L = 40'

VOLUME = $\left(\frac{30' + 18'}{2} \times 12'\right) \times 40'$

= (24' x 12') x 40'

= (288 sq. ft.) x 40' = 11,520 cu. ft. = 426.67 cu. yds.

Go on to the next page.

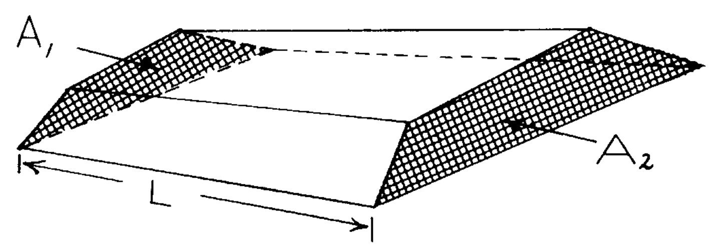
AVERAGE-END-AREA CALCULATIONS

The volumes of solids having different-sized end areas are calculated by multiplying the average of the two end areas by the length.

The volume of Figure Eight is calculated with the formula:

$$V = \frac{A_1 + A_2}{2} L$$

Figure Eight



<u>NOTE:</u> This formula is exact only in one case. Usually, it gives only approximate answers.

PROBLEM

11. Assume the data below apply to Figure Eight. What is the volume of Figure Eight?

A_1 = 50 S.F.

A_2 = 150 S.F.

L = 42 ft.

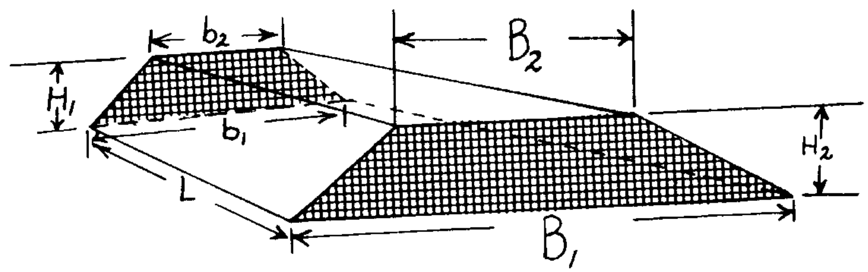
V = ________________ cu. ft.

Go to Problem 12

PROBLEM

12. Calculate the average end area and the volume:

b_1 = 25'
b_2 = 15'
H_1 = 8'
B_1 = 37.5'
B_2 = 23.9'
H_2 = 9.6'
L = 43.5'



Average end area = ________________ sq. ft.
0.01

Volume = ________________ cu. yds.
0.1

Right? Study "CYLINDRICAL SOLIDS". Mistakes? Compare your work to the calculations on the next page.

Calculations, Problems 11 and 12

The only new calculations to be made are the averages -- and you have done them before. The calculation information is shown below only so that you can check your own calculations quickly. You can always check the accuracy of your own work by working each calculation backwards.

Problem 11

$$V = \frac{A_1 + A_2}{2} L = \frac{50 \text{ S.F.} + 150 \text{ S.F.}}{2} \text{ x } 42 \text{ ft.} = 100 \text{ S.F. x } 42 \text{ ft.} = 4{,}200 \text{ cu. ft.}$$

Problem 12

$$A = \frac{B + b}{2} H$$

$$A_1 = \frac{25' + 15'}{2} \text{ x } 8' = 20' \text{ x } 8' = 160 \text{ S.F.}$$

$$A_2 = \frac{37.5' + 23.9'}{2} \text{ x } 9.6' = 30.7' \text{ x } 9.6' = 294.72 \text{ S.F.}$$

$$\text{Average End Area} = \frac{A_1 + A_2}{2} = \frac{160 \text{ S.F.} + 294.72 \text{ S.F.}}{2} = \frac{454.72 \text{ S.F.}}{2} = 227.36 \text{ S.F.}$$

$$V = 227.36 \text{ S.F. x } 43.5 \text{ ft.} = 9{,}890.16 \text{ C.F.} = \underline{366.3 \text{ cubic yards}}$$

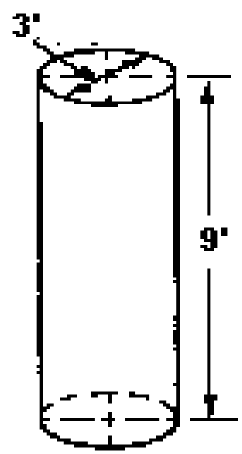
Go on to "CYLINDRICAL SOLIDS".

CYLINDRICAL SOLIDS

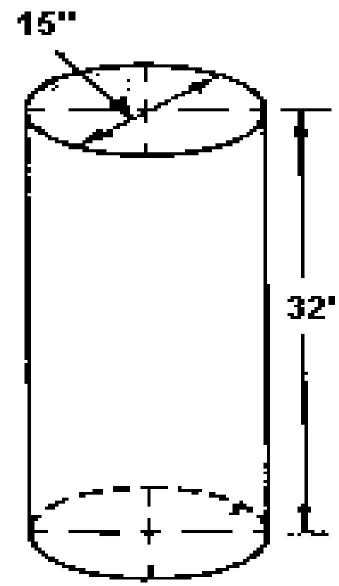
PROBLEMS

Determine the volumes of these cylinders in cubic yards. Round to 0.1.

13. V = ________________ C.Y.



14. V = ________________ C.Y.



Calculations, Problems 13 and 14

The volumes of circular columns having equal end areas are based on the formula: V = AH. You have to be able to calculate the areas of course.

Problem 13

The available data: D = 3', H = 9'

The formula: V = AH, so V = A x 9 ft. What is the value of A?

$A = \pi r^2$, and $r = \dfrac{D}{2} = \dfrac{3'}{2} = 1.5$ ft.

A = 3.142 (1.5' x 1.5') = 3.14 x 2.25 sq. ft. = 7.07 sq. ft.

V = 7.07 sq. ft. x 9 ft. = 63.63 cu. ft. = <u>2.4 cu. yd.</u>

Problem 14

The available data: D = 15" or 1.25 ft., H = 32 ft.

The formula: V = AH, but what is the value of A?

$A = \pi r^2$, and $r = \dfrac{D}{2} = \dfrac{1.25'}{2} = 0.625$ ft.

A = 3.142 (0.625' x 0.625') = 3.142 x 0.391 sq. ft. = 1.23 sq. ft.

V = 1.23 sq. ft. x 32 ft. = 39.36 cu. ft. = <u>1.5 cu. yd.</u>

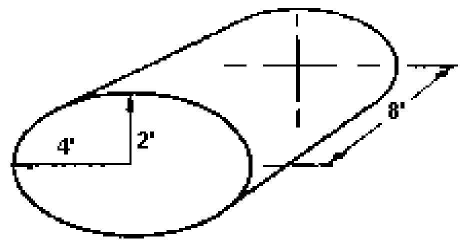
ELLIPTICAL SOLIDS

PROBLEMS

15.

V = ________________ cu. ft.
0.1

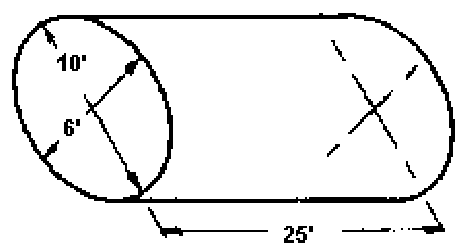
V = ________________ gals.
whole number



16.

V = ________________ cu. ft.
0.1

V = ________________ gals.
whole number



Calculations. Problems 15 and 16

As with cylinders having circular end areas, the volumes of cylinders with equal elliptical end areas are calculated by multiplying the area of one end by the height or length of the cylinder.

Problem 15

As with cylinders having circular end areas, the volumes of cylinders with equal elliptical end areas are calculated by multiplying the area of one end by the height or length of the cylinder.

The available data: R = 4', r = 2', L= 8'

The formula: $V = AL$ and $A = \pi(Rr)$

So, $V = \pi(Rr) \times L = \pi\ (4' \times 2') \times 8'$
$V = 3.14 \times (8 \text{ sq. ft.}) \times 8' = 200.96$ cu. ft.

Volume = 201.0 cu. ft. x 7.5 gals. per cu. ft.
= 1,507.5 gals. = <u>1,508 gals.</u>

Problem 16

The available data: D = 10', d = 6', L = 25'

The formula : $V = AL$, $A = \pi(Rr)$, $R = \frac{D}{2}$ and $r = \frac{d}{2}$

So, $V = \pi(Rr) \times L = \pi\ (5' \times 3') \times 25'$
$V = 3.14 \times (15 \text{ sq. ft.}) \times 25' = 1{,}177.5$ cu. ft.

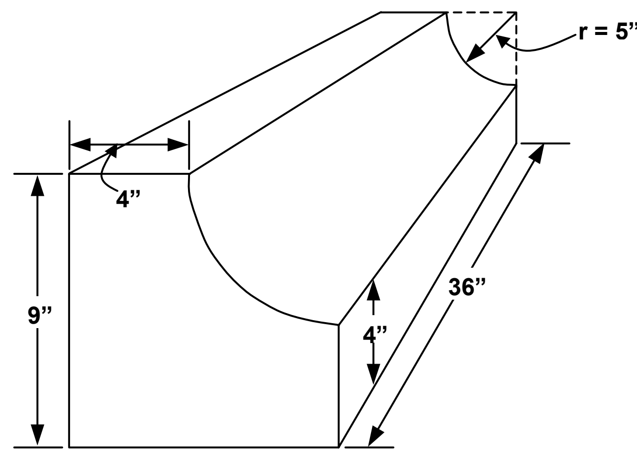
Volume = 1,177.5 cu. ft. x 7.5 gals. per cu. ft.
= 8,831.25 gals. = <u>8,831 gals.</u>

FILLET AREA SOLIDS

PROBLEM

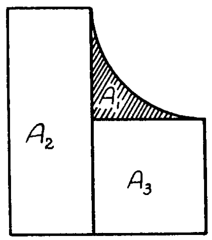
17.

V = ________________ cu. ft.
0.01



Calculations, Problem 17

As with cylinders, the volumes of fillet areas are calculated by multiplying end areas times heights. In Problem 17 the fillet area is not identified separately. The volume of the object includes the volume of the fillet area plus the volumes of the rectangular solids.



The shaded area is the fillet area. It is identified here as A_1. Areas A_2 and A_3 are rectangles. The formula is $V = (A_1 + A_2 + A_3)\ L$

Remember these characteristics of fillets:

The length of the side equals the length of the radius of the circle that would be formed. The length of the side equals half the length of the square that would be formed. The area equals one-fourth the "area of the square minus the area of the circle".

The available data: r = 5", L = 36", H = 9", Width of A_2 = 4" and Height of A_3 = 4"

As you can see, we have more data than we need. We could get the height of the solid or the widths of A_2 and A_3 if either the height or the two widths were unknown.

But, $V = (A_1 + A_2 + A_3)\ L$

And, $A_1 = \dfrac{(2r)^2 - \pi r^2}{4} = \dfrac{[(2)(5)]^2 - \pi(5)^2}{4} = \dfrac{100 \text{ sq. in.} - 78.50 \text{ sq. in.}}{4} = 5.375 \text{ sq. in.}$

A_2 = 4" x 9" = 36 sq. in. A_3 = 4" x 5" = 20 sq. in.

So, V = (5.375 sq. in. + 36 sq. in. + 20 sq. in.) x 36"
= 61.375 sq. in. x 36" = 2,209.500 cu. in. = 1.28 cu. ft.

Go on to "QUIZ ON SIMPLE SOLIDS".

QUIZ ON SIMPLE SOLIDS

All solids having equal end areas can be thought of as simple solids -- because their volumes are equal to one area times height or length.

The QUIZ summarizes certain area and volume characteristics of simple solids. See if you can score 85% or better.

1. V of cube = 729 cu. ft. A = 81 sq. ft. H = _______________

2. The end area of a parallelogram-shaped solid = 200 sq. ft. and Volume = 20,000 cu. ft.
Length = _______________

3. One end area and the height of a uniform cylinder are known. What is the formula for volume? _______________

4. The end area and length of a trapezoid-shaped solid are known. What is the formula for volume?
____

5. The end area and length of an irregularly-shaped solid are known. The opposite end areas are equal. What is the formula for volume? _______________

6. The end area of a fillet-shaped solid and the length are known. The two end areas are equal. What is the formula for volume? _______________

QUIZ, continued

7. What are two formulas for the volume of a cube? _______________ and _______________

8. A cubic foot = _______________cu. in.

9. A cubic yard = _______________ cu. ft.

10. A cubic foot = _______________ gals.

11. A square foot = _______________ sq. in.

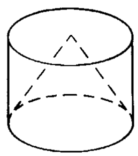
12. A square yard = _______________ sq. ft.

There are 13 possible correct answers. Did you get at least 11 right? Good. Keep going in this chapter.

Did you get three or four wrong? Study them again. Did you know the right answers but make mistakes in putting them down? If so, keep going. If not, you probably should study some of the previous sections that are giving you difficulty.

CONES

All cones are 33% the size of the cylinders they would make.

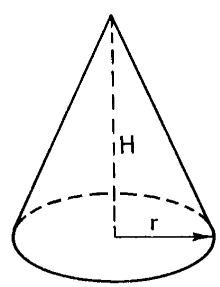


V = A (H / 3)   Volume = Area of the base of the cone times 0.33 of the height.

PROBLEM

13. Using the formula V = A (H /3), calculate the volume of this cone:

H = 12 ft.
r = 5 ft.



V = ________________ cu. ft.
0.01

Right? Simple, aren't they? Study the section titled CONES calculations for Problem 18 on the next page. Mistake? Study the calculations for Problem 18 on the next page.

Calculations, Problem 13

A $= \pi r^2 = 3.14\ (5' \times 5') = 3.14 \times 25$ sq. ft. = 78.50 sq. ft.

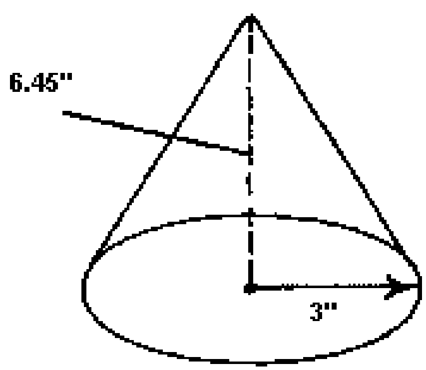
V = A (H / 3)

V = 78.50 sq. ft. x (12' / 3) = 78.50 sq. ft. x 4' = 314.00 cu. ft.

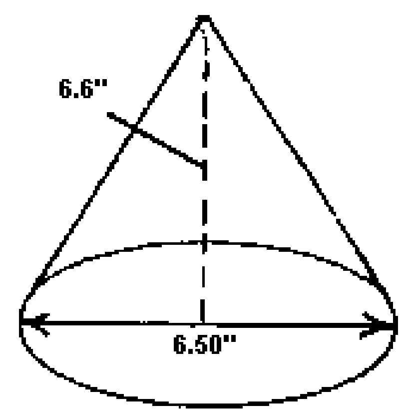
Try the next two problems.

PROBLEMS

14.
V = ________________ cu. in.
tenths

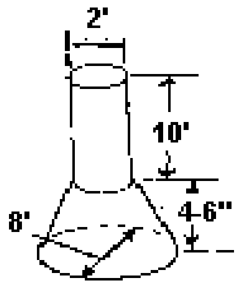


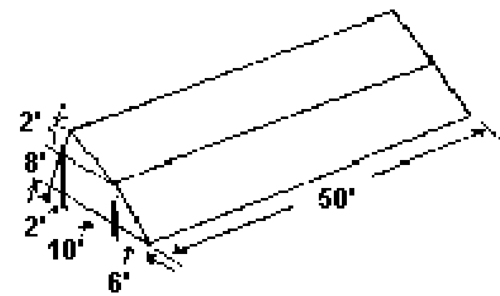
15.
V = ________________ cu. in.
tenths

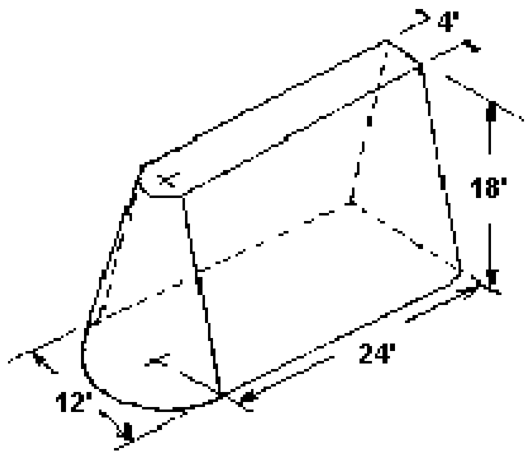


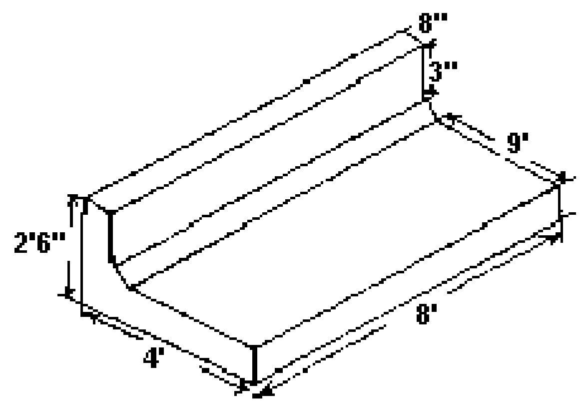
COMPOUND VOLUMES -- STRUCTURES

Some compound volumes are sketched below:





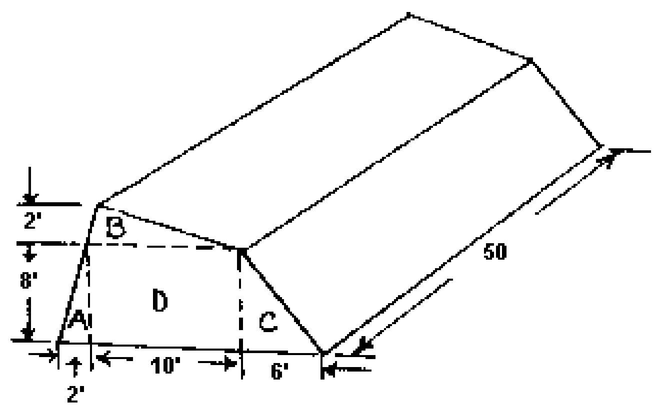




Compound volume calculations can be divided into different series of separate calculations. Total volumes are the result of adding all the part volumes.

PROBLEM

16. The compound figure below can be divided into four solids: three triangular solids -- A, B and C -- and one rectangular solid -- D. Calculate the volume.



V = ________________ cu. yds.
0.1

Calculations, Problem 16

Volume of A = Area of A x Length = $\frac{2' \times 8'}{2}$ x 50' = 400 cu. ft.

Volume of B = Area of B x Length = $\frac{2' \times 10'}{2}$ x 50' = 500 cu. ft.

Volume of C = Area of C x Length = $\frac{6' \times 8'}{2}$ x 50' = 1,200 cu. ft.

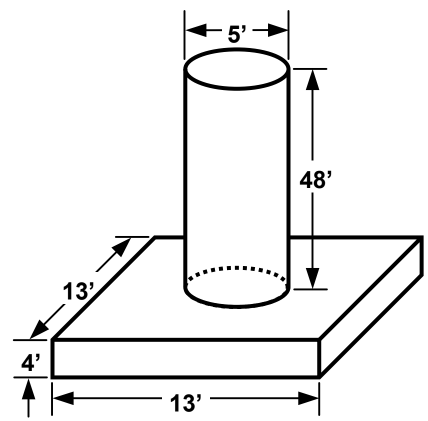
Volume of D = Length x Width x Height = 10' x 8' x 50' = 4,000 cu. ft.

Total Volume = Volumes of : A + B + C + D
= 400 cu. ft. + 500 cu. ft. + 1,200 cu. ft. + 4,000 cu. ft. = 6,100 cu. ft.
= <u>225.9 cu.yds.</u>

PROBLEM

17. Calculate the volume of this bridge bent.

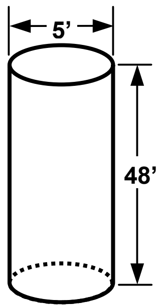
V = ________________ cu. yds.
0.01



Right? Try Problem 18. Wrong? Compare your calculations to those on the next page.

Calculations. Problem 17

The bent is composed of a cylinder and a rectangular solid.

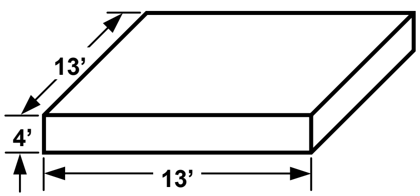


$$A = \pi r^2 = 3.14 \times (2.5 \text{ ft.})^2$$

$$= 3.14 \times 6.25 \text{ sq. ft.} = 19.625 \text{ sq. ft.}$$

$$V = AH = 19.625 \text{ sq. ft.} \times 48 \text{ ft.}$$

$$= 942.000 \text{ cu. ft.}$$

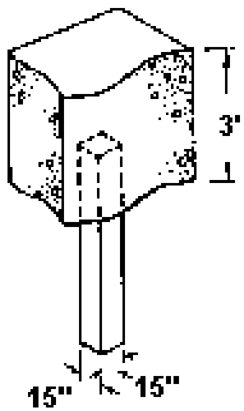


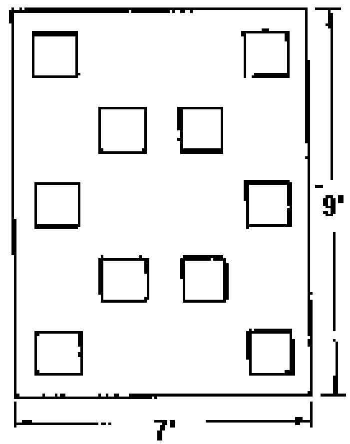
$$V = LWH = 13 \text{ ft.} \times 13 \text{ ft.} \times 4 \text{ ft.} = 676 \text{ cu. ft.}$$

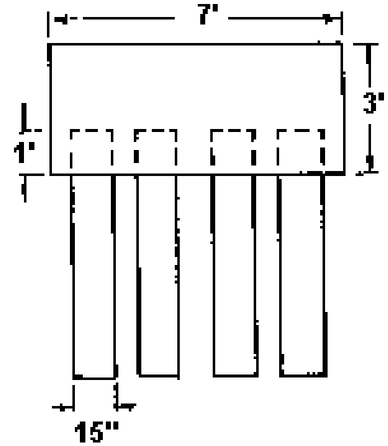
Volume of bent = Volume of cylinder + Volume of rectangular solid

Volume of bent = 942.000 C.F. + 676 cu. ft. = 1,618 cu. ft.
= <u>59.93 cu. yd.</u>

18. A concrete footing must be poured over 10 piles. How many cubic yards of concrete will be needed?

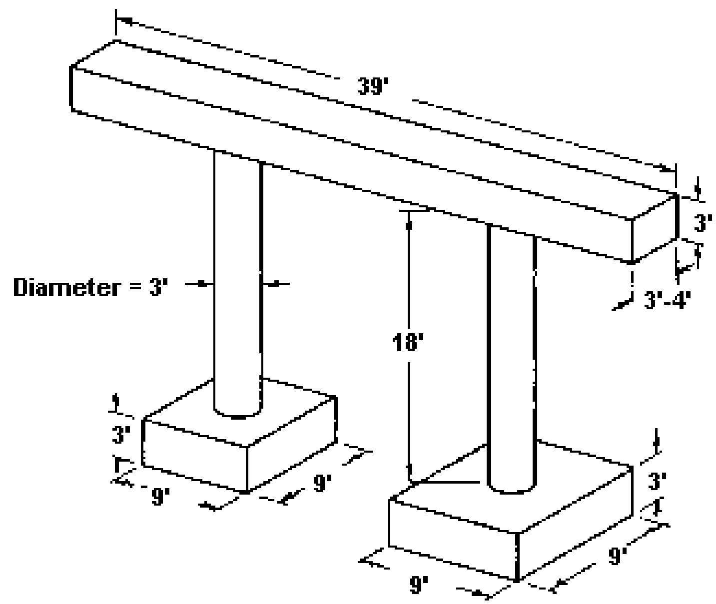






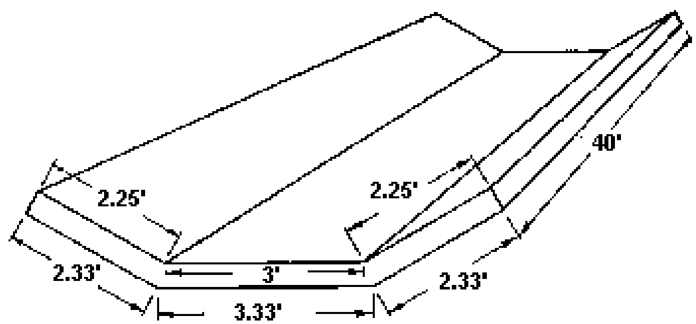
V = ________________ cu. yds.
0.01

19. Compute the volume of this bridge pier.



V = ________________ cu. yds.
0.01

20. How many square yards of concrete will be needed to pave 40 ft of this ditch?



NOTE: In diagram the walls are 0.25' thick.

Ditch surface area = _______________ sq. yds.
whole number

ANSWERS TO PROBLEMS

Page 7-3
1. 1728
   1
   7.5

Page 7-4
2. 9
   27
   1
3. 12, 000
4. 864
   0.5

Page 7-7
5. 6.67

Page 7-9
6. 12
7. 4.25
8. 0.13

Page 7-11
9. 77.78
10. 426.67

Page 7-13
11. 4,200

Page 7-14
12. 227.36
    366.3

Page 7-16
13. 2.4
14. 1.5

Page 7-18
15. 201.0
    1508
16. 1177.5
    8831

Page 7-20
17. 1.28

Page 7-22
1. 9 ft.
2. 100 ft.
3. V= AH
4. V= AL
5. V= AL
6. V= AL

Page 7-23
7. L x L x L and L^3
8. 1,728
9. 27
10. 7.5
11. 144
12. 9

Page 7-24
13. 314

Page 7-25
14. 60.8
15. 73.0

Page 7-27
16. 225.9

Page 7-28
17. 59.93

Page 7-30
18. 6.42

Page 7-31
19. 41.86

Page 7-32
20. 33

CHAPTER EIGHT

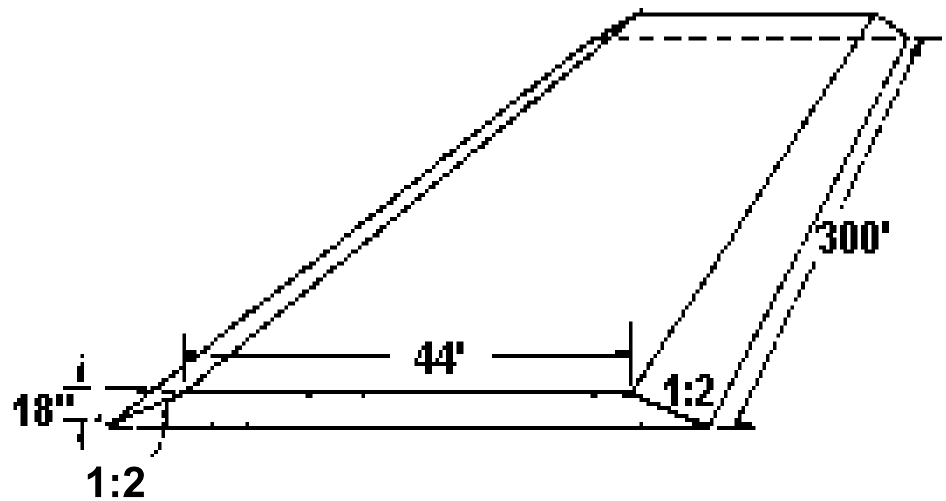
Highway Problems

CONTENTS

8
HIGHWAY PROBLEMS

This chapter includes problems and calculations only. Work as many as you like but do those most difficult for you. Check your answers by working backwards before comparing your calculations to those at the end of the chapter.

1. Calculate the cubic yards of granular materials placed below:

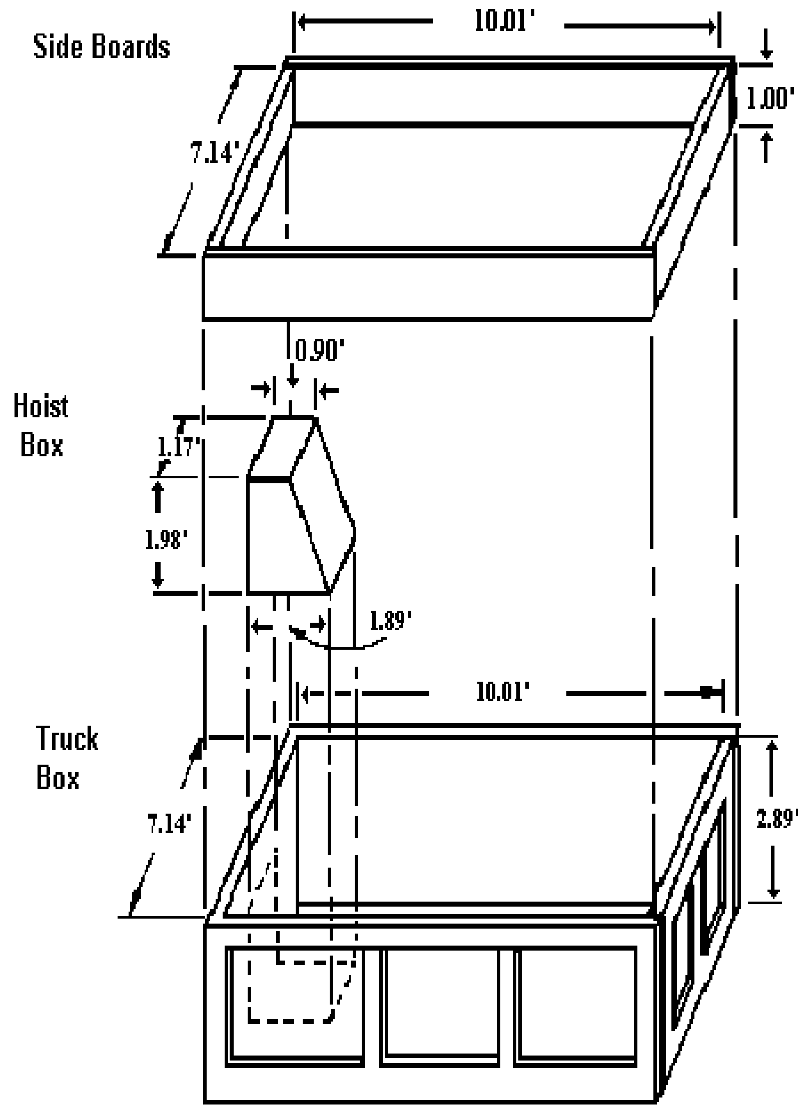


V = _______________ cubic yards
whole number

HIGHWAY PROBLEMS, continued

2. What is the volume of this truck body when the hoist box and side boards are in place?

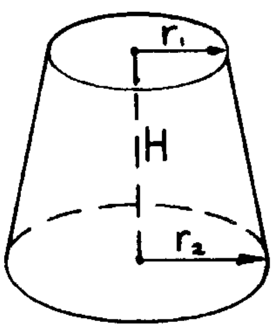
V = ________________ cu. ft.
0.1



3. If a sand sample contains 4.6% moisture -- based on the dry weight, how much additional wet sand should be added to an existing 1,300-pound load -- to yield 1,300 pounds of dry sand?

4. Solve for H.

$$V = 0.33(A_1 + A_2 + \sqrt{A_1 \times A_2})\ H$$



$V = 458.4$ cu. ft.

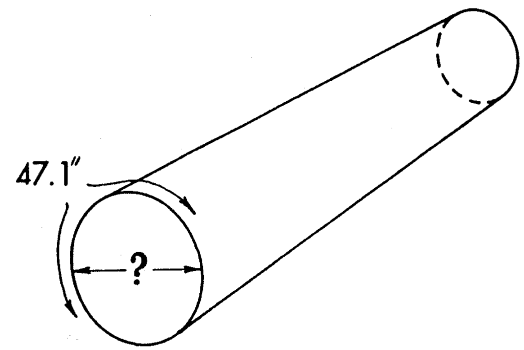
$r_1 = 3.5'$

$r_2 = 5'$

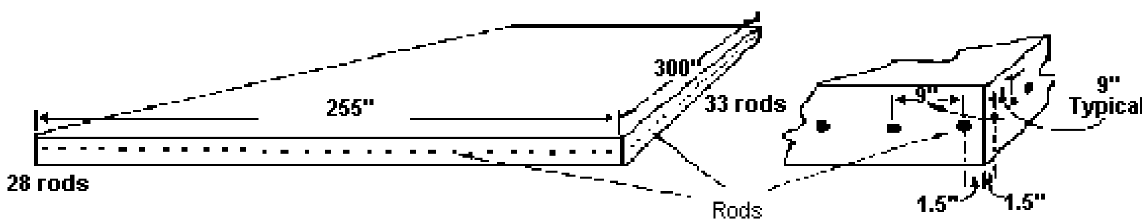
H = ________________ ft.
0.1

5. The circumference of a pipe is 47.1". What is the diameter?

D = _______________ inches
whole number



6. How many feet of reinforcement steel rods are required for the slab below?



All rods are spaced 9" center-to-center. #5 rod = 1.043 lbs./ft. How many total pounds of #5 rods are needed?

All rods are spaced 9" center-to-center. Rod = 1.043 lbs./ft. How many total pounds of rods are needed?

Total feet = _______________ ft.
0.1

Total weight = _______________ lbs.
whole number

7. If D = 12.35'

C = ________________ ft.
0.1

C = Circumference
D = Diameter

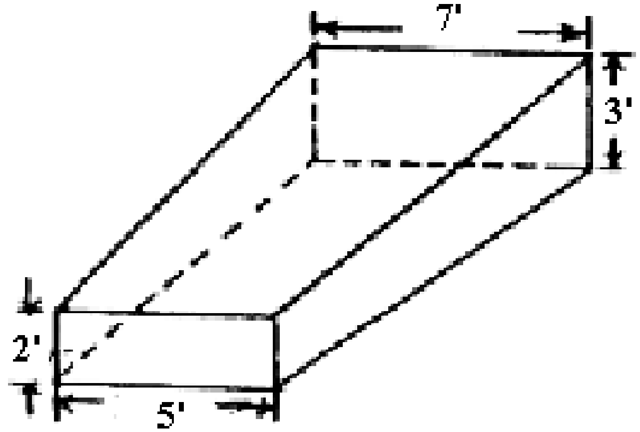
8. If C = 192.35'

D = ________________ ft.
0.1

C = Circumference
D = Diameter

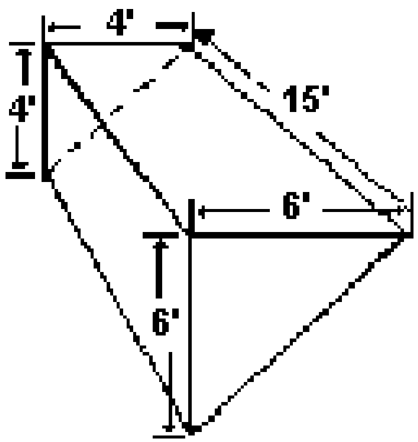
HIGHWAY PROBLEMS, continued

9. What is the average end area?



Average end area = ________________ sq. ft.
0.1

10. Calculate the average end area in square feet. Then compute the volume of cubic meters cubic yards.



Average end area = ________________ sq. ft.
0.1

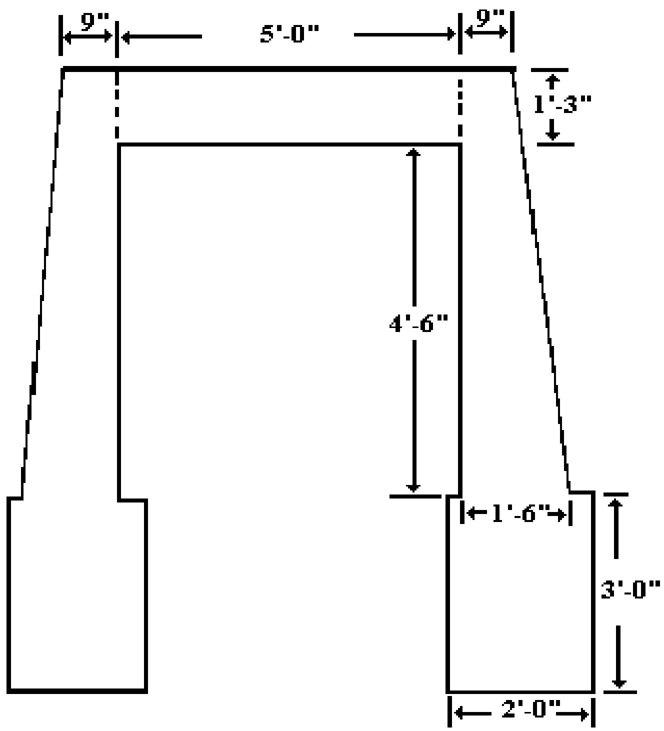
V = ________________ cubic yards
0.1

HIGHWAY PROBLEMS, continued

11. If you excavate from Station 200 + 54 to 201 + 36, forty feet wide and one foot below subgrade, how many cubic yards will be removed below subgrade?

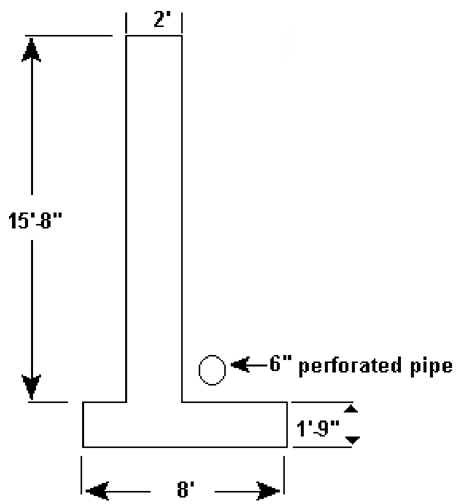
V = ________________ cubic yards
whole number

12. How many cubic yards of concrete are needed per foot?



V = ________________ cubic yard concrete/linear ft.
0.01

13. How many yards of concrete do you need to pour this cantilever retaining wall? It will be 0.071 mi. long. A 6" perforated pipe covered by select materials will be placed along the entire length of the wall. How long will the pipe be in feet?

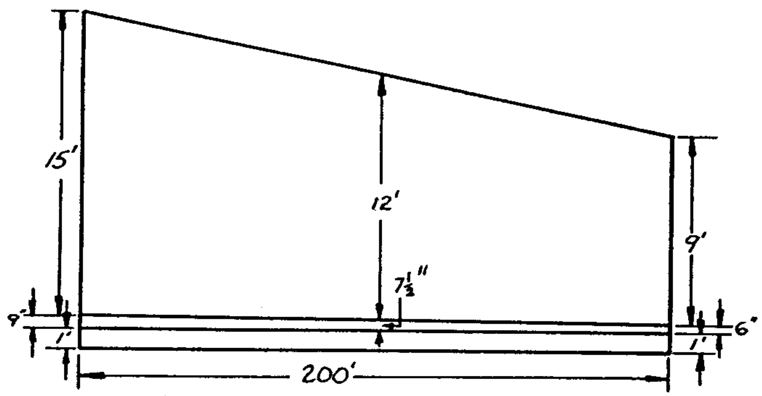


End area = ________________ sq. ft.
0.001

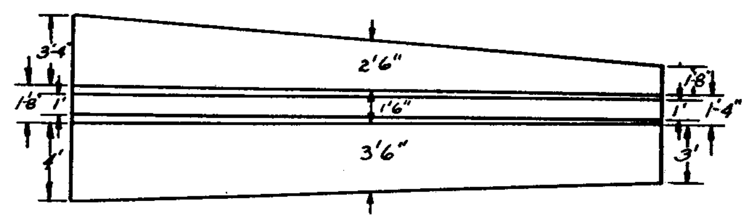
Pipe length = ________________ feet
0.01

V = ________________ cubic yards
0.01

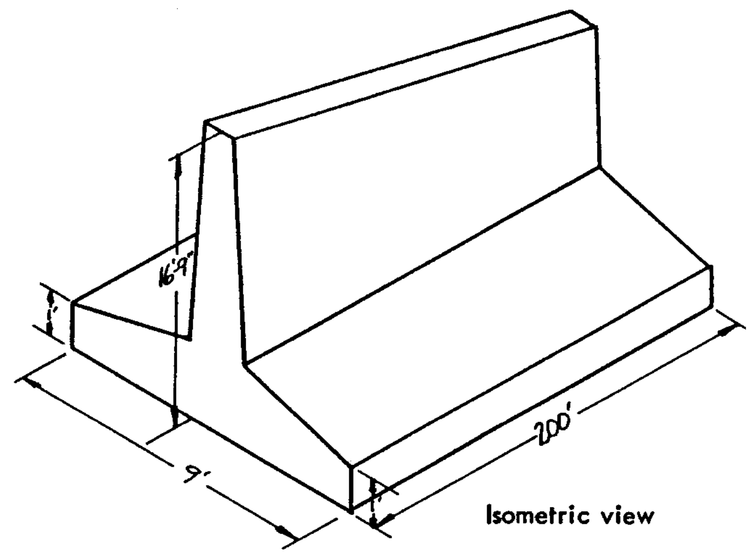
13. Calculate the volume of portland cement concrete used to construct the retaining wall shown in the drawings below:



Profile view



Plan view



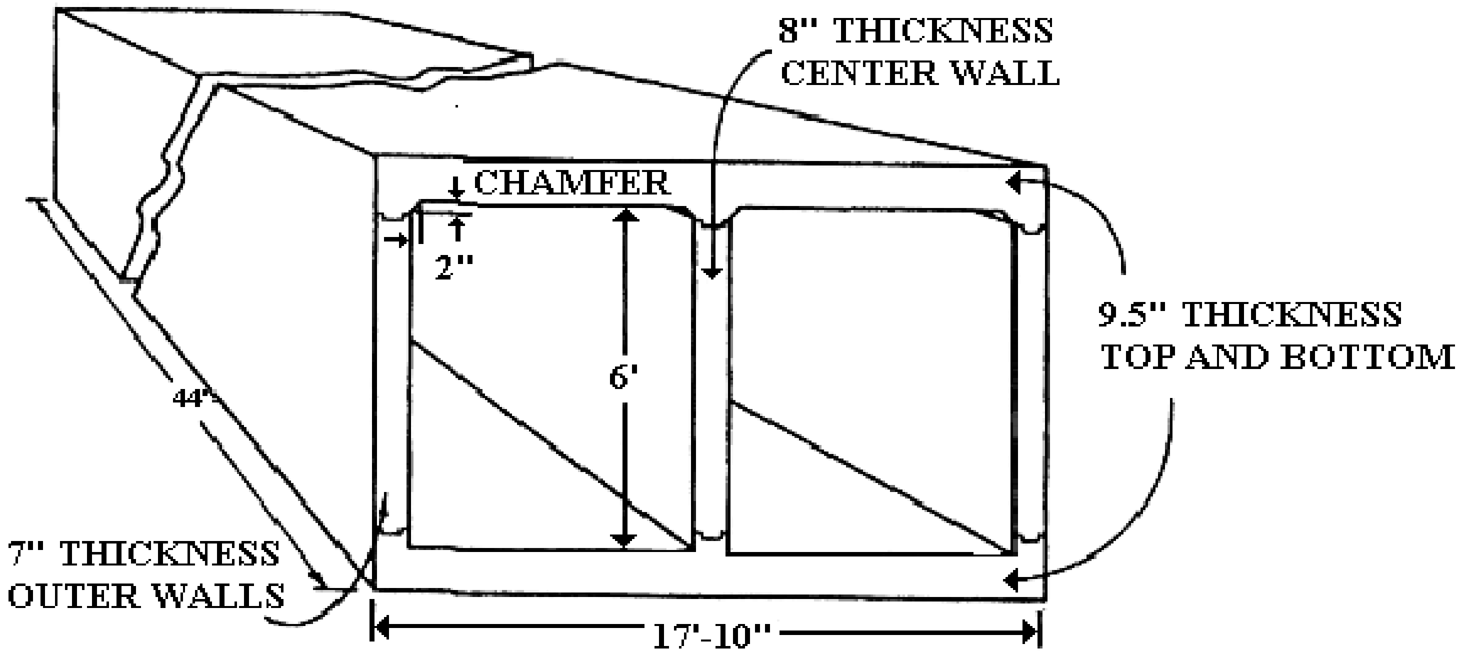
Isometric view

Answer = ________________ cubic yards

0.01

Workspace For Problem 14

15. Compute the cubic yards of concrete needed for the barrels in this box culvert:

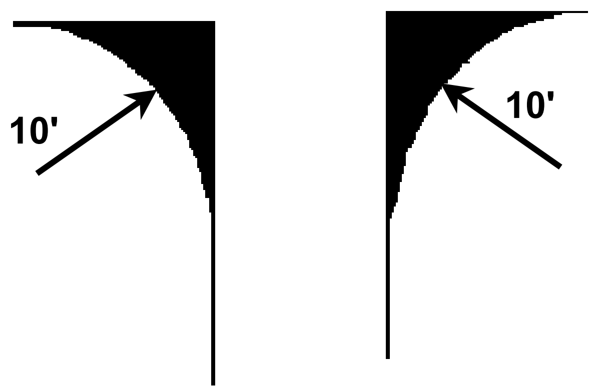


V = ________________ cubic yards
0.01

HIGHWAY PROBLEMS, continued

Workspace For Problem 15

16. A city intersection is being improved. The shaded area is pavement that must be removed. How many square yards of pavement must be removed?



A = _______________ square yards
whole number

17. What is the safe bearing value f the timber piling driven according to these data?

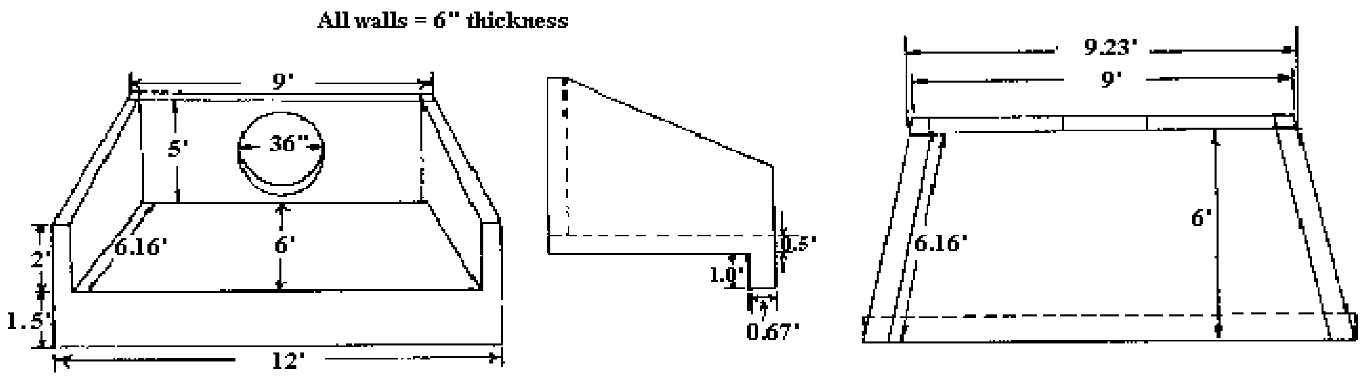
E	=	Energy blow of hammer	=	17.6 foot-tons
S	=	Average penetration per blow recorded for the last 10 blows	=	0.29 inches
R	=	Safe bearing value in tons	=	????
P	=	Weight of pile	=	18 tons

Customary Units Equation:

$$R = \frac{2E}{S + 0.1 + 0.001P}$$

$$R = \frac{\rule{4cm}{0.4pt}}{0.1} \text{ tons}$$

18. Calculate the number of cubic yards of concrete needed for the structure shown below:



V = _______________ cubic yards
0.1

Workspace For Problem 18

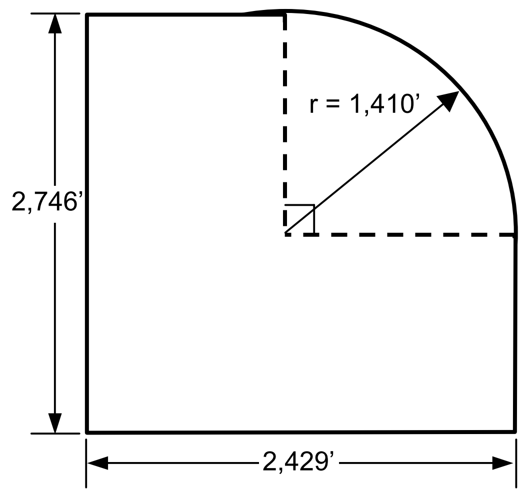
19. How many tons of dry fertilizer is needed to cover a strip 80 feet wide and 0.75 miles long if the rate of application is 875 pounds per acre?

Fertilizer = ________________ Tons
0.1

20. A tank truck weighs 5 tons empty. Loaded, it weighs 14.3 tons. How much water is it carrying? One m. gallon of water weighs 8.3 pounds.

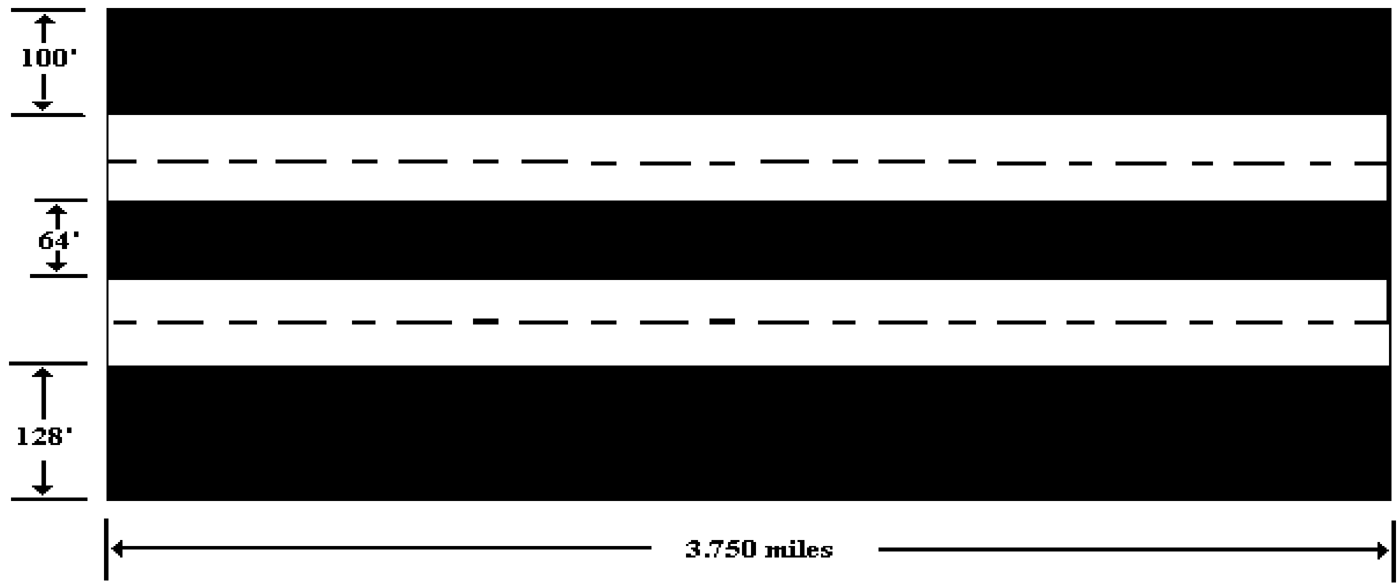
Water = ________________ gallons
0.1

21. Find the number of acres in the plot of land below.



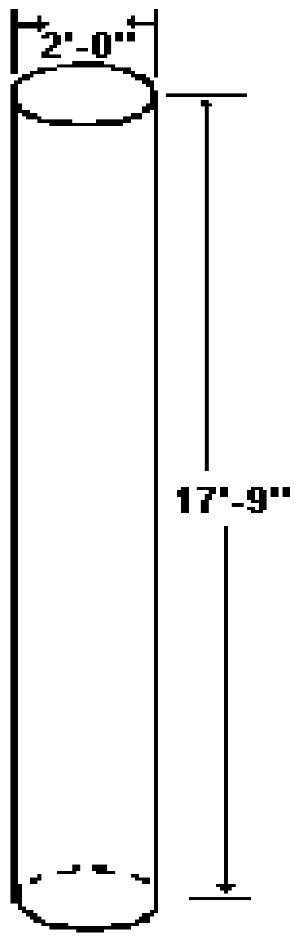
A = ________________ acres
0.001

22. How many pounds of grass seed are needed to cover the shaded areas in the sketch below? Seed is normally applied at 66 lbs. of seed per acre.



Grass seed = _______________ lbs.
whole number

23. Calculate the number of cubic yards of concrete needed to construct the concrete bridge column shown below.



V = ________________ cubic yards
0.01

24. Calculate the actual quantities of sand, gravel, water and cement required for cubic yard of concrete. One bag of cement yields 0.16 cubic yards of concrete. 500 g of wet sand weigh 480 g when dried. 1,000g of wet gravel weigh 990 g when dried. The quantities for a one-bag batch are shown below.

Cement = 94 lbs.

Sand = 160 lbs. dry

Gravel = 334 lbs. dry

Water = 5.0 gals.

Cement = ________________ lbs.
whole number

Sand = ________________ lbs.
whole number

Gravel = ________________ lbs.
whole number

Water = ________________ gals.
whole number

ANSWER TO QUESTIONS

Page 8-2
1. 783

Page 8-3
2. 274.8

Page 8-4
3. 59.8

Page 8-5
5. 15
6. 1,447.5

Page 8-6
7. 38.8
8. 61.26

Page 8-7
9. 15.5
10. 13.0
    7.2

Page 8-8
11. 121
12. 1.16

Page 8-9
13. 45.333
    374.9
    629.42

Page 8-10
14. 188.38

Page 8-12
15. 64.05

Page 8-14
16. 5

Page 8-15
17. 61.8

Page 8-16
18. 90.3

Page 8-18
19. 3.2
20. 2.2

Page 8-19
20. 143.310

Page 8-20
22. 8,758

Page 8-21
23. 2.06

Page 8-22
24. 588
    1,050
    2,125
    31.5

CHAPTER NINE

Quiz

There are 43 questions in the following quiz. You probably won't get them all right the first time -- unless you learned much more than you were expected to.

Try for 85 percent. That would be 37 right answers. If you make it, you will have no difficulty with the qualification examination. There are no "reversed questions" in that examination. Everything is straightforward, based on practical inspection problems.

Some of the following questions are reversed -- to make the quiz more interesting and to give you a better understanding of the relationships of measurements to areas and areas to volumes.

Use a scratch pad to help you "think through" some of the questions. Some of your answers will be expressed differently than those following. Count your answers right if the meanings are the same.

9

QUIZ

1. If an answer is needed to 0.1 (tenths) of a square yard, preliminary calculations are rounded to _______________.

2. Round this number to 0.1 (tenths): 2.3509 ____________________

3. Round this number to 0.01 (hundredths): 21.6666 ____________________

4. If you were calculating for a final answer in 0.1 (tenths), how would you use this number: 21.6666
   Select one below.

   A. 21.66
   B. 21.667
   C. 21.666
   D. 21.67

5. If the area were the length times the height, what formula would you use for determining the height from the area and length?____________________

QUIZ, continued

6. If $A = BH/2$ what is H when you know B and A? ____________________

7. $\frac{\text{cubic yards}}{\text{yards}} =$ ____________________

8. square feet x feet = ____________________

9. square feet ÷ feet = ____________________

10. Feet x inches per foot = ____________________

11. $\frac{B + b}{2}$ H is the formula for ____________________

12. $\frac{BH}{2} + LH + \frac{BH}{2}$ would give you what? ____________________

13. $2r =$ ____________________

14. $\pi D =$ ____________________

QUIZ, continued

15. πr^2 = ____________________

16. A / r^2 = ____________________

17. π = ____________________

18. $\pi r^2 H$ is the formula for ____________________

19. The area of a circle is calculated by what formula? ____________________

20. The area of a 90° fillet is calculated by what formula? ____________________

21. The volume of a 90° fillet is the area times the ____________________.

22. $(\pi Rr)H$ is the formula for ____________________.

23. $\frac{A_1 + A_2}{2} L$ is the formula for ____________________.

QUIZ, continued

24. What is the square root of 1.44? ____________________

25. What is the square root of 14.4? ____________________

26. One cubic foot = ________ cu. in.

27. One cubic yard = ________ cu. ft.

28. A cylinder containing 750.0 gallons has a capacity of ________ cubic feet.

29. L^3 is the formula for ____________________.

30. $L^3/2$ is the formula for ____________________.

31. The height of a parallelogram is the _______________ distance between one base line and the other.

32. The area of a parallelogram is calculated by what formula? ____________________

QUIZ, continued

33. The volume of a solid parallelogram is the area times the ____________________.

34. $(\frac{B + b}{2}H)L$, would give you what? ____________________

35. To calculate the areas of the trapezoids, first divide the figures into _______________ and _______________.

36. A right angle has how many degrees? ____________________

37. A circle has how many degrees? ____________________

38. What percent of 68.4 is 27.36? ____________________

39. What is the value of R when this equation is solved? (NOTE: Round to tenths)

    $R = \frac{2(12.1)}{0.32 + 0.1}$ R = ____________________

QUIZ, continued

40. If you have two cross-section end areas, how do you find the volume? ____________________

41. If the end area of a cone is 100 sq. ft. and the height is 12 ft., what is its volume? ____________________

42. If the volume of half a cone is 1,200 cu. ft., and the height is 12 ft., what is the area of its base? ____________________

43. (LWH) - ($\pi r^2 W$) is the formula for what?____________________

ANSWERS TO QUIZ

Page 9-2

1. 0.01(Hundredths)
2. 2.4
3. 21.67
4 21.67
5 H = A / L

Page 9-3

6. H = 2A/B
7. Square Yards
8. cubic feet
9. Feet
10. Inches
11. Trapezoid areas
12. Area of a trapezoid
13. Diameter of a circle
14. Circumference of a circle

Page 9-4

15. Area of a circle
16. 3.14 (π)
17. 3.14
18. Volume of a cylinder
19. $A = \pi r^2$
20. $A = \frac{(2r)^2 - (\pi r^2)}{4}$
21. Height or length
22. Volume of an elliptical cylinder
23. average end areas

Page 9-5

24. 1.2
25. 3.79
26. 1,728
27. 27
28. 100
29. Volume of a cube
30. Volume of half a cube
31. Vertical or perpendicular
32. A = BH

Page 9-6

33. Length, height or depth
34. Volumes of a trapezoidal solid
35. Rectangles and triangles
36. 90
37. 360
38. 40%
39. 57.6

Page 9-7

40. Multiply the average of the end areas by the length
41. 400 cubic feet
42. 600 square feet
43. Volume of a rectangular solid with a cylindrical opening

Did you get 37 or more right? Very good! Some of those questions are hard to interpret. Check the questions on which you made mistakes to see if you should review any of the sections in the course, or if you just (1) misunderstood the questions or (2) were not fully careful in writing your answers.

<u>HOW ABOUT THAT?</u>

You have completed

CONSTRUCTION MATH!

We hope it proved to be beneficial to you.

Keep this book as your personal property. Use it for review and reference purposes.

CONVERSION FACTORS

TYPE	CUSTOMARY UNIT	MULTIPLY BY	METRIC EQUIVALENT
LENGTH	inches	25.400 000	mm
	inches	0.025 400	m
	feet	0.304 800	m
	yards	0.914 400	m
	miles	1609.344 000	m
	miles	1.609 344	km
AREA	square inches	645.160 000	mm^2
	square feet	0.092 903	m^2
	square yard	0.836 127	m^2
	acres	4046.873 000	m^2
	square miles	2.589 988	km^2
	hectare	10 000	m^2
VOLUME	cubic feet	0.028 317	m^3
	cubic yard	0.764 555	m^3
	gallon (fluid)	3.785 412	L

TYPE	CUSTOMARY UNIT	MULTIPLY BY	METRIC EQUIVALENT
Mass	pound	0.453 592	kg
	ton	907.184 700	kg
	ton	0.907 184 700	metric ton or t
	ounce	0.028 350	kg
Force	pound (force)	4.448 222	N
	ton-force	8.896 443	N
Stress	pound/inch^2 (psi)	6894.757 000	Pa
	kips / in^2	6.894 757	N/mm^2
Velocity	fps	0.304 800	m/s
	mph	0.447 040	m/s
	mph	1.609 344	km/h

METRIC (INCH-POUND) TERMS, SYMBOLS (ABBREVIATIONS) AND EXAMPLES

Term	Symbol (Abbreviation)	Example
centimeter (inch)	cm (in.)	3 cm or 0.02 cm (1.18 in. 78.74 10^{-6} in.)
millimeter (Inches)	mm (in. or ″)	50.8 mm (2 in. or 2")
meter (feet)	m (ft. or _)	0.6096 m (2 ft. or 2')
meter (yards)	m (yds.)	4.572 m (5 yds.)
kilometer (Miles)	km (mi.)	16.09 km (10 mi.)
kilogram (Pounds)	kg (lbs.)	78.70 kg (173.5 lbs.)
Metric Ton (Tons)	t (Tns.)	19.4 t (21.4 Tns.)
Hours (Hours)	h (hrs.)	7.4 h (7.4 hrs.)
Liter (Gallons)	L (gals.)	4725.33 L (1248.3 gals.)
Square millimeters (Square Inches)	mm^2 (sq. in.)	1290.32 mm^2 (2 sq. In.)
Square meters (Square yards)	m^2 (sq. Yds. or S.Y.)	4.18 m^2 (5 sq. Yds. or S.Y.)
Square kilometers (Square miles)	km^2 (sq. mi.)	15.54 km^2 (6 sq. mi.)
Hectare (Acres)	ha (Ac.)	2.14 ha (3 Ac.)
Average (Average)	Avg. (Avg.)	--------
Cubic millimeters (Cubic inches)	mm^3 (cu. in.)	16 387.06 mm^3 (1 cu. in.)
Cubic meters (Cubic feet)	m^3 (cu. Ft. or C.F.)	0.226 m^3 (8 cu. ft. or 8 C.F.)
Cubic meters (Cubic yards)	m^3 (cu. yds. or C.Y.)	165.14 m^3 (9 cu. Yds. or C.Y.)

Add	+
Subtract	-
Multiply	x · ( ) ( ) A B
Divide	÷ / —— /‾‾‾
Ratio	: / ——
Proportion	: :
Equal	=
Percent	%
Average	Avg.
Circumference	C
Diameter	D
Pi	π
Radius	r or R
Length	L
Height	H
Width	W
Base, large	B
Base, small	b

Length Measures

Foot	Yard	Mile
12 in.	36 in.	
	3 ft.	5280 ft.
		1760 yds.

Volume Measures

Cubic Foot	Cubic Yard
1728 cu. in.	27 cu. ft.

Weight Measures

Kilogram	Pound	Ton
1000 grams	16 ozs.	2000 lbs.

Liquid Measures

Cubic Foot	Gallon
7.48 gals.	0.1337 cu. ft.
62.4 lbs.	8.3 lbs.

Area Measures

Square Foot	Square Yard	Acre
144 sq. in.	9 sq. ft.	43,560 sq. ft.

Term	Abbreviation	Example
Inches	in. or "	2 in. or 2"
Feet	ft. or '	2 ft. or 2'
Yards	yds.	5 yds.
Miles	mi.	10 mi.
Grams	g	3.48 g
Ounces	oz.	12.4 oz.
Pounds	lbs.	173.5 lbs.
Tons	Tns	21.4 Tns
Gallons	gals.	1248.3 gals.
Hours	hrs.	7.4 hrs.

Decimal Places	Decimal Numbers	Expression
One	0.1	Tenth
Two	0.01	Hundredth
Three	0.001	Thousandth
Four	0.0001	Ten-Thousandth

$$\text{Liquid asphalt application rate (in gallons per square yards)} = \frac{\text{Number of gallons used}}{\text{Area covered in square yards}}$$

$$\text{Pump discharge rate (gallons per hours)} = \frac{\text{Number of gallons used}}{\text{Elapsed time in hours}}$$

$$\text{Average} = \frac{\text{Total of items added}}{\text{Number of items}}$$

Term	**Abbreviation**	**Example**
Square inches	**sq. in.**	**2 sq. in.**
Square feet	**sq. ft. of S.F.**	**7 sq. ft. or 7 S. F.**
Square yards	**sq.yds. or S.Y.**	**5 sq. yds. or 5 S.Y.**
Acres	**Ac.**	**3 Ac.**
Cubic inches	**cu. in.**	**1 cu. in.**
Cubic feet	**cu. ft. or C.F.**	**8 cu. ft. or 8 C.F.**
Cubic yards	**cu. yds. or C.Y.**	**9 cu. yds. or 9 C.Y.**

Multiply	Times	To Find
Cubic Yards	46,656	Cubic Inches
Cubic Yards	202.0	Gallons
Grams	0.03527	Ounces
Ounces	28.35	Grams
Ounces	0.0625	Pounds
Pounds	453.6	Grams
Tons	2000	Pounds
Pounds	0.0005	Tons
Gallons	8.3	Pounds of Water
Gallons	0.1337	Cubic Feet
Gallons	231	Cubic Inches
Pounds of Water	0.1198	Gallons
Miles per Hour	88	Feet Per Minute
Miles Per Hour	1.467	Feet Per Second

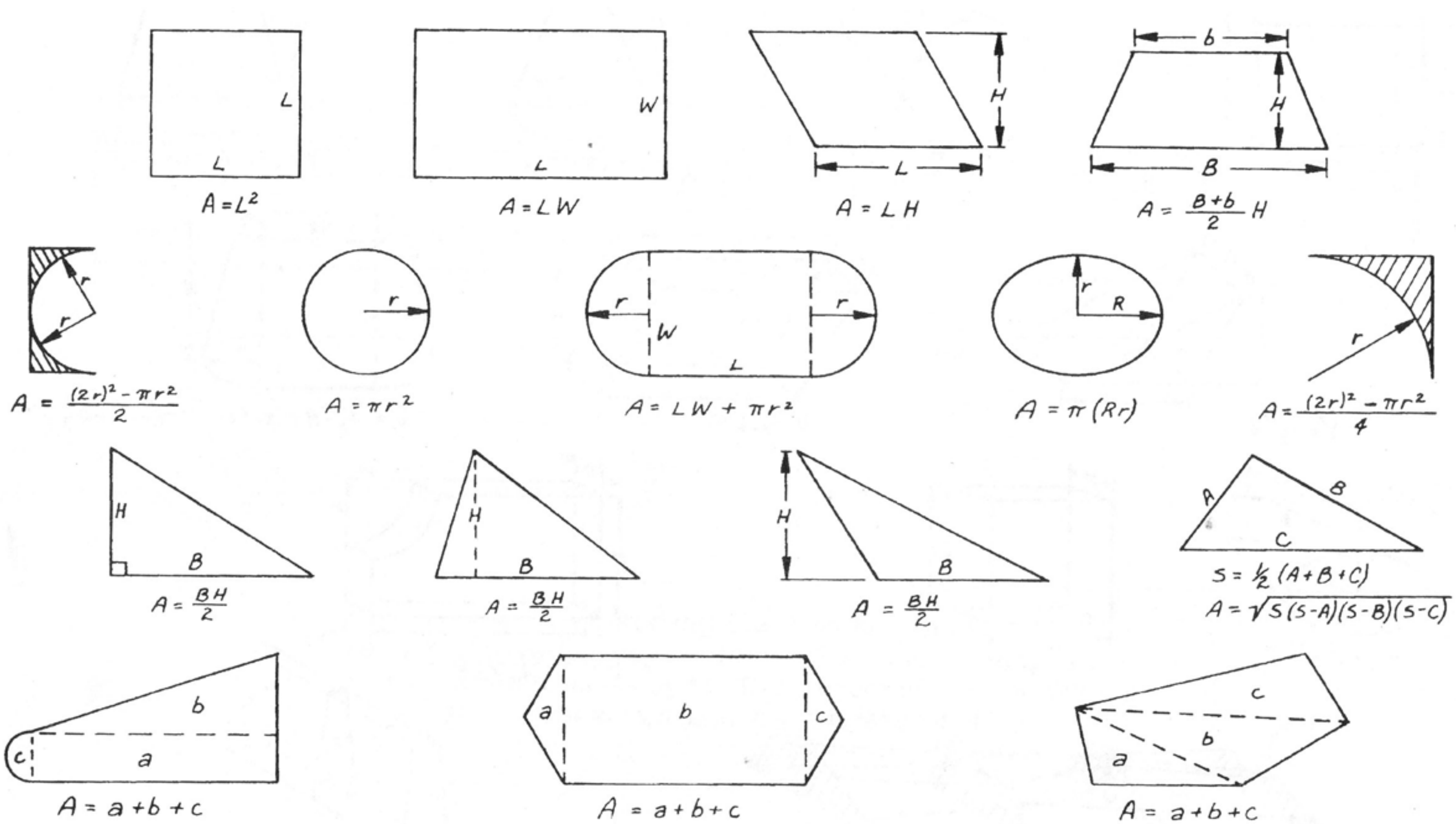
Multiply	Times	To Find
Feet	12	Inches
Feet	0.3333	Yards
Miles	5280	Feet
Miles	1760	Yards
Square Feet	0.1111	Square Yards
Acre	43,560	Square Feet
Acre	4840	Square Yards
Square Miles	640	Acres
Square Yards	9	Square Feet
Cubic Feet	62.4	Pounds of Water
Cubic Feet	1728	Cubic Inches
Cubic Feet	0.03704	Cubic Yards
Cubic Feet	7.48	Gallons
Cubic Inches	.0005787	Cubic Feet
Cubic Inches	.00002143	Cubic Yard
Cubic Yards	27	Cubic Feet

$$\text{Slope Ratio} = \frac{\text{Vertical}}{\text{Horizontal}}$$

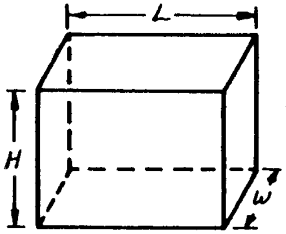
$$\text{Percent grade} = \frac{\text{Vertical rise or fall}}{\text{Horizontal Distance}} \text{ X } 100$$

$$\text{Percent moisture (based on dry weight)} = \frac{\text{Wet weight – dry weight}}{\text{Dry weight}} \text{ X } 100$$

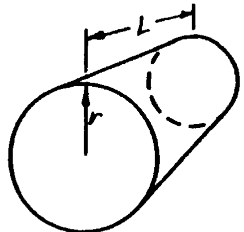
AREA FORMULAS



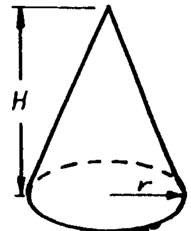
VOLUME FORMULAS



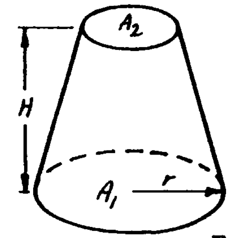
$V = LWH$



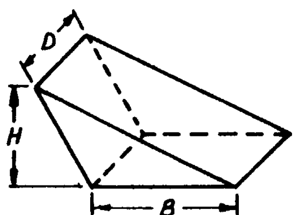
$V = (\pi r^2)L$



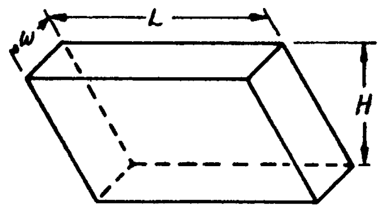
$V = (\pi r^2)\frac{H}{3}$



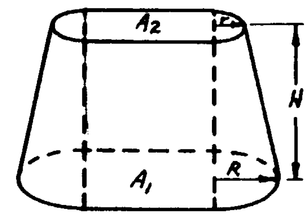
$V = \left[A_1 + A_2 + \sqrt{A_1 \cdot A_2}\right]\frac{H}{3}$



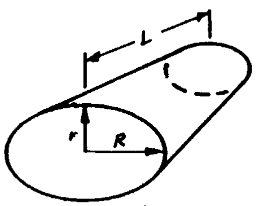
$V = \frac{BH}{2}D$



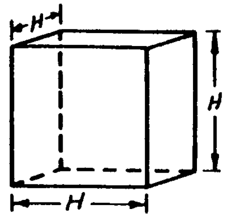
$V = LWH$



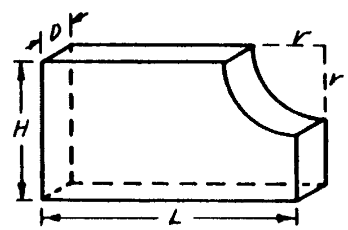
$V = \frac{A_1 + A_2}{2}H + \left[\pi r^2 + \pi R^2 + \sqrt{\pi r^2 \cdot \pi R^2}\right]\frac{H}{3}$



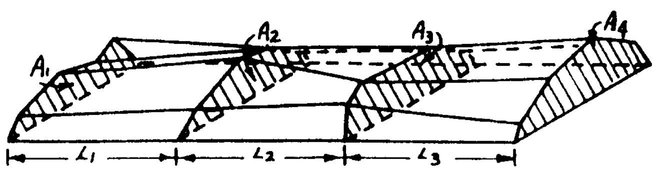
$V = \pi(Rr)L$



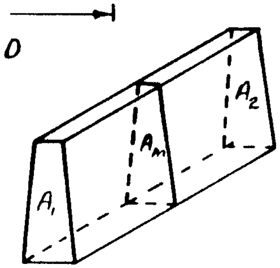
$V = H^3$



$V = \left[LH - \left(\frac{\pi r^2}{4}\right)\right]D$



$V = \left(\frac{A_1 + A_2}{2}\right)L_1 + \left(\frac{A_2 + A_3}{2}\right)L_2 + \left(\frac{A_3 + A_4}{2}\right)L_3$



$V = \left(\frac{A_1 + A_2 + 4A_m}{6}\right)L$